

D2 June 11 Solutions

1. (a)

	A	B	C	D	E	F
A	-	19	11	23	20	37
B	19	-	8	42	17	32
C	11	8	-	34	9	26
D	23	42	34	-	27	31
E	20	17	9	27	-	17
F	37	32	26	31	17	-

Table of least distances

(b)

$$A \rightarrow C \rightarrow B \rightarrow E \rightarrow F \rightarrow D \rightarrow A$$

$$11 + 8 + 17 + 17 + 31 + 23$$

Route: ACBCEFDA

$$\text{Length} = \underline{\underline{107 \text{ km}}}$$

① ② ⑤ ③ ④

	A	B	C	D	E	F
A	11	19	11	23	20	37
B	19	-	8	42	17	32
C	11	8	-	34	9	26
D	23	42	34	-	27	31
E	20	17	9	27	-	17
F	37	32	26	31	17	-

Table of least distances

$$R_{\text{MST}} = BC, CE, EF, ED = 8 + 9 + 27 + 17$$

$$= 61$$

Lower bound = $61 + AC + AB$

$$= 61 + 19 + 11$$

$$= \underline{\underline{91 \text{ km}}}$$

2. (a)

	A	B	C	Dummy	Supply
1	31	29	32	0	20
2	22	33	27	0	22
3	25	27	32	0	20
4	23	26	38	0	38
Demand	35	25	30	10	

Table 1

(b)

	A	B	C	D
1	20			
2	15	7		
3		18	2	
4			28	10

Table 2
31 42 47 9

	A	B	C	D
1	X	-13	-15	-9
2	X	X	-11	0
3	9	X	X	6
4	1	-7	X	X

Table 3

(c)

You may not need to use all these tables

	A	B	C	D
1	- θ		θ	
2	+ θ	- θ		
3		+ θ	- θ	
4				

$$I_{1C} = 32 - 47 - 0 = -15$$

$$I_{2D} = 0 - 9 + 9 = 0$$

$$I_{3A} = 25 + 15 - 31 = 9$$

$$I_{3D} = 0 - 9 + 15 = 6$$

Entering cell: 1C

Exiting cell: 3C

$$\theta = 2$$

	A	B	C	D
1	18		2	
2	17	5		
3		20		
4			28	10

	A	B	C	D
1				
2				
3				
4				

	A	B	C	D
1				
2				
3				
4				

	A	B	C	D
1				
2				
3				
4				

	A	B	C	D
1				
2				
3				
4				

Improved solution

Leave blank

3. (a) $P - 7x + z + 4s = 320$

(b)

b.v.	x	y	z	r	s	t	Value
r	$-\frac{1}{2}$	0	2	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	5
t	$\frac{1}{2}$	0	1	0	$-\frac{1}{4}$	1	4
P	-7	0	1	0	4	0	320

$\theta = 10 / \frac{1}{2} = -20$
 $\theta = 5 / \frac{1}{4} = 10$
 $\theta = 4 / \frac{1}{4} = 8$

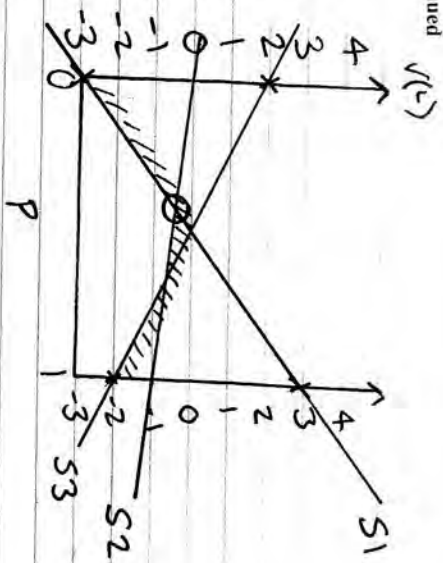


You may not need to use all of these tableaux

b.v.	x	y	z	r	s	t	Value	Row Ops
r	$-\frac{1}{2}$	0	2	1	$-\frac{1}{2}$	0	10	
y	$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	5	
t	1	0	2	0	$-\frac{1}{2}$	2	8	$\times 2$
P	-7	0	1	0	4	0	320	

b.v.	x	y	z	r	s	t	Value	Row Ops
r	0	0	3	1	$-\frac{3}{4}$	1	14	$+\frac{1}{2} \times \text{III}$
y	0	1	$-\frac{1}{4}$	0	$\frac{1}{2}$	-1	1	$-\frac{1}{2} \times \text{III}$
x	1	0	2	0	$-\frac{1}{2}$	2	8	
P	0	0	15	0	$\frac{1}{2}$	14	376	$+7 \times \text{III}$

b.v.	x	y	z	r	s	t	Value	Row Ops
P								



Optimal point where $6p - 3 = -p$

$7p = 3$

$p = \frac{3}{7}$

Optimal strategy for Laura:

- Play 1 never
 - Play 2 prob $\frac{3}{7}$
 - Play 3 prob $\frac{4}{7}$
- Value of game to Laura = $-\frac{3}{7}$.

5 (a) $a = 1$, $b = 5$, $c = 13$, value of initial flow: 49

(b)

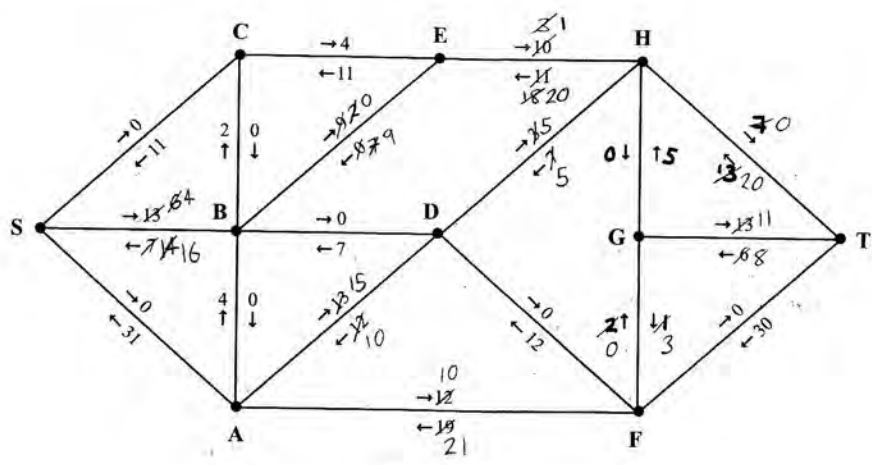


Diagram 1

Question 5 continued

(c) SB EHT - 7
SB EHD AFG T - 2 (OR SBCEHDAFGT - 2)

(d) Value of maximum flow: 58

(e)

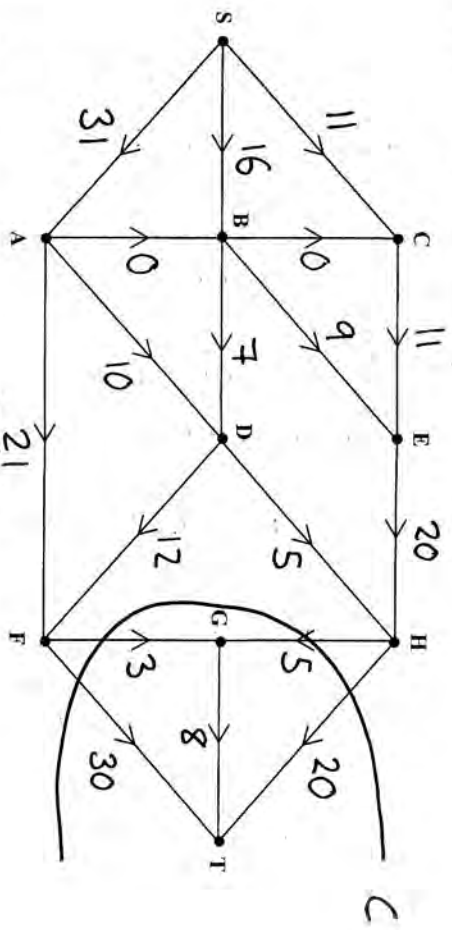


Diagram 2

(f) Cut C (through HT, Hg, qT, FT) = 58
max flow = minimum cut

(Total 15 marks)

	Task A	Task B	Task C
Worker P	27	31	25
Worker Q	26	30	34
Worker R	35	29	32

Table 1

$$\text{Let } x_{ij} = \begin{cases} 1, & \text{if worker } i \text{ does task } j \\ 0, & \text{otherwise} \end{cases}$$

$$i \in \{P, Q, R\} \quad j \in \{A, B, C\}$$

Linear programming problem:

$$\begin{aligned} \text{Minimise Cost} = & 27x_{PA} + 31x_{PB} + 25x_{PC} \\ & + 26x_{QA} + 30x_{QB} + 34x_{QC} \\ & + 35x_{RA} + 29x_{RB} + 32x_{RC} \end{aligned}$$

$$\text{Subject to: } \sum x_{Pj} = 1$$

$$\sum x_{Qj} = 1$$

$$\sum x_{Rj} = 1$$

$$\sum x_{iA} = 1$$

$$\sum x_{iB} = 1$$

$$\sum x_{iC} = 1$$

Handwritten lines for calculations or notes.

	Task A	Task B	Task C
Worker P	33	37	31
Worker Q	32	36	40
Worker R	41	35	38

Table 2

	Task A	Task B	Task C
Worker P	8	4	10
Worker Q	9	5	1
Worker R	0	6	3

Subtract all from 41

(b)

(Total 9 marks)

Q6

7. (a)

Stage	State	Action	Destination	Value
0	H	H - London	London	$36 - 5 = 31^*$
	I	I - London	London	$38 - 4 = 34^*$
1	F	FH	H	$29 - 6 + 31 = 54$
		FI	I	$29 - 7 + 34 = 56^*$
	G	GH	H	$27 - 5 + 31 = 53$
		GI	I	$27 - 6 + 34 = 55^*$
2	C	CF CF	F	$42 - 6 + 56 = 92^*$
		CQ	Q	$42 - 5 + 55 = 92^*$
	D	DF	F	$41 - 6 + 56 = 91$
		DQ	Q	$41 - 3 + 55 = 93^*$
	E	EF	F	$39 - 4 + 56 = 91^*$
		EQ	Q	$39 - 4 + 55 = 90$
3	A	AC	C	$22 - 5 + 92 = 109$
		AD	D	$22 - 4 + 93 = 111^*$
		AE	E	$22 - 2 + 91 = 111^*$
	B	BC	C	$17 - 4 + 92 = 105$
		BD	D	$17 - 4 + 93 = 106^*$
		BE	E	$17 - 3 + 91 = 105$
4	London	London - A	A	$-5 + 111 = 106^*$
	London	London - B	B	$-3 + 106 = 103$

Handwritten lines for calculations or notes.

