Solutions

1.	(a)	$x = 9$, $y = 11$	B1, B1	2
		1B1: cao (permit B1 if 2 correct answers, but transposed)		
		2B1: cao		
	(b)	AC DC DT ET	B2,1,0	2
		1B1: correct (condone one error – omission or extra)		
		2B1: all correct (no omissions or extras)		
	(c)	36	B1	1
		1B1: cao		
	(d)	$C_1 = 49$, $C_2 = 48$, $C_3 = 39$	B1, B1, B1	3
		1B1: cao		
		2B1: cao		
		3B1: cao		
	(e)	e.g. SAECT	B1	1
		1B1: A correct route (flow value of 1 given)		
	(f)	maximum flow = minimum cut		
		cut through DT, DC, AC and AE	M1A1	2
		1M1: Must have attempted (e) and made an attempt at a cut.		
		1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of fours.		

[11]

2.	(a)	A walk is a **finite sequence of arcs** such that the **end vertex of one arc is the start vertex of the next**.	B2,1,0	2
		1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous.		
		2B1: A good clear complete answer: End vertex = start vertex + finite.		
	(b)	A tour is a walk that visits **every vertex, returning to its stating vertex**.	B2,1,0	2
		1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous.		
		2B1: A good clear complete answer: Every vertex + return to start.		

<u>From the D1 and D2 glossaries</u>

<u>D1</u>

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, <u>and in which no vertex appears more than once</u>.

A **cycle (circuit)** is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

<u>D2</u>

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

[4]

3.	(a)	Total supply > total demand	B2,1,0	2

(b) Adds 0, 0 and 5 to the dummy column — B2,1,0 — 2

(c) — B1 — 1

	L	E	D
A	35	20	
B		40	5

(d) — M1A1

		80	70	20
		L	E	D
0	A	35	20	
–20	B		40	5

$I_{AD} = 0 - 0 - 20 = -20$

$I_{BL} = 60 + 20 - 80 = 0$ — A1 — 3

M1

	L	E	D
A	35	$20 - \theta$	θ
B		$40 + \theta$	$5 - \theta$

$\theta = 5$; entering square is AD; exiting square is BD — A1ft — 2

B1ft

		80	70	0
		L	E	D
0	A	35	15	5
–20	B		45	

$I_{BL} = 60 + 20 - 80 = 0$

$I_{BD} = 0 + 20 - 0 = 20$ — B1ft — 2

(e) Cost is (£) 6100 — B1 — 1

[13]

4. (a) Maximin : we seek a route where the shortest arc used is a great as possible.
Minimax : we seek a route where the longest arc used is a small as possible. — B2,1,0 — 2

(b)

Stage	State	Action	Dest.	Value	
	G	GR	R	132*	
1	H	HR	R	175*	M1A1
	I	IR	R	139*	
	D	DG	G	min (175,132) = 132	M1A1
		DH	H	min (160,175) = 160*	
2	E	EG	G	min (162,132) = 132	
		EH	H	min (144,175) = 144*	A1
		EI	I	min (102,139) = 102	
	F	FH	H	min (145,175) = 145*	
		FI	I	min (210,139) = 139	
	A	AD	D	min (185,160) = 160*	
		AE	E	min (279,144) = 144	M1A1ft
3	B	BD	D	min (119,160) = 119	
		BE	E	min (250,144) = 144*	A1ft
		BF	F	min (123,145) = 123	
	C	CE	E	min (240,144) = 144	
		CF	F	min (170,145) = 145*	
	L	LA	A	min (155,160) = 155*	A1ft
4		LB	B	min (190,144) = 144	
		LC	C	min (148,145) = 145	

Maximin route: LADHR A1ft 5

[12]

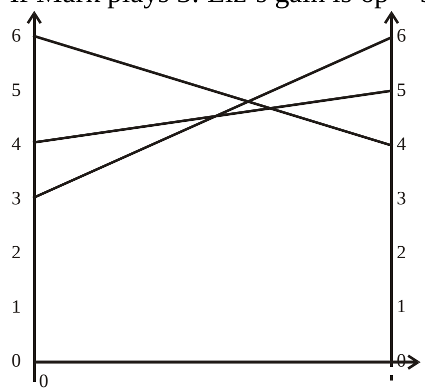
5. (a) For each row the element in column x must be less than the element in column y. B2,1,01 2

(b) Row minimum {2,4,3} row maximin = 4 M1
Column maximum {6,5,6} column minimax = 5 A1
$4 \neq 5$ so not stable A1 3

(c) Row 3 dominates row 1, so matrix reduces to B1

	M1	M2	M3
L2	4	5	6
L3	6	4	3

Let Liz play 2 with probability p and 3 with probability (1– p)
If Mark plays 1: Liz's gain is 4p + 6 (1– p) = 6 – 2p
If Mark plays 2: Liz's gain is 5p + 4 (1– p) = 4 + p M1
If Mark plays 3: Liz's gain is 6p + 3 (1– p) = 3 + 3p A1 3



B2,1,0 2

$4 + p = 6 - 2p$ M1A1

$p = \frac{2}{3}$ A1ftA1 4

(d) Liz should play row 1 – never, row 2 – $\frac{2}{3}$ of the time,

row 3 – $\frac{1}{3}$ of the time

and the value of the game is $4\frac{2}{3}$ to her. B1

Row 3 no longer dominates row 1 and so row 1 can not be deleted.
Use Simplex (linear programming). B1 2

[16]

6. (a) Since maximising, subtract all elements from some n ≥ 53

$$\begin{bmatrix} 5 & 4 & 11 & 11 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 6 & 3 & 7 & 10 \end{bmatrix}$$

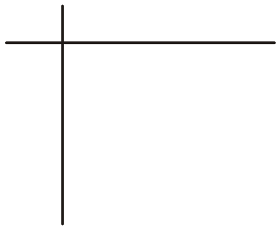
M1A1 2

Reduce rows $\begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 3 & 0 & 4 & 7 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 4 & 0 & 0 \\ 2 & 0 & 3 & 2 \\ 3 & 0 & 2 & 4 \end{bmatrix}$

M1A1ft 2

Minimum element 1

M1

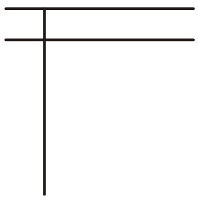


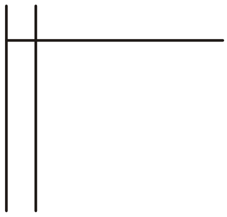
A1ft

$$\begin{bmatrix} 0 & 0 & 4 & 3 \\ 0 & 5 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$$

A1ft 3

M1





A1ftA1ft 3

(b)

$$\begin{bmatrix} 0 & 1 & 4 & 3 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 3 & 2 \\ 1 & 6 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

M1A1ft 2

M1A1 2

Joe	A	A
Min-Seong	C	D
Olivia	D	B
Robert	B	C

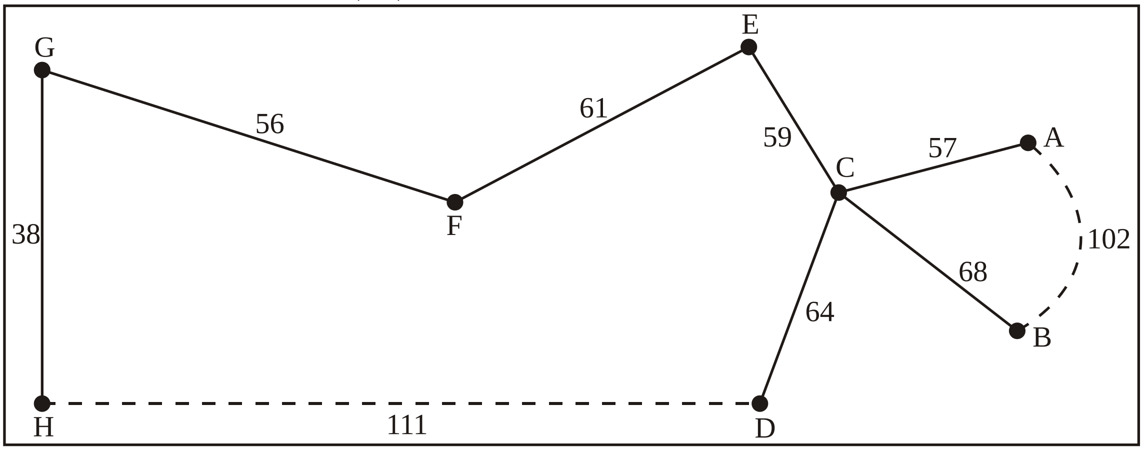
Value £197 000

[14]

7. (a) GH(38) GF(56) CA(57) EC(59) FE(61) CD(64) CB(68) — M1A1ft — 2

(b) $2 \times 403 = 806$ (km) — B1 — 1

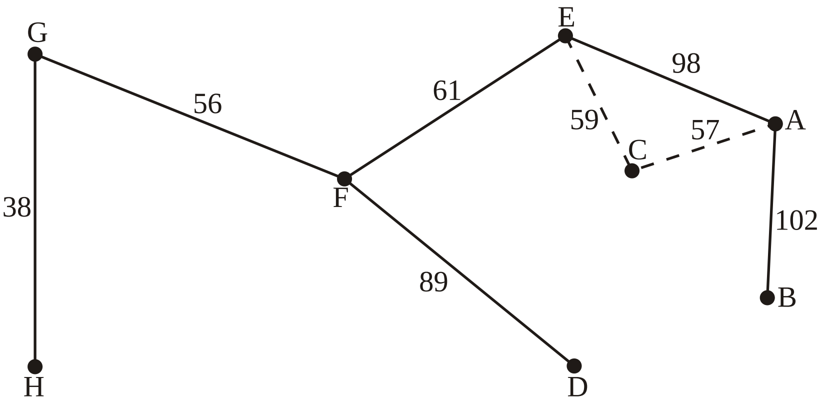
(c) e.g. DH saves 167 — M1A1
AB saves 23
$806 - 190 = 616$ (km) — A1



A1 — 4

(d) eg A B C E F G H D C A — M1A1
B C A E F G H D B
$68 + 57 + 98 + 61 + 56 + 38 + 111 + 108 = 597$ (km) — A1 — 3

(e) Delete C



M1A1M1A1ft — 4

(f) RMST weight = 444
Lower bound = $444 + 59 + 57 = 560$ (km)
$560 < \text{length} \leq 597$ — B2,1,0 — 2

[16]

8. (a)

b.v.	x	y	z	R	s	t	Value
r	4	$\frac{7}{3}$	$\frac{5}{2}$	1	0	0	64
s	1	3	0	0	1	0	16
t	4	2	2	0	0	1	60
P	–5	$-\frac{7}{2}$	–4	0	0	0	0

b.v.	x	y	z	R	s	t	Value	Row ops	
r	0	$\frac{1}{3}$	$\frac{1}{2}$	1	0	–1	4	$R_1 - 4R_3$	M1A1
s	0	$\frac{5}{2}$	$-\frac{1}{2}$	0	1	$-\frac{1}{4}$	1	$R_2 - R_3$	M1A1ftA1
x	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{4}$	15	$R_3 \div 4$	
P	0	–1	$-\frac{3}{2}$	0	0	$\frac{5}{4}$	75	$R_4 + 5R_3$	

b.v.	x	y	z	R	s	t	Value	Row ops		
z	0	$\frac{2}{3}$	1	2	0	–2	8	$R_1 \div \frac{1}{2}$	M1A1ft	
s	0	$\frac{17}{6}$	0	1	1	$-\frac{5}{4}$	5	$R_2 + \frac{1}{2}R_1$	M1A1	9
x	1	$\frac{1}{6}$	0	–1	0	$\frac{5}{4}$	11	$R_3 - \frac{1}{2}R_1$		
P	0	0	0	3	0	$-\frac{7}{4}$	87	$R_4 + \frac{3}{2}R_1$		

(b) There is still negative numbers in the profit row. B1 1

[10]