Solutions

1. (a) A game in which the gain to one player is equal to the loss of the other B2, 1, 0 2

(b) If there is a <u>stable solution</u>(s) a_{ij} in a game, the <u>location</u> of this stable solution is called the saddle point. B2, 1, 0 2

It is the point(s) where row maximum = column maximum.

[4]

2. Subtract all terms from some $n \geq 35$, e.g.35

4	11	3	0
19	25	16	13
16	21	15	14
17	20	14	12

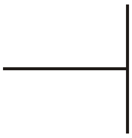
M1

A1 2

Reducing rows then columns

2	4	2	0
4	5	2	0
0	0	0	0
3	1	1	0

B1

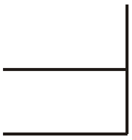


Minimum uncovered 1

1	3	1	0
3	4	1	0
0	0	0	1
2	0	0	0

M1

A1 ft 3



Minimum uncovered 1

0	2	0	0
2	3	0	0
0	0	0	2
2	0	0	1

M1

A1 ft

e.g. matching	*D* - *A*		*A*		*M*		*S*	A1 ft	
	H – *S*	*or*	*S*	*or*	*S*	*or*	*M*		
	K - *M*		*L*		*A*		*A*		
	T - *L*		*M*		*L*		*L*	A1	4

Total 88 points

[9]

3. (a) (i) Minimum connector using Prim: *AC*, *CB*, *CD*, *CE* M1 A1

Length = 98 + 74 + 82 + 103 = 357 {1, 3, 2, 4, 5}

So upper bound = 2 × 357 = 714 M1 A1 4

(ii) *A* $\underline{(98)}$ *C* $\underline{(74)}$ *B* $\underline{(131)}$ *D* (134) *E* $\underline{(115)}$ *A* M1 A1

Length = 98 + 74 + 131 + 134 + 115 = 552 A1 3

(b) Residual minimum connector is *AC*, *CB*, *CD* M1

Length 254 A1

Lower bound = 254 + 103 + 115 = 472 M1 A1 4

(c) 472 ≤ solution ≤ 552 B1 ft 1

[12]

4. (a)

$$\begin{array}{cc} & \text{row min} \\ \begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix} & \begin{array}{c} -4 \\ -2 \quad \leftarrow \text{max} \end{array} \end{array}$$

$$\begin{array}{cccc} \text{Col. max} & 2 & 1 & 3 \\ & & \uparrow & \\ & & \text{min} & \end{array}$$

M1 A1

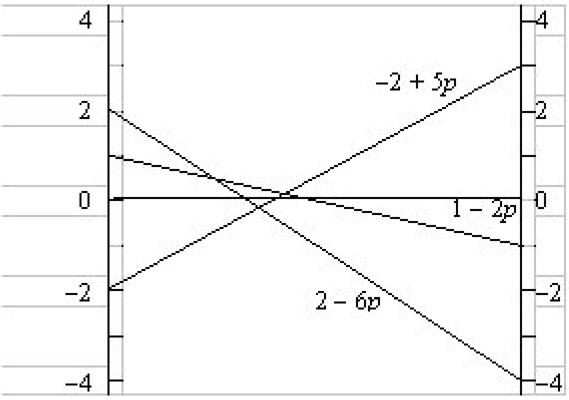
$-2 \neq 1$ ∴ not stable A1 3

(b) Let Emma play R_1 with probability p

If Freddie plays C_1, Emma's winnings are $-4p + 2(1 - p) = 2 - 6p$

C_2, Emmas winnings are $-p + 1(1 - p) = 1 - 2p$ M1 A1

C_3, Emma's winnings are $3p - 2(1 - p) = -2 + 5p$ A1 3



M1 A1 ft 2

Need intersection of $2 - 6p$ and $-2 + 5p$ M1

$2 - 6p = -2 + 5p,$

$4 = 11p,$

$p = \dfrac{4}{11}$ A1

So Emma should play R_1 with probability $\frac{4}{11}$

R_2 with probability $\frac{7}{11}$ A1 ft 3

The value of the game is $-\frac{2}{11}$ to Emma

(c) Value to Freddie $\frac{2}{11}$, matrix $\begin{pmatrix} 4 & -2 \\ 1 & -1 \\ -3 & 2 \end{pmatrix}$ B1 ft B1, B1 3

[14]

5. (a) Idea of many supply and demand points and many units to be moved. Costs are variable and dependent upon the supply and demand points, need to minimise costs. Practical costs proportional to number of units B2, 1, 0 2

(b) Supply = 120 Demand = 110 so not balanced B1 1

(c) Adds 0, 0, 0, 10 to column *f* M1 A1

M1 A1

	d	*e*	*f*
A	45		
B	5	30	
C		30	10

Cost 545 B1 ft 5

(d) $R_1 = 0$ $R_2 = -1$ $R_3 = -3$

$k_1 = 5$ $k_2 = 7$ $k_3 = 3$ M1 A1

$Ae = 3 - 0 - 7 = -4$

$Af = 0 - 0 - 3 = -3$ M1 A1 ft

$Bf = 0 + 1 - 3 = -2$ A1 ft 5

$Cd = 2 + 3 - 5 = 0$

(e) $Ae^+ \to Be^- \to Bd^+ \to Ad^-$ send 30 M1 A1 ft

	d	*e*	*f*
A	15	30	
B	35		
C		30	10

depM1

A1 ft

Cost 425 A1 5

[18]

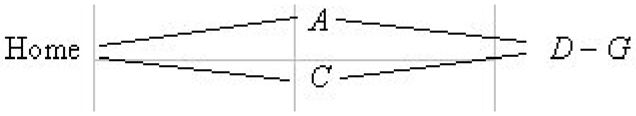
6. (a) Stage – Number of weeks to finish B1

State – Show being attended B1

Action – Next journey to undertake B1 3

(b) eg

Stage	State	Action	Value		
1	*F* *G* *H*	*F* – Home *G* – Home *H* – Home	500 – 80 = 420 * 700 – 90 = 610 * 600 – 70 = 530 *	M1 A1	
2	*D*	*DF* *DG* *DH*	1500 – 200 + 420 = 1720 1500 – 160 + 610 = 1950 * 1500 – 120 + 530 = 1910	M1 A1ft A1 ft	
	E	*EF* *EG* *EH*	1300 – 170 + 420 = 1550 1300 – 100 + 610 = 1810 * 1300 – 110 + 530 = 1720	A1	
3	A	*AD* *AE*	900 – 180 + 1950 = 2670 * 900 – 150 + 1810 = 2560	M1 A1 ft	
	B	*BD* *BE*	800 – 140 + 1950 = 2610 * 800 – 120 + 1810 = 2490	A1 ft	
	C	*CD* *CE*	1000 – 200 + 1950 = 2750 * 1000 – 210 + 1810 = 2600	A1	
4	Home	Home – *A* Home – *B* Home – *C*	–70 + 2670 = 2600 * –80 + 2610 = 2530 –150 + 2750 = 2600 *	M1 A1	12

(c) 

B2 ft 1 ft 0

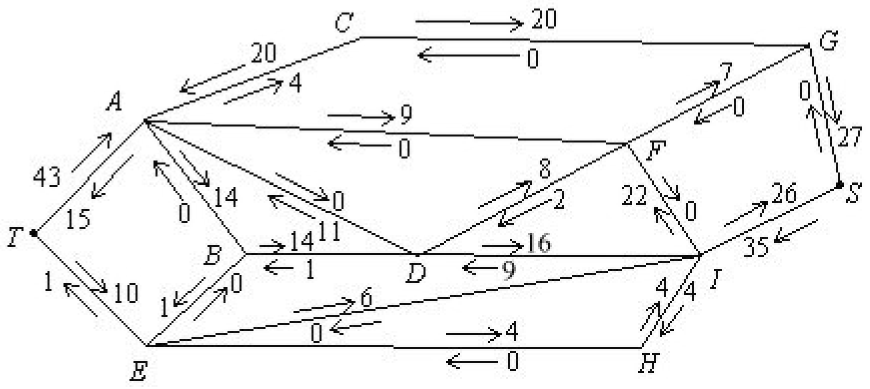
Total profit £2 600 B1 ft 3

[18]

7. (a) *x* = 9, *y* = 16 B1 B1 2

(b) Initial flow = 53 – Either finds a flow-augmenting route or demonstrates not enough saturated arcs for a minimum cut B1 B1 2

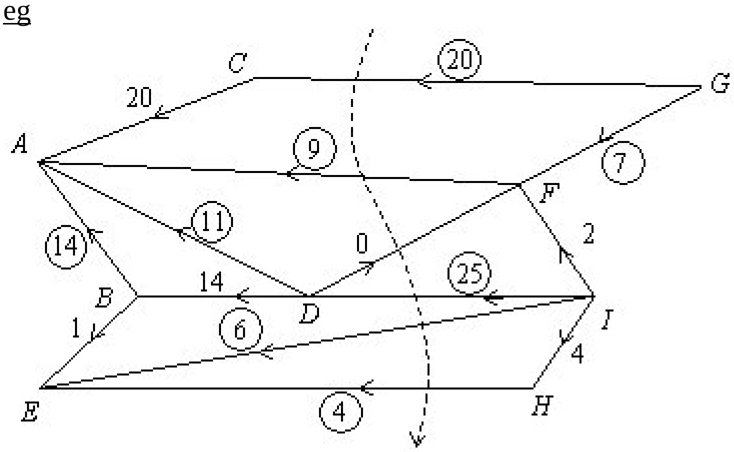
(c)



M1A1 2

e.g. *IDA* – 9 A1

		IFDA – 2	A1	
		max flow – 64	B1	3
	(d)	eg	M1 A1	2



	(e)	Max flow – min cut	M1	
		Finds a cut *GC*, *AF*, *DF*, *DJ*, *EI*, *EH* value 64	A1	2
		Note: must not use supersource or supersink arcs.		

[13]

8.	(a)	Yes, there are no negative values in the profit row	B1	1
	(b)	$p = 63, x = 0, y = 7, z = 0, r = \frac{9}{2,}, s = \frac{2}{3}, t = 0$	M1, A1, A1,	3
	(c)	$\frac{63}{7} = 9$	M1, A1	2

[6]

9.	(a)	$C_1 = 7 + 14 + 0 + 14 = 35$	B1	
		$C_2 = 7 + 14 + 5 = 26$	B1	
		$C_3 = 8 + 9 + 6 + 8 = 31$	B1	3
	(b)	Either Min cut = Max flow and we have a flow of 26 and a cut of 26 or C2 is through saturated arcs	B1	1
	(c)	Using EJ (capacity 5) e. g – will increase flow by 1– ie increase it to 27 since only one more unit can leave E.	M1 A1	
		- BEJL – 1		
		Using FH (capacity 3) e. g.– will increase flow by 2 – ie increase it to 28 since only two more units can leave F.		
		- BFHJL - 2		
		Thus choose option 2 add FH capacity 3.	A1	3

[7]

10. (a) Maximise $P = 50x + 80y + 60z$ B1

subject to $x + y + 2z \le 30$

$x + 2y + z \le 40$

$3x + 2y + z \le 50$ B3, 2, 1,0 4

where $x, y, z \ge 0$

(b) Initialising tableau B1ft M1

bv	x	y	z	r	s	t	value
r	1	1	2	1	0	0	30
s	1	②	1	0	1	0	40
t	3	2	1	0	0	1	50
p	–50	–80	–60	0	0	0	0

chooses correct pivot, divides R_2 by 2 A1 ft

states correct row operation $R_1 – R_2$, $R_3 – 2R_2$, $R_4 + 80R_2$, $R_2 \div 2$ A1 4

(c) The solution found after one iteration has a stack of 10 units of black per day B2, 1, 0 2

(d) (i)

bv	x	y	z	r	s	t	value
r	$\frac{1}{2}$	0	$\textcircled{\frac{3}{2}}$	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	–1	1	10
p	–10	0	–20	0	40	0	1600

(given)

bv	x	y	z	r	s	t	value
z	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	$6\frac{2}{3}$
y	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	$16\frac{2}{3}$
t	2	0	0	0	–1	1	10
p	$-3\frac{1}{3}$	0	0	$13\frac{1}{3}$	$33\frac{1}{3}$	0	$1733\frac{1}{3}$

$R_1 \div \frac{3}{2}$ M1 A1

$R_2 - \frac{1}{2} R_1$

R_3 – no change M1 A1 4

$R_4 + 20R$

(ii) not optimal, a negative value in profit row B1ft

(iii) $x = 0$ $y = 16\frac{2}{3}$ $z = 6\frac{2}{3}$ M1 A1ft

$p = £1733.33$ $r = 0, s = 0, t = 10$ A1ft 4

[18]