Solutions

1. (a)

	A(I)	A(II)
B(I)	3	–4
B(II)	–2	1
B(III)	–5	4

B2, 1, 0 2

(b) e.g. matrix becomes

	A(I)	A(II)
B(I)	9	2
B(II)	4	7
B(III)	1	10

M1

Defines variables (–including non-zero constants) B1

e.g. maximise $P = V$

subject to
$v - 9q_1 - 4q_2 - q_3 + r = 0$
$v - 2q_1 - 7q_2 - 10q_3 + s = 0$
$q_1 + q_2 + q_3 + t = 1$

OR

e.g. minimise $P = x_1 + x_2 + x_3$ where $x_i = \frac{q_i}{v}$

subject to
$9x_1 + 4x_2 - x_3 + r = 1$
$2x_1 - 7x_2 - 10x_3 + s = 1$ A2 ft, 1 ft, 0 4

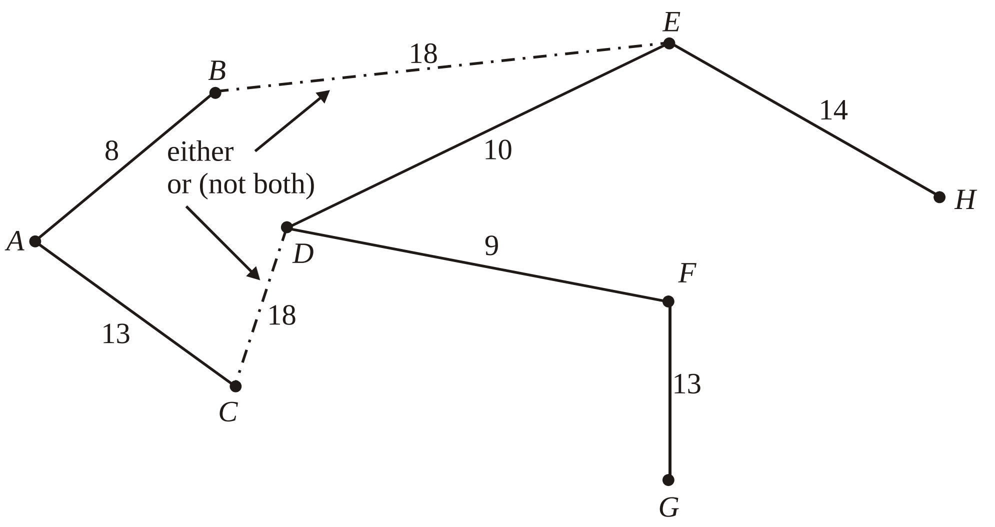
OR

e.g. maximise $P = V$
$v - 8q_1 - 3q_2 + R = 0$
$v - 8q_1 - 3q_2 + S = 0$

[6]

2. (a) In the *practical* TSP each vertex must be visited *at least once* B1
In the *classical* TSP each vertex must be visited *exactly once* B1 2

(b) AB, DF, DE, (reject EF), $\left\{\begin{matrix} FG \\ AC \end{matrix}\right\} EH \left\{\begin{matrix} DC \\ \text{or} \\ BE \end{matrix}\right\}$ M1 A1



B1 3

(c) Initial upper bound = 2 × 85 = 170 km M1 A1 2

(d) e.g. when *CD* is part of the tree
use *GH* (saving 26) and *BD* (saving 19) giving new u. b. of 125 km M1
Tour *A B D E H G F D C A* A1 3

(or e.g. when *BE* is part of the tree
use *CG* (saving 40) giving new upper bound of 130 km;
Tour *A B E H E D F G C A*)

[10]

3. (a) (i) Either *rows then columns* giving

	I	II	III	IV
C	0	22	16	4
J	1	20	24	0
N	1	18	18	0
S	1	23	26	0

then

	I	II	III	IV
C	0	4	0	4
J	1	2	8	0
N	1	0	2	0
S	1	5	10	0

M1, A1, A1 3

3 lines only needed (or) least element 1 so

	I	II	III	IV
C	0	4	0	5
J	0	1	7	0
N	1	0	2	1
S	0	4	9	0

or

	I	II	III	IV
C	0	5	0	5
J	0	2	7	0
N	0	0	1	0
S	0	5	9	0

M1, A1, A1 3

Alternative

(a) (i) or *columns then rows* giving

	I	II	III	IV
C	1	2	0	6
J	2	0	8	2
N	4	0	4	4
S	0	1	8	0

(then no change) M1, A1

3 lines only needed and *either* row 1 *or* column 3

if row 1: least uncovered 2

	I	II	III	IV
C	1	4	0	6
J	0	0	6	0
N	2	0	2	2
S	0	3	8	0

if column 3: least uncovered 1

	I	II	III	IV
C	0	2	0	5
J	1	0	8	1
N	3	0	4	3
S	0	2	9	0

Then least uncovered 1 — M1 A1 M1 A1 — 6

	I	II	III	IV
C	0	3	0	5
J	0	0	7	0
N	2	0	3	2
S	0	3	9	0

(ii) *C* – III, *J* – I or IV, *N* – II, *S* – IV or I — M1 A1

83 minutes ∴ 11.23 a.m. — M1 A1 — 4

(b) Subtracting all entries from some $n \geq 36$ (stated) — M1

e.g. subtractions from 36

	I	II	III	IV
C	24	2	8	20
J	23	4	0	24
N	21	4	4	22
S	25	3	0	26

M1, A2,1,0 — 3

[13]

4. (a) Player *A*: row minimums are –1, 0, –3 so maximin choice is play II — M1 A1

Player *B*: column maximums are 2, 3, 3 so minimax choice is play I — M1 A1 — 4

(b) Since *A*'s maximin (0) ≠ *B*'s minimax (2) there is no stable solution — B1 — 1

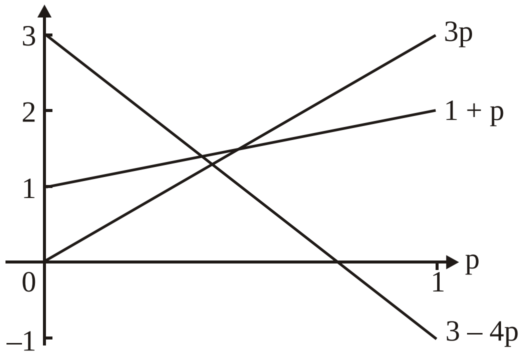
(c) For player *A* row II dominates row III, so *A* will *now* play III — B2, 1, 0 — 2

(d) Let *A* play I with probability p and II with probability $(1 - p)$

If *B* plays I, *A*'s expected winnings are $2p + (1 - p) = 1 + p$

If *B* plays II, *A*'s expected winnings are $-p + 3(1 - p) = 3 - 4p$ — M1, A2, 1, 0 — 3

If *B* plays III, *A*'s expected winnings are $3p$



M1

$3 - 4p = 3p \Rightarrow \underline{p = \tfrac{3}{7}}$ A1

A should play I with probability $\tfrac{3}{7}$

A should play II with probability $\tfrac{4}{7}$ A1

and never play III

The value of the game is $\underline{\tfrac{9}{7}}$ <u>to *A*</u> A1 ft 4

[14]

5. (a) e.g.

	D	E	F
A	6		
B	0	5	
C		4	4

<u>or</u>

	D	E	F
A	6	0	
B		5	
C		4	4

M1 A1

<u>cost £470</u> A1 3

(b)

$S_A = 0, S_B = 0, S_C = -10$
$D_D = 20, D_E = 30, D_F = 40$
$I_{AE} = 40 - 30 = 10$
$I_{AF} = 10 - 40 = -30$
$I_{BF} = 40 - 40 = 0$
$I_{CD} = 10 - 10 = 0$

$S_A = 0, S_B = -10, S_C = -20$
$D_D = 20, D_E = 40, D_F = 50$ M1 A1
$I_{AF} = 10 - 50 = -40$
$I_{BD} = 20 - 10 = 10$
$I_{BF} = 40 - 40 = 0$
$I_{CD} = 10 - 0 = 10$ M1 A1 4

Choose *AF* as entering route

$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow BE(-) \rightarrow BD(+) \rightarrow AD(-)$
Exiting route *CF* $\theta = 4$

$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow AE(-)$
Exiting route *AE* $\theta = 0$ M1 A1 ft

	D	E	F
A	2		4
B	4	1	
C		8	

	D	E	F
A	6		0
B		5	
C		4	4

A1 3

$S_A = 0, S_B = 0, S_C = -10$
$D_D = 20, D_E = 30, D_F = 10$
$I_{AE} = 10, I_{BF} = 30,$
$I_{CD} = 0, I_{CF} = 30$
$\therefore$ optimal, cost £350

$S_A = 0, S_B = 30, S_C = 20$
$D_D = 20, D_E = 0, D_F = 10$
$I_{AE} = 40, I_{BD} = -30,$
$I_{BF} = 20, I_{CD} = -30$ M1 A1 A1
$CD(+) \rightarrow AD(-) \rightarrow AF(+) \rightarrow CF(-)$
$\theta = 4$

	D	E	F
A	2		4
B		5	
C	4	4	

$S_A = 0$, $S_B = 0$, $S_C = -10$
$D_D = 20$, $D_E = 30$, $D_F = 10$
$I_{AE} = 10$, $I_{BD} = 0$, $I_{BF} = 30$, $I_{CF} = 30$
∴ optimal, cost £350 A1 7

[14]

6. (a) Total cost = 2 × 40 + 350 + 200 = £630 M1 A1 2

(b)

Stage	Demand	State	Action	Destination	Value		
(2) Oct	(5)	(1)	(4)	(0)	(590 + 200 = 790)		
		(2)	(3) (4)	(0) (1)	280 + 200 = 480 630 + 240 = 870	M1 A1	
		(3)	(2) 3 4	0 1 2	320 + 200 = 520 320 + 240 = 560 670 + 80 = 750	M1 A1	4
3 Sept	3	0	4	1	550 + 790 = 1340	M1 A1	
		1	3 4	1 2	240 + 790 = 1030 590 + 480 = 1070	M1 A1 ft	
4 Aug	3	0	3 4	0 1	200 + 1340 = 1540 550 + 1030 = 1580	M1 A1 ft	6

Month	August	September	October	November
Make	3	4	4	2

M1 A1

cost = £1540 A1 ft 3

(c) Profit per cycle = 13 × 1400 = 18 200 Cost of Kim's time = £2000
Cost of production = £1540 B1

∴ Total profit = 18 200 – 3540 M1
= £14 660 A1 ft 3

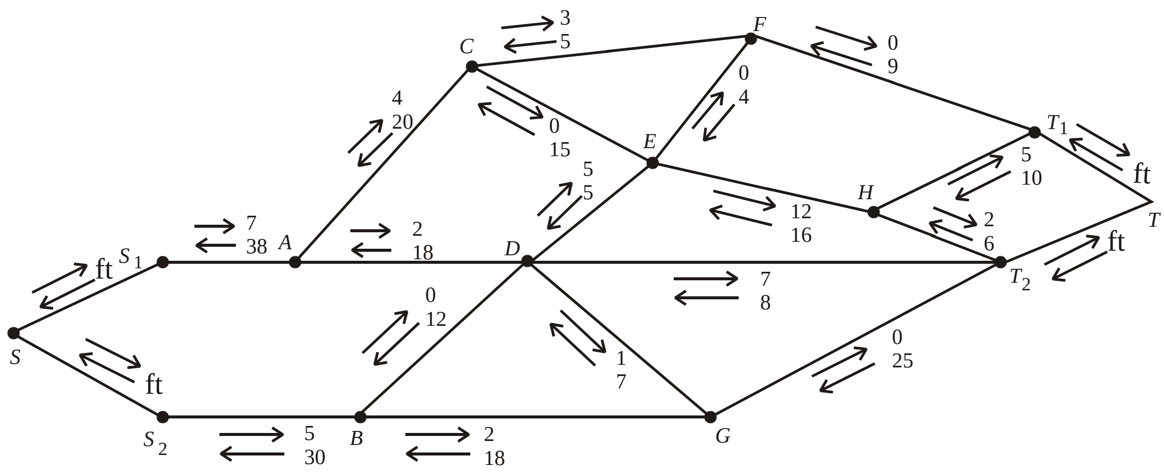
[18]

7. (a) Adds S and T and arcs M1
$SS_1 \geq 45$, $SS_2 \geq 35$, $T_1T \geq 24$, $T_2T \geq 58$ A1 2

(b) Using conservation of flow through vertices $x = 16$ and $y = 7$ B1 B1 2

(c) $C_1 = 86$, $C_2 = 81$ B1 B2 3

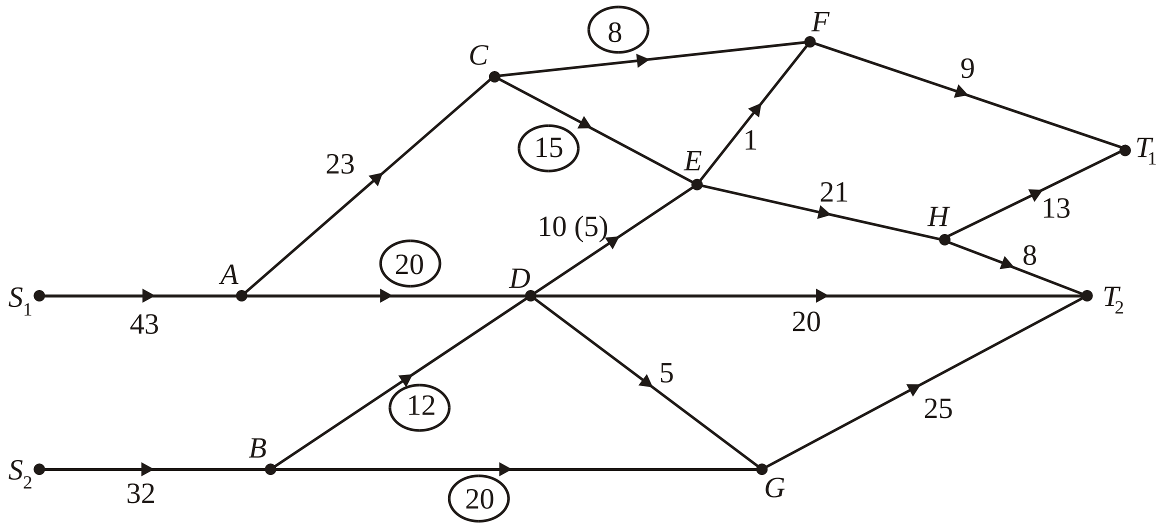
(d)



M1 A1
dM1

e.g. $S\,S_1\,A\,D\,E\,H\,T_2\,T$ – 2	A1	
$S\,S_1\,A\,C\,F\,E\,H\,T_1\,T$ – 3	A1	
$S\,S_2\,B\,G\,D\,T_2\,T$ – 2	A1	6

(e) e.g.:



M1 A1

<u>Flow 75</u> A1 3

(f) Max flow – min cut theorem cut through dM1

CF, *CE*, *AD*, *BD*, *BG* (value 75) A1 2

[18]

8. (a) $2x + 3y + 4z \leq 8$ B1

$3x + 3y + z \leq 10$ B1

$P = 8x + 9y + 5z$ B1 3

(b)

↓ (column y)

b.v	x	y	z	r	s	Value
r	2	(3)	4	1	0	8
s	3	3	1	0	1	10
P	–8	–9	–5	0	0	0

↓ (column x)

b.v	x	y	z	r	s	Value	
y	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{\ }{3}$	$R_1 \div 3$
s	(1)	0	–3	–1	1	2	$R_2 - 3R_1$
P	–2	0	7	3	0	24	$R_3 + 9R_1$

b.v	x	y	z	r	s	Value		
y	0	1	$\frac{10}{3}$	1	$-\frac{2}{3}$	$\frac{4}{3}$	$R_1 - \frac{2}{3}R_2$	M1 A1
x	1	0	–3	–1	1	2		M1
P	0	0	1	1	2	28	$R_3 + 2R_2$	A1 8

(c) $P = 28$ M1

$x = 2$, $y = \frac{4}{3}$ A1

$z = 0$, $r = 0$, $s = 0$ A1 3

[14]