D2 June 09

(Question 1 continued)

1)

	1	2	3	4
J	15	11	14	12
M	13	8	17	13
R	14	9	13	15
D	0	0	0	0

dummy needed as rows must equal columns, same number of salespeople to departments.

Reduce Rows

	1	2	3	4	
J	4	0	3	1	−11
M	5	0	9	5	−8
R	5	0	4	6	−9
D	0	0	0	0	×

2 lines ∴ not optimal
Smallest uncovered = 1

	1	2	3	4
J	3	0	2	0
M	4	0	8	4
R	4	0	3	5
D	0	1	0	0

3 lines ∴ not optimal
Smallest uncovered = 3

	1	2	3	4
J	3	3	2	0
M	1	0	5	1
R	1	0	0	2
D	0	4	0	0

4 lines ∴ Optimal

Allocation

Jess does 4 (12)

Matt does 2 (8)

Rachel does 3 (13)

(d) Minimum cost: £33

Leave blank

(Question 2 continued)

a) classical every node visited once in a tour. Practical every node visited at <u>least</u> once.

	A	B	C	D	E	F
~~A~~	~~-~~	~~77~~	~~34~~	~~56~~	~~67~~	~~21~~
~~B~~	~~77~~	~~-~~	~~58~~	~~58~~	~~36~~	~~74~~
C	34	58	-	73	70	42
D	56	58	73	-	68	38
E	67	36	70	68	-	71
F	21	74	42	38	71	-

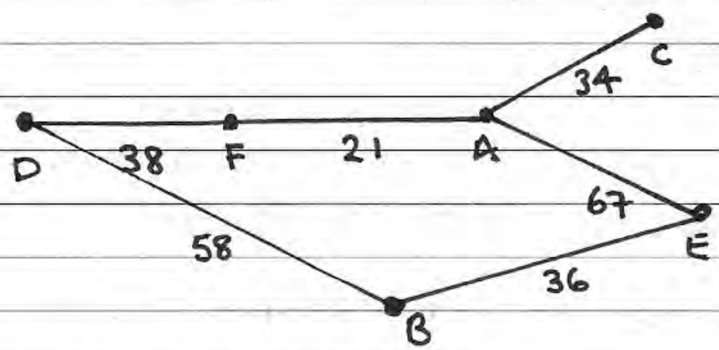
b) A F D B E C A <u>257</u>

21 38 58 36 70 34

c) 257, better upper bound as it is smaller ∴ closer to the optimal solution

d) Prim's AF (21) RMST = 160
AC (34) + BE (36)
FD (38) + BD (58)
AE (67) 254 = lower bound

e) 254, better lower bound as it is bigger ∴ closer to the optimal solution



lower bound not possible

∴ 254 < optimal route ≤ 257

Q2

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3.

	B plays 1	B plays 2	B plays 3	Row min
A plays 1	–5	6	–3	-5
A plays 2	1	–4	13	-4
A plays 3	–2	3	–1	-2
Column max	1	6	13	

a) Column minimax 1 $\neq$ Row maximin
$\therefore$ no stable solution

b) Column 3 dominates column 1 $\therefore$ column 3 can be deleted from the table as player B should never play 3 using a play-safe strategy.

c)

	B1	B2
A1	-5	6
A2	1	-4
A3	-2	3

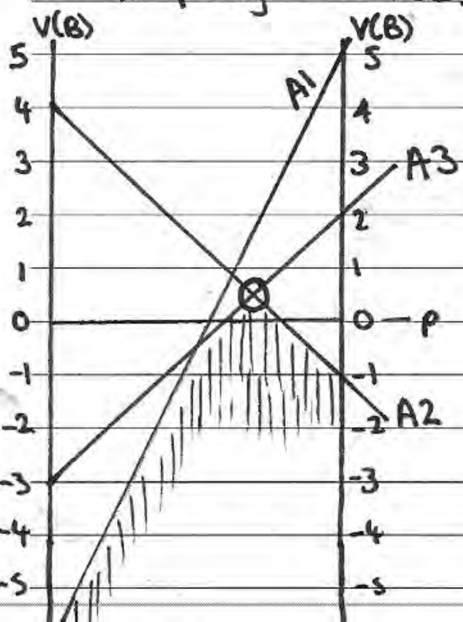
Invert matrix

	A1	A2	A3
B1	5	-1	2
B2	-6	4	-3

let B play 1 prob = p
let B play 2 prob = 1-p

If A plays 1 $V(B) = 5p - 6(1-p) = -6 + 11p$
A plays 2 $V(B) = -p + 4(1-p) = 4 - 5p$
A plays 3 $V(B) = 2p - 3(1-p) = -3 + 5p$

0	1
-6	5
4	-1
-3	2



A2/A3 ; A should never play 1 if B plays an optimal play-safe mixed strategy

$4 - 5p = -3 + 5p \Rightarrow 10p = 7 \quad p = \frac{7}{10}$

$\therefore$ B should play 1 prob = $\frac{7}{10}$
B should play 2 Prob = $\frac{3}{10}$
B should play 3 Never

$V(B)$
$4 - 5(\frac{7}{10})$
$= 0.5$

4 a) Value of cut C_1: $10+2+10+3+9 = 34$

Value of cut C_2: $7+5+8+25 = 45$

(b)

SBFGT (2)

Cut through SA, SB, SC now only passes through saturated arcs $\therefore$ mincut $= 7+10+11$

$\therefore$ min cut $= 28$

$\therefore$ by min cut-max flow theorem max flow $= 28$

$\therefore$ flow of 28 is maximal.

5) $x = 0$ $r = 0$ $P = 10$

$y = 0$ $s = 4$

$z = 2$ $t = 2$

b) $P - 2x - 4y + \frac{5}{4}r = 10$

6. (a) balanced ⇒ supply = demand

(b)

	A	B	C	
X	16	6		22
Y		9	8	17
Z			15	15
	16	15	23	

You may not need to use all of these tables

	A	B	C
X	16 $-\theta$	6 $+\theta$	
Y		9 $-\theta$	8 $+\theta$
Z	$+\theta$		15 $-\theta$

$\theta = 9$
YB = exiting cell

	A	B	C
X	7	15	
Y			17
Z	9		6

		17	8	20
		A	B	C
0	X	17	8	7
-5	Y	16	12	15
-11	Z	6	10	9

Shadow Costs

	A	B	C
X	X	X	-13
Y	4	9	X
Z	X	13	X

Improvement Indices

	A	B	C
X	7 $-\theta$	15	$+\theta$
Y			17
Z	9 $+\theta$		6 $-\theta$

entering cell = XC

$\theta = 6$

exiting cell = ZC

	A	B	C
X	1	15	6
Y			17
Z	15		

£ 524

7. (a)

Stage	State (in £1000s)	Action (in £1000s)	Destination (in £1000s)	Value (in £1000s)
1	250	250	0	300 *
	200	200	0	240 *
	150	150	0	180 *
	100	100	0	120 *
	50	50	0	60 *
	0	0	0	0 *
2	250	250	0	280 + 0 = 280
		200	50	235 + 60 = 295
		150	100	190 + 120 = 310 *
		100	150	125 + 180 = 305
		50	200	65 + 240 = 305
		0	250	0 + 300 = 300
	200	200	0	235 + 0 = 235
		150	50	190 + 60 = 250 *
		100	100	125 + 120 = 245
		50	150	65 + 180 = 245
		0	200	0 + 240 = 240
	150	150	0	190 + 0 = 190 *
		100	50	125 + 60 = 185
		50	100	65 + 120 = 185
		0	150	0 + 180 = 180
	100	100	0	125 + 0 = 125 *
		50	100	65 + 60 = 125 *
		0	150	0 + 120 = 120
	50	50	0	65 + 0 = 65 *
		0	50	0 + 60 = 60
	0	0	0	0 + 0 = 0 *

Stage	State (in £1000s)	Action (in £1000s)	Destination (in £1000s)	Value (in £1000s)
3	250	250	0	300 + 0 = 300
		200	50	230 + 65 = 295
		150	100	170 + 125 = 295
		100	150	110 + 190 = 300
		50	200	55 + 250 = 305
		0	250	0 + 310 = 310 *

Maximum income: 310 000

Scheme	1	2	3
Amount to be invested (in £1000s)	100	150	0

(b) i) Stage → scheme to be Invested in

ii) Stage → how much is available to Invest in scheme

iii) Action → how much is Invested in the scheme.

8. let v = value of original game to laura

+6 to every element so that $\bar{V} = v+6$ is the value of new game to Laura

	S1	S2	S3
L1	4	14	5
L2	13	10	3
L3	7	1	10

let Laura play 1 prob $= p_1$
Laura play 2 prob $= p_2$
Laura play 3 prob $= p_3$

Objective, maximise $P = V$ so that $P - V = 0$

Subject to

$$\bar{V} - 4p_1 - 13p_2 - 7p_3 \leq 0$$
$$\bar{V} - 14p_1 - 10p_2 - p_3 \leq 0$$
$$\bar{V} - 5p_1 - 3p_2 - 10p_3 \leq 0$$
$$p_1 + p_2 + p_3 \leq 1$$
$$p_1, p_2, p_3 \geq 0$$