D2 June 2002 (Adapted)

1.

	A	B	C	D	E	F
A	0	20	30	32	12	15
B	20	0	10	25	32	16
C	30	10	0	15	35	19
D	32	25	15	0	20	34
E	12	32	35	20	0	16
F	15	16	19	34	16	0

b) $A \,_{12}\, E \,_{16}\, F \,_{16}\, B \,_{10}\, C \,_{15}\, D \,_{32}\, A$ 101

c) AEFBCDA is not possible on the networ
the tour would be

AEFBCDEA

d) $B \,_{10}\, C \,_{15}\, D \,_{20}\, E \,_{12}\, A \,_{15}\, F \,_{16}\, B$ 88

2)

	B1	B2	B3	B4	Row Min
A1	-4	-5	-2	4	-5
A2	-1	1	-1	2	-1
A3	0	5	-2	-4	-4
A4	-1	3	-1	1	-1
Column max	0	5	-1	4	

Column Minimax (-
=
Row Maximin (-1

∴ Stable solution exists

b) Saddle points A2/B3 and A4/B3

c) V(B) = 1

3) minimax

Stage	State	Action	Dest.	Value
1	D	DT	T	max(8 , X) = 8 *
	E	ET	T	max(10, X) = 10 *
	F	FT	T	max(6, X) = 6 *
2	A	AD	D	max(7 , 8) = 8 *
		AE	E	max(8 , 10) = 10
	B	BE	E	max(9 , 10) = 10
		BF	F	max(3 , 6) = 6 *
	C	CE	E	max(6 , 10) = 10
		CF	F	max(9 , 6) = 9 *
3	S	SA	A	max(9 , 8) = 9
		SB	B	max(7 , 6) = 7 *
		SC	C	max(6 , 9) = 9

SBFT (7)

4)

	B1	B2	B3
A1	3	5	4
A2	1	4	2
A3	6	3	7

- Row 1 Dominates Row 2 ∴ Row 2 can be deleted as player A should never play 2
- Column 3 dominates column 1 ∴ Column 3 can be deleted as player B should never play 3.

	B1	B2
A1	3	5
A3	6	3

let A play 1 prob $= p$
A play 2 prob $= 1-p$

0	1
6	3
3	5

If B plays 1 $\quad V(A) = 3p + 6(1-p) = 6-3p$
B plays 2 $\quad V(A) = 5p + 3(1-p) = 3+2p$

$6-3p = 3+2p \Rightarrow 5p = 3 \Rightarrow p = \frac{3}{5}$

optimal play-safe strategy for A

play 1 prob $= \frac{3}{5}$ play 3 prob $= \frac{2}{5}$ $V(A) = \frac{21}{5}$.
play 2 prob $= 0$!

Invert matrix so pay-off is for B.

	A1	A3
B1	-3	-6
B2	-5	-3

B plays 1 prob $= q$
B plays 2 prob $= 1-q$

If A plays 1 $\quad V(B) = -3q - 5(1-q) = 2q-5$
A plays 3 $\quad V(B) = -6q - 3(1-q) = -3q-3$

$2q-5 = -3q-3 \Rightarrow 5q = 2 \quad q = \frac{2}{5}$

$\therefore$ B should play 1 prob $= \frac{2}{5}$
B should play 2 prob $= \frac{3}{5}$
B should play 3 prob $= 0$

$V(B) = 2\left(\frac{2}{5}\right) - 5 = -\frac{21}{5}$

5)

	J1	J2	J3	J4
M1	14	5	8	7
M2	2	12	6	5
M3	7	8	3	9
M4	2	4	6	10

Reduce Rows

9	0	3	2	−5
0	10	4	3	−2
4	5	0	6	−3
0	2	4	8	−2

Reduce Columns

9	0	3	0
0	10	4	1
4	5	0	4
0	2	4	6
x	x	x	−2

3 lines
∴ not optimal
smallest uncovered
= 1

10	0	4	0
0	9	4	0
4	4	0	3
0	1	4	5

4 lines needed to cover zeros
∴ optimal

	0		0
0			0
		0	
0			

M1 − J2 (5)
M2 − J4 (5)
M3 − J3 (3)
M4 − J1 (2)

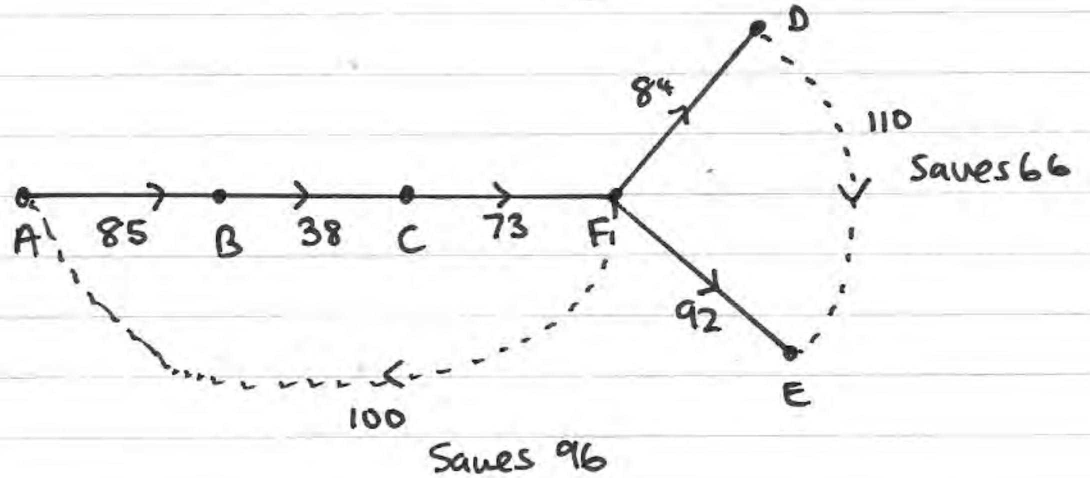
15

6)

	A	B	C	D	E	F
A	–	85	110	175	108	100
B	85	–	38	175	160	93
C	110	38	–	148	156	73
D	175	175	148	–	110	84
E	108	160	156	110	–	92
F	100	93	73	84	92	–

AB(85)
BC (38)
CF (73)
FD (84)
FE (92)
372

∴ upper bound = 744



$A_{85} B_{38} C_{73} F_{84} D_{110} E_{92} F_{100} A$ 582

short cut so ABCFDEFCBA 678

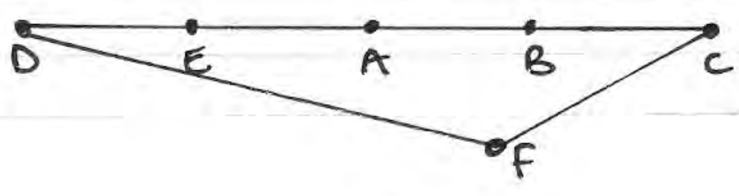
using short cut DE.

c) Prim's

AB(85)
BC (38)
AE (108)
ED (110)

RMST = 341
+ FC (73)
+ FD (84)
498

Lower bound = 498



498 ≤ optimal Solution < 582

∴ = 498

7)

	B_1	B_2	B_3	Supply
F_1	10	4	11	35
F_2	12	5	8	25
F_3	9	6	7	15
Demand	20	25	30	75

balanced supply = demand

NWC

20	15		35
	10	15	25
		15	15
20	25	30	

Initial Solution

shadow Costs

	10	4	7
0	10	4	11
1	12	5	8
0	9	6	7

Improvement Indices

X	X	4
1	X	X
-1	2	X

$20^{-\theta}$	$15^{+\theta}$	
	$10^{-\theta}$	$15^{+\theta}$
$+\theta$		$15^{-\theta}$

entering Cell = $F_3 B_1$

$\theta = 10$

exiting Cell = $F_2 B_2$

10	25	
		25
10		5

	10	4	8
0	10	4	11
0	12	5	8
-1	9	6	7

Shadow Costs

X	X	3
2	1	X
X	3	X

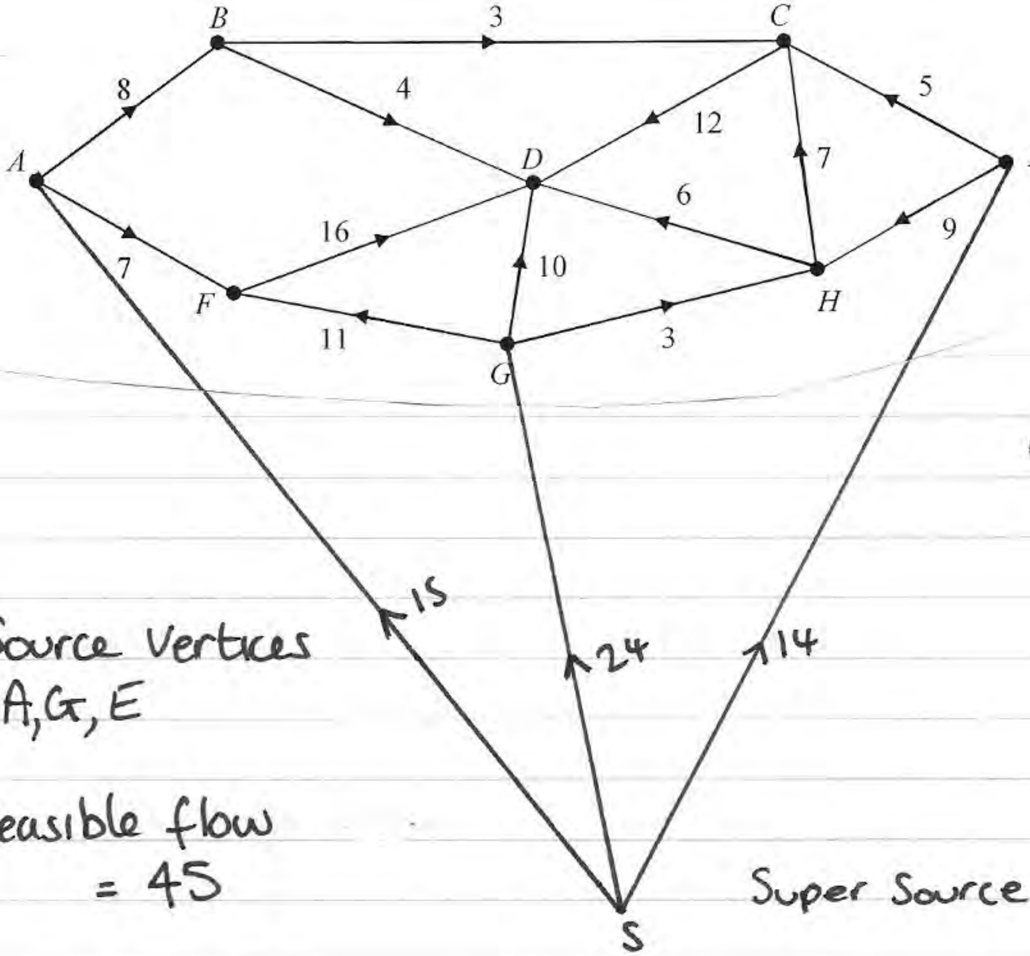
Improvement Indices

∴ no negative Improvement Index ∴ Solution is optimal

	B_1	B_2	B_3
F_1	10	25	
F_2			25
F_3	10		5

Cost = 525

8

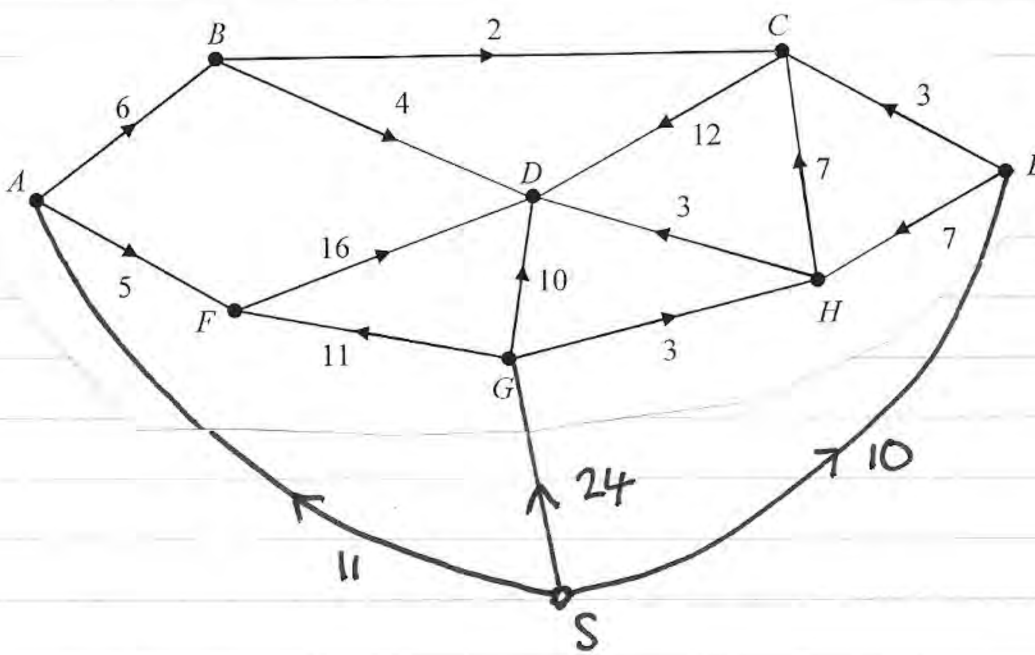


Capacity

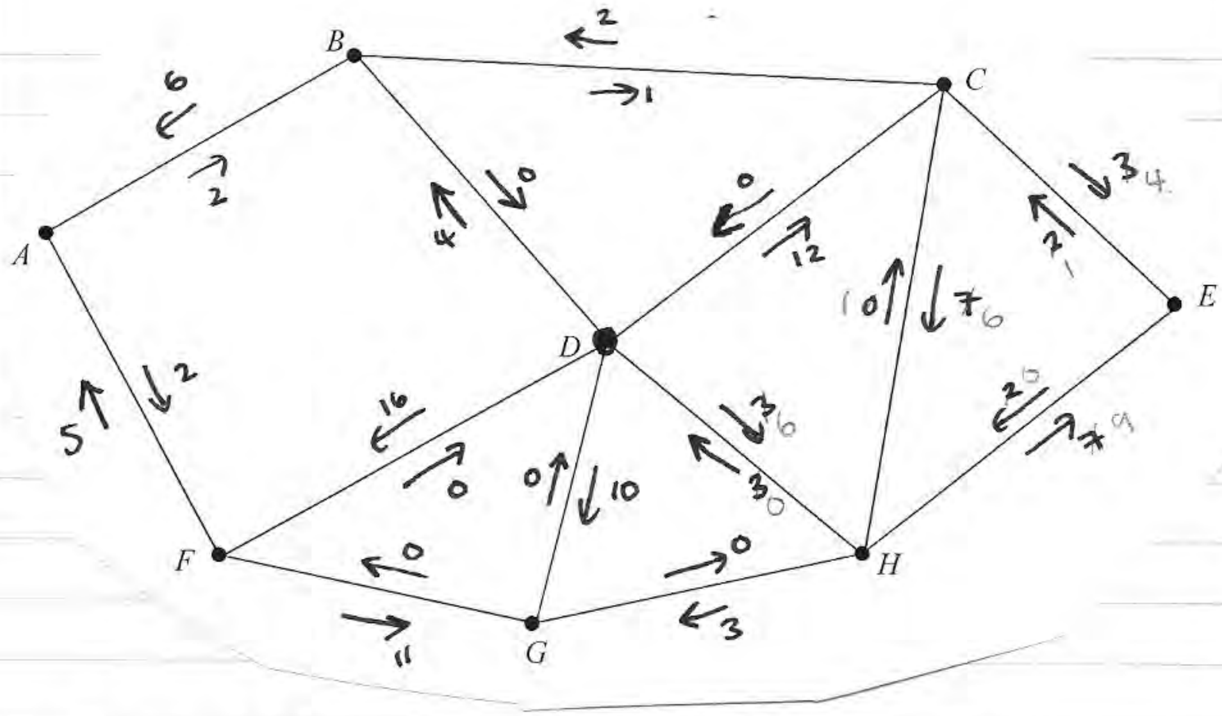
a) Source Vertices
A, G, E

b) feasible flow
= 45

Super Source S

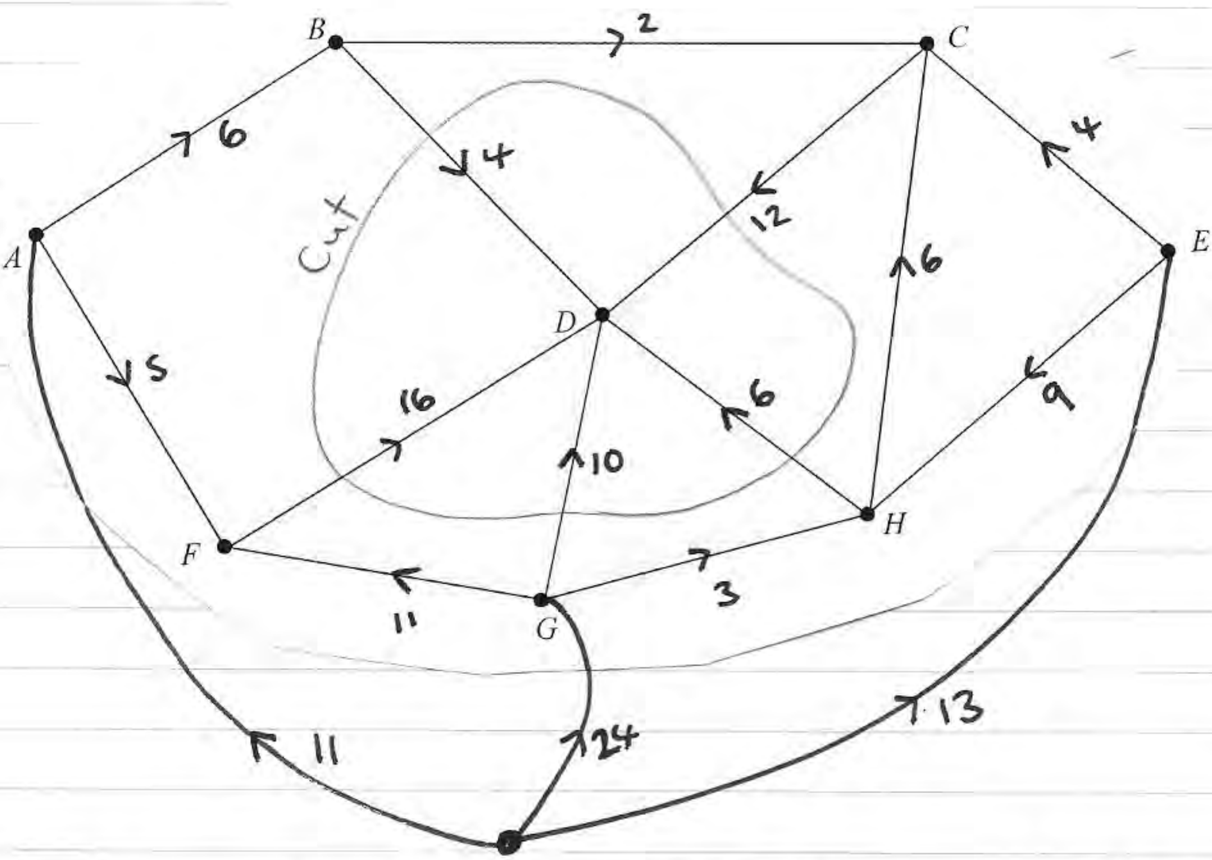


flow



EHD (2)
ECHD (1)
$\overline{\ \ }$
3

total flow increased to <u>48</u>



Cut through BD, BC, DH, DG, DF splits network into 2 halves one containing sink the other the sources, all arcs saturated ∴ min cut = 48 max flow - min cut theorem ⇒ max flow = 48 ∴ max

9)

		Processing	Blending	Packing	Profit (£100)
x	Morning blend	3	1	2	4
y	Afternoon blend	2	3	4	5
z	Evening blend	4	2	3	3
		≤ 35	≤ 20	≤ 24	

Objective is to maximise profit P, (£100)

$P = 4x + 5y + 3z$

Subject to

$$3x + 2y + 4z \leq 35$$
$$x + 3y + 2z \leq 20$$
$$2x + 4y + 3z \leq 24$$
$$x, y, z \geq 0$$

Basic variable	x	y	z	r	s	t	Value	
r	3	2	4	1	0	0	35	$\theta = 35 \div 2 = 17.5$
s	1	3	2	0	1	0	20	$\theta = 20 \div 3 = 6\frac{2}{3}$
t	2	$\boxed{4}$	3	0	0	1	24	$\theta = 24 \div 4 = 6$ *
P	−4	−5	−3	0	0	0	0	

Increasing y

Basic variable	x	y	z	r	s	t	Value	
	3	2	4	1	0	0	35	R_1
	1	3	2	0	1	0	20	R_2
	$1/2$	$\boxed{1}$	$3/4$	0	0	$1/4$	6	$R3 \div 4$
	-4	-5	-3	0	0	0	0	R_4

$\theta = 11.5$
θ -ve
$\theta = 12$

Basic variable	x	y	z	r	s	t	Value	
r	$\boxed{2}$	0	5/2	1	0	-1/2	23	R1-2R3
S	-1/2	0	-1/4	0	1	-3/4	2	R2-3R3
y	1/2	1	3/4	0	0	1/4	6	R3
P	-3/2	0	3/4	0	0	5/4	30	R4+5R3

Increasing x

Basic variable	x	y	z	r	s	t	Value	
	1	0	5/4	1/2	0	-1/4	23/2	R1÷2
	-1/2	0	-1/4	0	1	-3/4	2	R2
	1/2	1	3/4	0	0	1/4	6	R3
	-3/2	0	3/4	0	0	5/4	30	R4

Basic variable	x	y	z	r	s	t	Value	
x	1	0	5/4	1/2	0	-1/4	23/2	R1
S	0	0	3/8	1/4	1	-7/8	31/4	R2+$\frac{1}{2}$R1
y	0	1	1/8	-1/4	0	3/8	1/4	R3-$\frac{1}{2}$R1
P	0	0	21/8	3/4	0	7/8	189/4	R4+$\frac{3}{2}$R1

$x = 23/2$ $\quad r = 0$ $\quad P = 47.25$
$y = 1/4$ $\quad S = 31/4$ $\quad = £4725$
$z = 0$ $\quad t = 0$

c) Slack in S, which was used in the equation created from the constraints in the time for blending ∴
do not increase time available for blending but do increase time available for processing and packing as all the time is currently being used in this solution

10) No negative values in the profit row ∴ optimal

b) $x = 1$ $\quad r = 2/3$ $\quad P = 11$

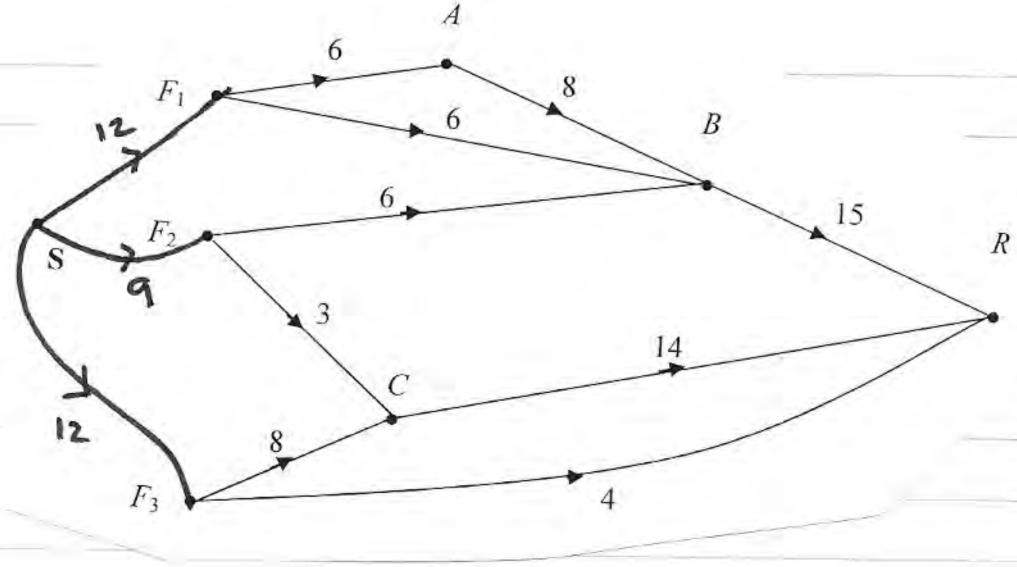
$y = 1/3$ $\quad s = 0$

$z = 0$ $\quad t = 0$

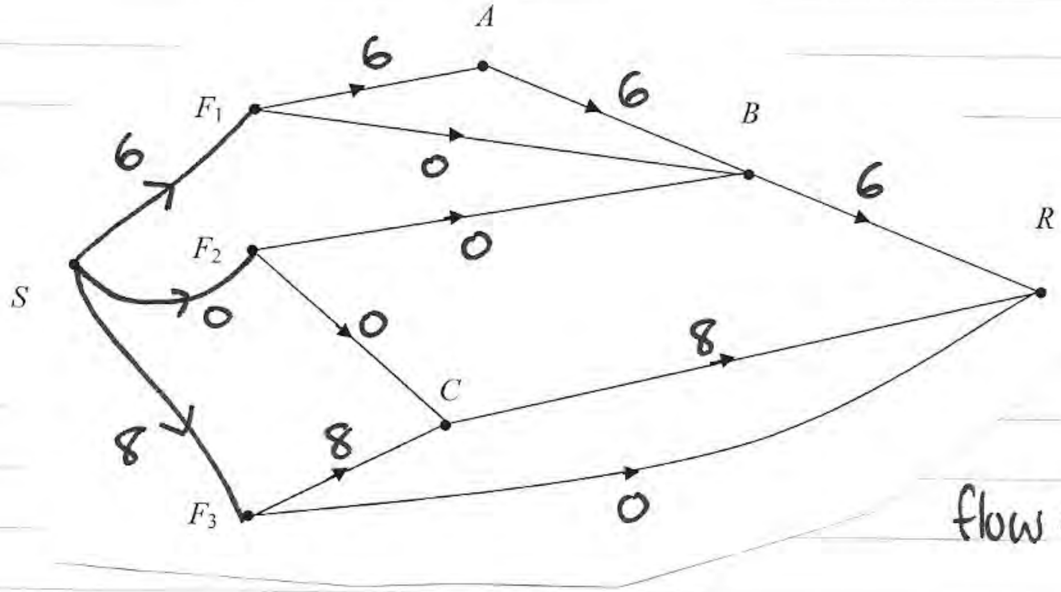
c) $P + z + s + t = 11 \Rightarrow P = 11 - z - s - t$

So increasing z, s or t would reduce the profit

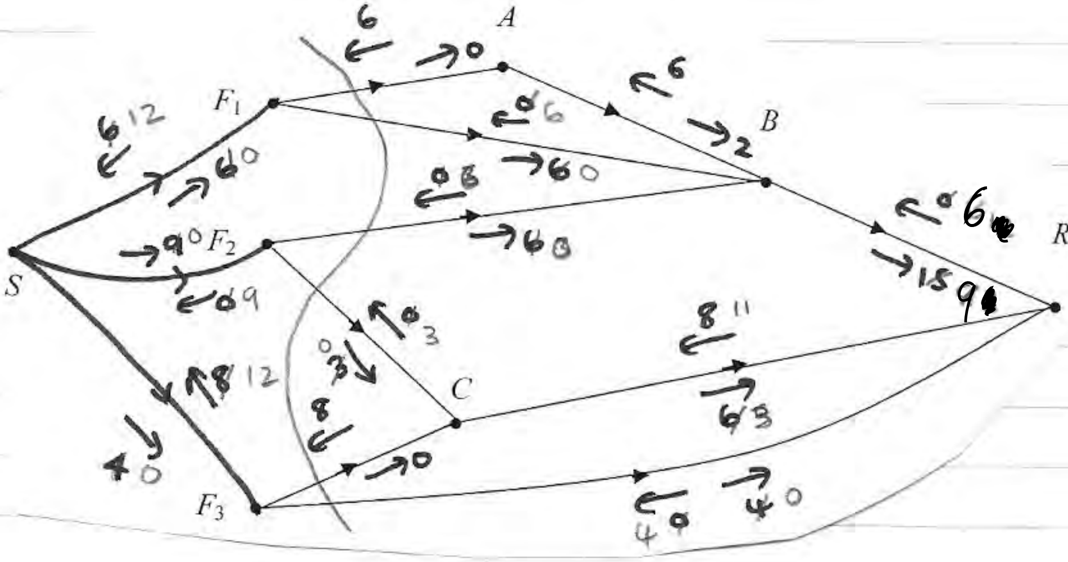
11



$SF_1ABR = 6$ $SF_3CR = 8$



flow = 14



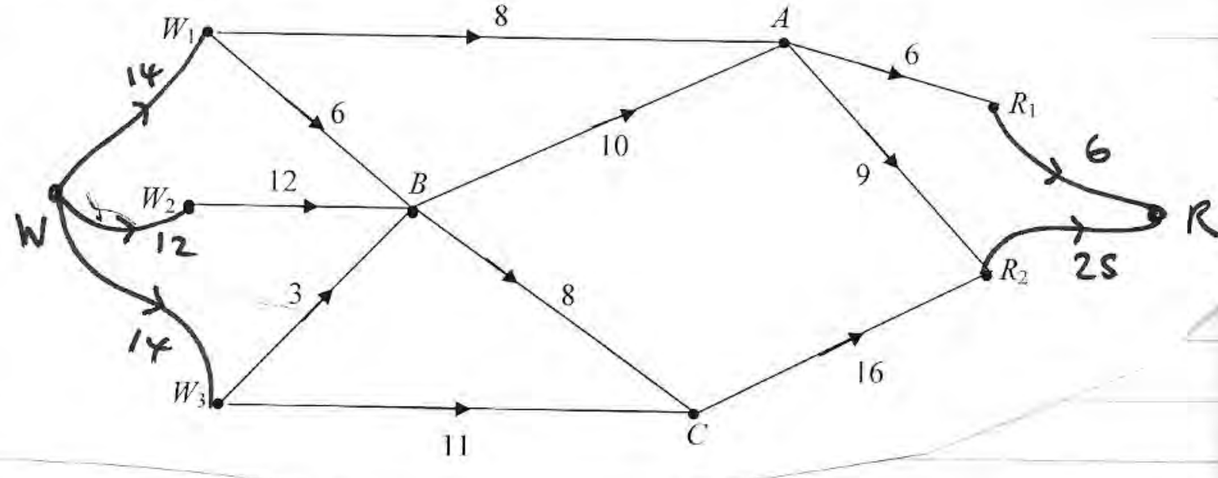
c) i) SF_1BR (6)
SF_2BR (3)
SF_3R (4)
SF_2CR (3)
16

flow now 30

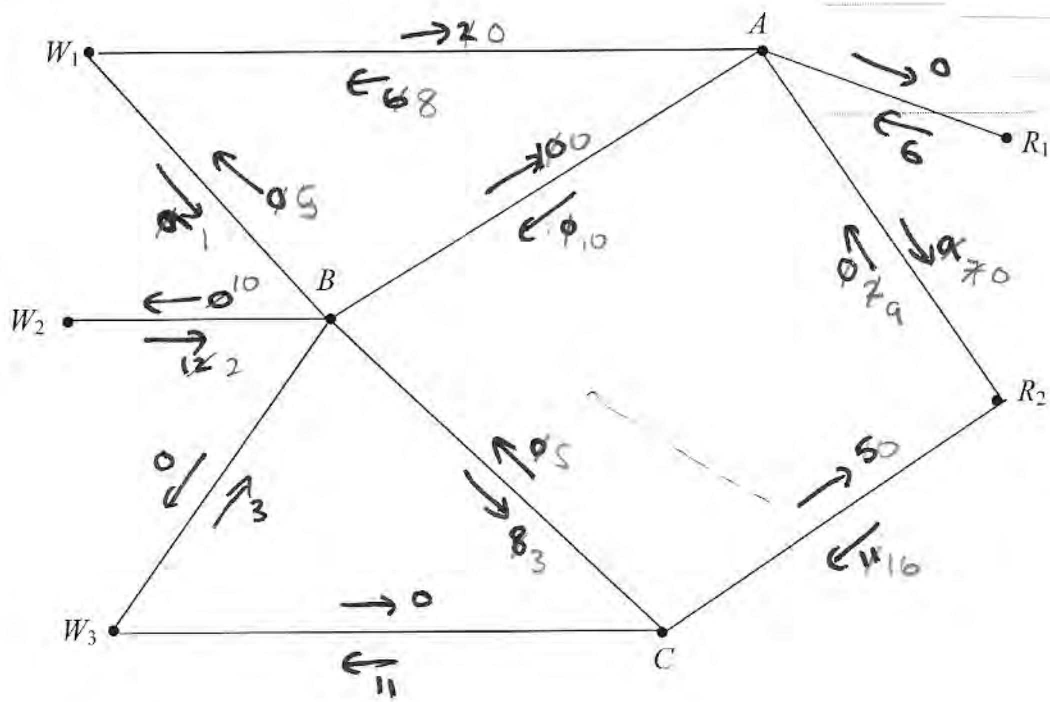
ii) Cut through F_1A, F_1B, F_2B, F_2C, F_3C, F_3R

only passes through saturated arcs ∴ min cut = 30 by maxflow-mincut theorem
max flow = 30 ∴ maximal.

12



$WW_1AR_1R = 6$ $WW_3CR_2R = 11$



WW_1AR_2R (2)
WW_1BCR_2R (5)
WW_2BAR_2R (7)

flow now
31

d) flow through $B = 15$