

1.

Figure 1

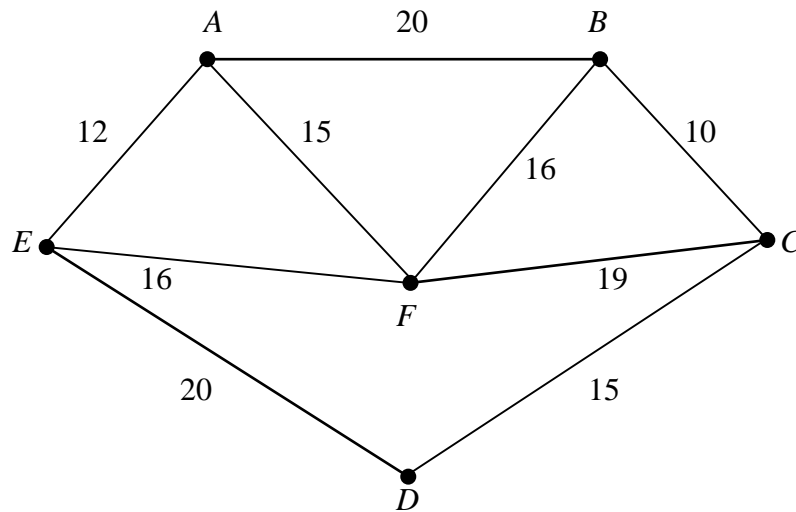


Figure 1 shows a network of roads connecting six villages A , B , C , D , E and F . The lengths of the roads are given in km.

(a) Complete the table in the answer booklet, in which the entries are the shortest distances between pairs of villages. You should do this by inspection.

(2)

The table can now be taken to represent a complete network.

(b) Use the nearest-neighbour algorithm, starting at A , on your completed table in part (a). Obtain an upper bound to the length of a tour in this complete network, which starts and finishes at A and visits every village exactly once.

(3)

(c) Interpret your answer in part (b) in terms of the original network of roads connecting the six villages.

(1)

(d) By choosing a different vertex as your starting point, use the nearest-neighbour algorithm to obtain a shorter tour than that found in part (b). State the tour and its length.

(2)

2. A two-person zero-sum game is represented by the following pay-off matrix for player A.

		<i>B</i>			
		I	II	III	IV
<i>A</i>	I	-4	-5	-2	4
	II	-1	1	-1	2
	III	0	5	-2	-4
	IV	-1	3	-1	1

(a) Determine the play-safe strategy for each player.

(4)

(b) Verify that there is a stable solution and determine the saddle points.

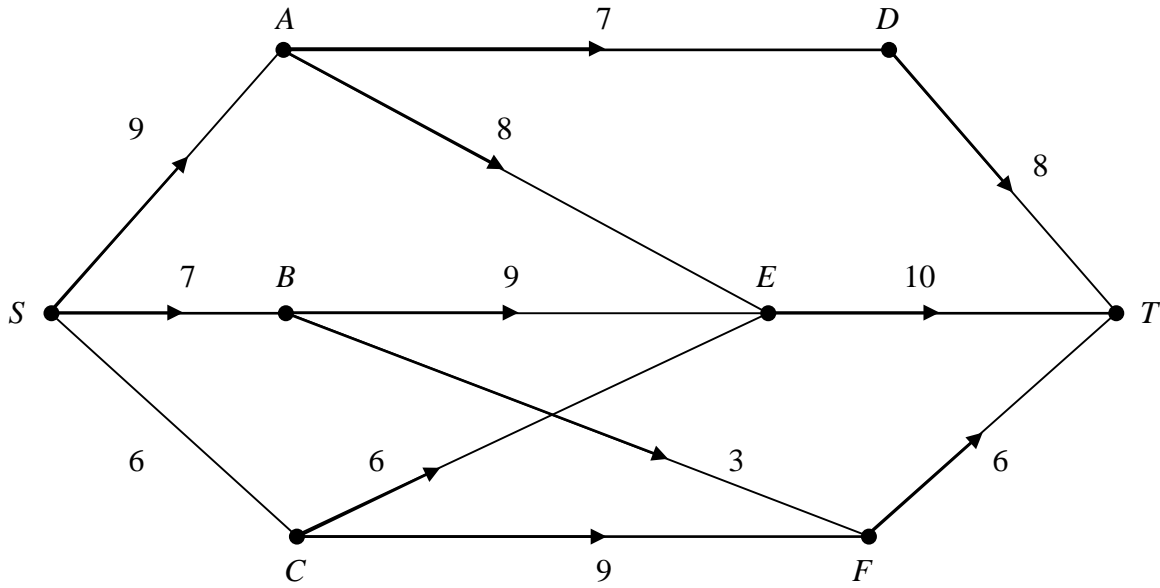
(3)

(c) State the value of the game to *B*.

(1)

3.

Figure 2



The network in Fig. 2 shows possible routes that an aircraft can take from S to T . The numbers on the directed arcs give the amount of fuel used on that part of the route, in appropriate units. The airline wishes to choose the route for which the maximum amount of fuel used on any part of the route is as small as possible. This is the minimax route.

(a) Complete the table in the answer booklet.

(8)

(b) Hence obtain the minimax route from S to T and state the maximum amount of fuel used on any part of this route.

(2)

4. Andrew (A) and Barbara (B) play a zero-sum game. This game is represented by the following pay-off matrix for Andrew.

$$A \begin{matrix} & \begin{matrix} B \\ \begin{pmatrix} 3 & 5 & 4 \\ 1 & 4 & 2 \\ 6 & 3 & 7 \end{pmatrix} \end{matrix} \end{matrix}$$

- (a) Explain why this matrix may be reduced to

$$\begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}.$$

- (b) Hence find the best strategy for each player and the value of the game.

(8)

5. An engineering company has 4 machines available and 4 jobs to be completed. Each machine is to be assigned to one job. The time, in hours, required by each machine to complete each job is shown in the table below.

	Job 1	Job 2	Job 3	Job 4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

Use the Hungarian algorithm, *reducing rows first*, to obtain the allocation of machines to jobs which minimises the total time required. State this minimum time.

(11)

6. The table below shows the distances, in km, between six towns A, B, C, D, E and F .

	A	B	C	D	E	F
A	–	85	110	175	108	100
B	85	–	38	175	160	93
C	110	38	–	148	156	73
D	175	175	148	–	110	84
E	108	160	156	110	–	92
F	100	93	73	84	92	–

(a) Starting from A , use Prim's algorithm to find a minimum connector and draw the minimum spanning tree. You must make your method clear by stating the order in which the arcs are selected.

(4)

(b) (i) Using your answer to part (a) obtain an initial upper bound for the solution of the travelling salesman problem.

(ii) Use a short cut to reduce the upper bound to a value less than 680.

(4)

(c) Starting by deleting F , find a lower bound for the solution of the travelling salesman problem.

(4)

7. A steel manufacturer has 3 factories F_1 , F_2 and F_3 which can produce 35, 25 and 15 kilotonnes of steel per year, respectively. Three businesses B_1 , B_2 and B_3 have annual requirements of 20, 25 and 30 kilotonnes respectively. The table below shows the cost C_{ij} in appropriate units, of transporting one kilotonne of steel from factory F_i to business B_j .

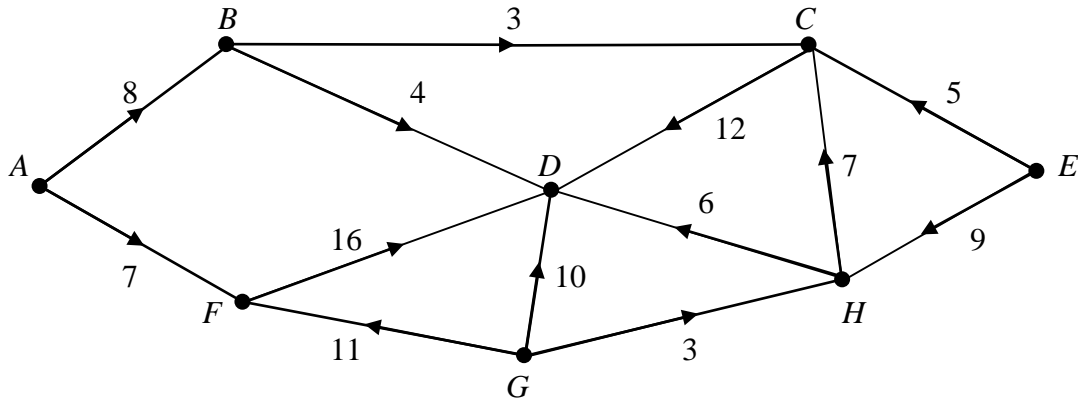
		Business		
		B_1	B_2	B_3
Factory	F_1	10	4	11
	F_2	12	5	8
	F_3	9	6	7

The manufacturer wishes to transport the steel to the businesses at minimum total cost.

- (a) Write down the transportation pattern obtained by using the North-West corner rule. (2)
- (b) Calculate all of the improvement indices I_{ij} , and hence show that this pattern is not optimal. (5)
- (c) Use the stepping-stone method to obtain an improved solution. (3)
- (d) Show that the transportation pattern obtained in part (c) is optimal and find its cost.

8.

Figure 4



The network in Fig. 4 models a drainage system. The number on each arc indicates the capacity of that arc, in litres per second.

(a) Write down the source vertices.

(2)

Figure 5

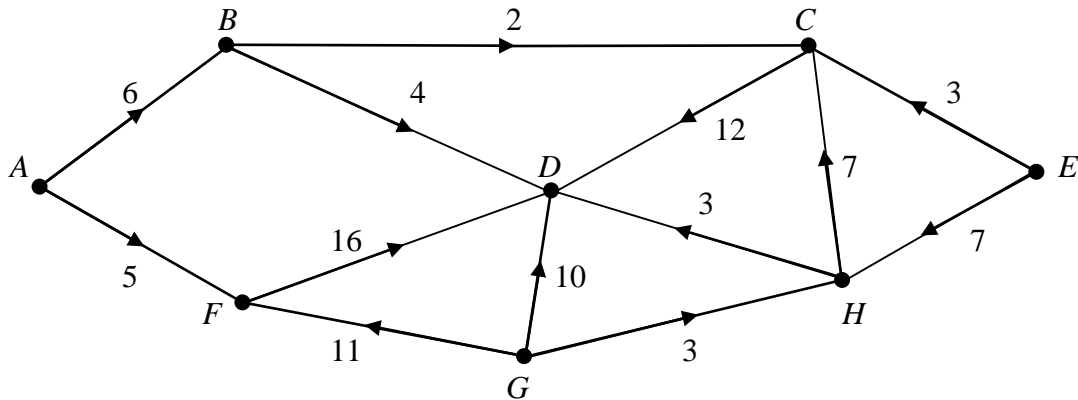


Figure 5 shows a feasible flow through the same network.

(b) State the value of the feasible flow shown in Fig. 5.

(1)

Taking the flow in Fig. 5 as your initial flow pattern,

(c) use the labelling procedure on Diagram 1 to find a maximum flow through this network. You should list each flow-augmenting route you use, together with its flow.

(6)

(d) Show the maximal flow on Diagram 2 and state its value.

(3)

(e) Prove that your flow is maximal.

(2)

9. T42 Co. Ltd produces three different blends of tea, Morning, Afternoon and Evening. The teas must be processed, blended and then packed for distribution. The table below shows the time taken, in hours, for each stage of the production of a tonne of tea. It also shows the profit, in hundreds of pounds, on each tonne.

	Processing	Blending	Packing	Profit (£100)
Morning blend	3	1	2	4
Afternoon blend	2	3	4	5
Evening blend	4	2	3	3

The total times available each week for processing, blending and packing are 35, 20 and 24 hours respectively. T42 Co. Ltd wishes to maximise the weekly profit.

Let x , y and z be the number of tonnes of Morning, Afternoon and Evening blend produced each week.

- (a) Formulate the above situation as a linear programming problem, listing clearly the objective function, and the constraints as inequalities.

(4)

An initial Simplex tableau for the above situation is

Basic variable	x	y	z	r	s	t	Value
r	3	2	4	1	0	0	35
s	1	3	2	0	1	0	20
t	2	4	3	0	0	1	24
P	-4	-5	-3	0	0	0	0

- (b) Solve this linear programming problem using the Simplex algorithm. Take the most negative number in the profit row to indicate the pivot column at each stage.

(11)

T42 Co. Ltd wishes to increase its profit further and is prepared to increase the time available for processing or blending or packing or any two of these three.

- (c) Use your answer to part (b) to advise the company as to which stage(s) it should increase the time available.

(2)

10. While solving a maximizing linear programming problem, the following tableau was obtained.

Basic variable	x	y	z	r	s	t	Value
r	0	0	$1\frac{2}{3}$	1	0	$-\frac{1}{6}$	$\frac{2}{3}$
y	0	1	$3\frac{1}{3}$	0	1	$-\frac{1}{3}$	$\frac{1}{3}$
x	1	0	-3	0	-1	$\frac{1}{2}$	1
P	0	0	1	0	1	1	11

(a) Explain why this is an optimal tableau.

(1)

(b) Write down the optimal solution of this problem, stating the value of every variable.

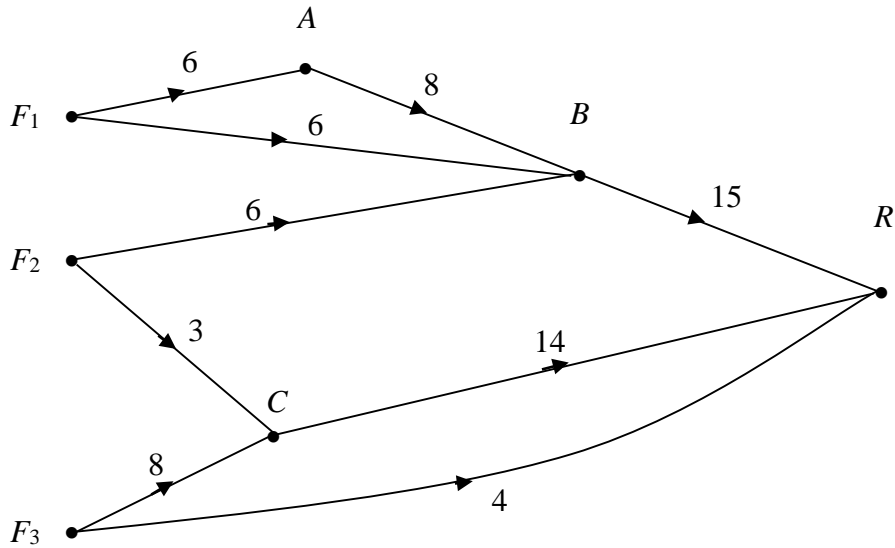
(3)

(c) Write down the profit equation from the tableau. Use it to explain why changing the value of any of the non-basic variables will decrease the value of P .

(2)

11. A company wishes to transport its products from 3 factories F_1 , F_2 and F_3 to a single retail outlet R . The capacities of the possible routes, in van loads per day, are shown in Fig. 5.

Figure 5



(a) On Diagram 1 in the answer booklet add a supersource S to obtain a capacitated network with a single source and a single sink. State the minimum capacity of each arc you have added. (2)

(b) (i) State the maximum flow along $S F_1 A B R$ and $S F_3 C R$.

(ii) Show these maximum flows on Diagram 2 in the answer booklet, using numbers in circles. (2)

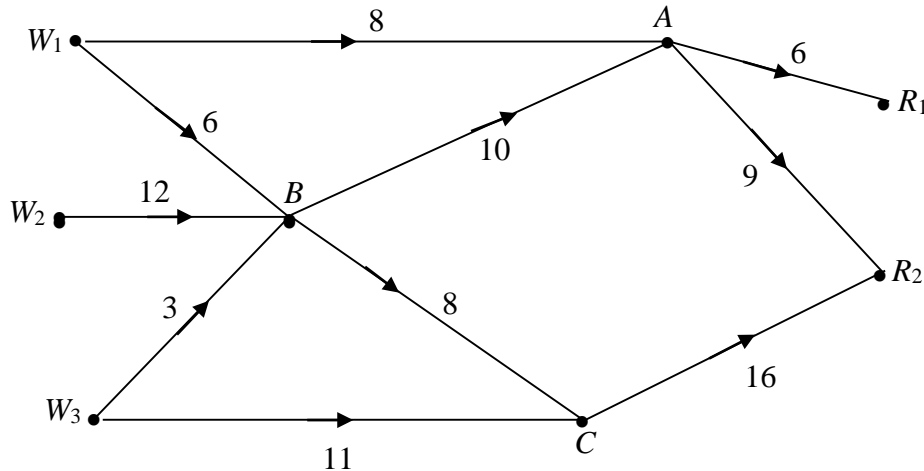
Taking your answer to part (b)(ii) as the initial flow pattern,

(c) (i) use the labelling procedure to find a maximum flow from S to R . Your working should be shown on Diagram 3. List each flow-augmenting route you find together with its flow.

(ii) Prove that your final flow is maximal. (7)

12.

Figure 2



A company has 3 warehouses W_1 , W_2 , and W_3 . It needs to transport the goods stored there to 2 retail outlets R_1 and R_2 . The capacities of the possible routes, in van loads per day, are shown in Fig 2. Warehouses W_1 , W_2 and W_3 have 14, 12 and 14 van loads respectively available per day and retail outlets R_1 and R_2 can accept 6 and 25 van loads respectively per day.

(a) On Diagram 1 on the answer sheet add a supersource W , a supersink R and the appropriate directed arcs to obtain a single-source, single-sink capacitated network. State the minimum capacity of each arc you have added.

(3)

(b) State the maximum flow along

- (i) $W \rightarrow W_1 \rightarrow A \rightarrow R_1 \rightarrow R$, (ii) $W \rightarrow W_3 \rightarrow C \rightarrow R_2 \rightarrow R$.

(2)

(c) Taking your answers to part (b) as the initial flow pattern, use the labelling procedure to obtain a maximum flow through the network from W to R . Show your working on Diagram 2. List each flow-augmenting route you use, together with its flow.

(5)

(d) From your final flow pattern, determine the number of van loads passing through B each day.

(1)