D2 Jan 2006 (adapted for new spec)

1. A theme park has four sites, A, B, C and D, on which to put kiosks. Each kiosk will sell a different type of refreshment. The income from each kiosk depends upon what it sells and where it is located. The table below shows the expected daily income, in pounds, from each kiosk at each site.

	Hot dogs and beef burgers (H)	Ice cream (I)	Popcorn, candyfloss and drinks (P)	Snacks and hot drinks (S)
Site A	267	272	276	261
Site B	264	271	278	263
Site C	267	273	275	263
Site D	261	269	274	257

Reducing rows first, use the Hungarian algorithm to determine a site for each kiosk in order to maximise the total income. State the site for each kiosk and the total expected income. You must make your method clear and show the table after each stage.

(Total 13 marks)

2. An engineering firm makes motors. They can make up to five in any one month, but if they make more than four they have to hire additional premises at a cost of £500 per month. They can store up to two motors for £100 per motor per month. The overhead costs are £200 in any month in which work is done.
Motors are delivered to buyers at the end of each month. There are no motors in stock at the beginning of May and there should be none in stock after the September delivery.

The order book for motors is:

Month	May	June	July	August	September
Number of motors	3	3	7	5	4

Use dynamic programming to determine the production schedule that minimises the costs, showing your working in the table provided below.

Stage (month)	State (Number in store at start of month)	Action (Number made in month)	Destinatio n (Number in store at end of month)	Value (cost)

Production schedule

Month	**May**	**June**	**July**	**August**	**September**
Number to be made					

Total cost: ....

(Total 12 marks)

3. Three depots, F, G and H, supply petrol to three service stations, S, T and U. The table gives the cost, in pounds, of transporting 1000 litres of petrol from each depot to each service station.

	S	T	U
F	23	31	46
G	35	38	51
H	41	50	63

F, G and H have stocks of 540 000, 789 000 and 673 000 litres respectively.

S, T and U require 257 000, 348 000 and 412 000 litres respectively. The total cost of transporting the petrol is to be minimised.

Formulate this problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 8 marks)

4. The following minimising transportation problem is to be solved.

	J	K	Supply
A	12	15	9
B	8	17	13
C	4	9	12
Demand	9	11	

(a) Complete the table below.

	J	K	L	Supply
A	12	15		9
B	8	17		13
C	4	9		12
Demand	9	11		34

(2)

(b) Explain why an extra demand column was added to the table above.

(2)

A possible north-west corner solution is:

	J	K	L
A	9	0	
B		11	2
C			12

(c) Explain why it was necessary to place a zero in the first row of the second column.

(1)

After three iterations of the stepping-stone method the table becomes:

	J	K	L
A		8	1
B			13
C	9	3	

(d) Taking the most negative improvement index as the entering square for the stepping stone method, solve the transportation problem. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal.

(11) (Total 16 marks)

5. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	–2	1	3	–1
A plays 2	–1	3	2	1
A plays 3	–4	2	0	–1
A plays 4	1	–2	–1	3

(a) Verify that there is no stable solution to this game.

(3)

(b) Explain why the 4 × 4 game above may be reduced to the following 3 × 3 game.

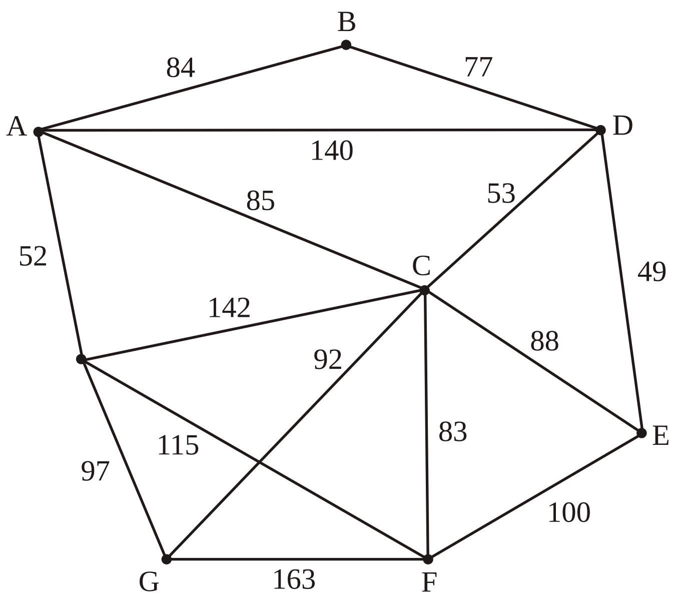
–2	1	3
–1	3	2
1	–2	–1

(2)

(c) Formulate the 3 × 3 game as a linear programming problem for player A. Write the constraints as inequalities. Define your variables clearly.

(8)
(Total 13 marks)

6. The network in the figure above, shows the distances in km, along the roads between eight towns, A, B, C, D, E, F, G and H. Keith has a shop in each town and needs to visit each one. He wishes to travel a minimum distance and his route should start and finish at A.



By deleting D, a lower bound for the length of the route was found to be 586 km.
By deleting F, a lower bound for the length of the route was found to be 590 km.

(a) By deleting C, find another lower bound for the length of the route. State which is the best lower bound of the three, giving a reason for your answer.

(5)

(b) By inspection complete the table of least distances.

(4)

	A	B	C	D	E	F	G	H
A	-	84	85	138	173		149	52
B	84	-	130	77	126	213	222	136
C	85	130	-	53	88	83	92	
D	138	77	53	-	49			190
E	173	126	88	49	-	100	180	215
F		213	83		100	-	163	115
G	149	222	92		180	163	-	97
H	52	136		190	215	115	97	-

The table can now be taken to represent a complete network.

The nearest neighbour algorithm was used to obtain upper bounds for the length of the route: Starting at D, an upper bound for the length of the route was found to be 838 km.
Starting at F, an upper bound for the length of the route was found to be 707 km.

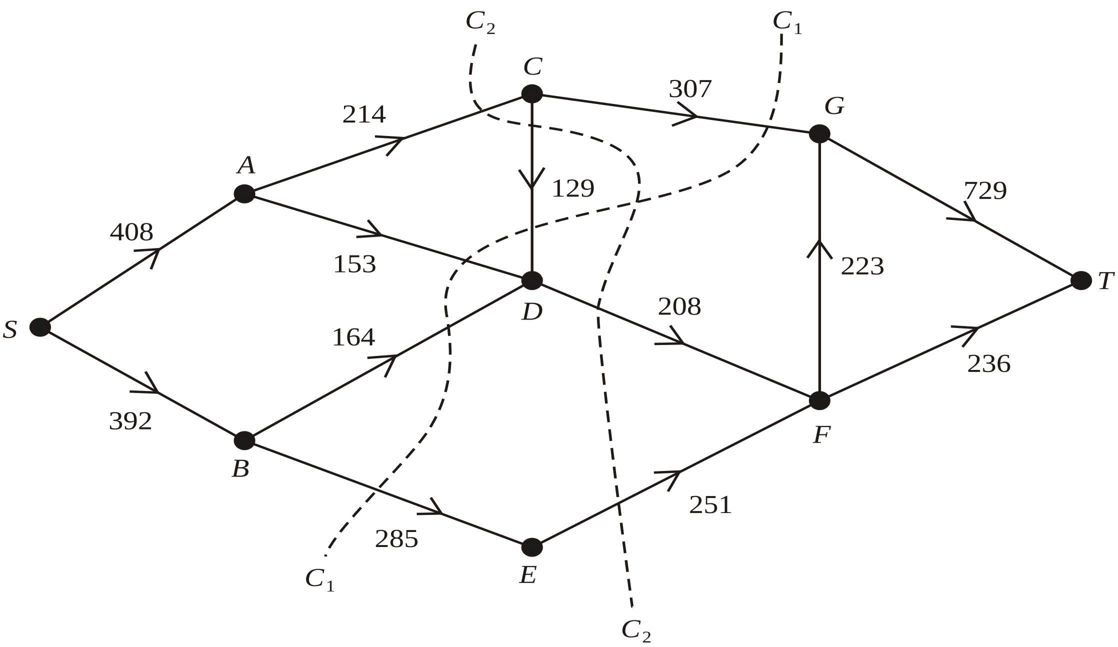
(c) Starting at C, use the nearest neighbour algorithm to obtain another upper bound for the length of the route. State which is the best upper bound of the three, giving a reason for your answer.

(4) (Total 13 marks)

7. (a) Define the terms (i) cut, (ii) minimum cut,

as applied to a directed network flow.

(2)



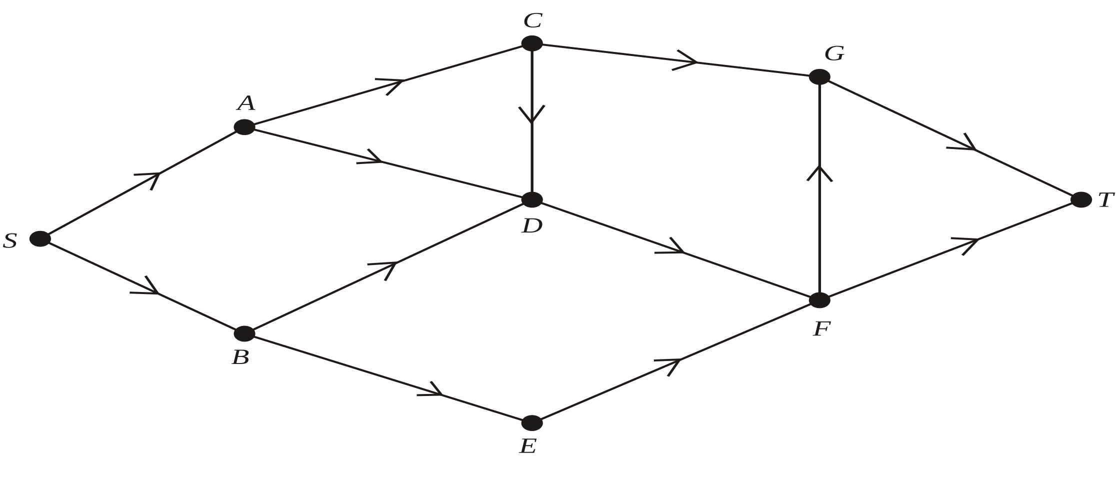
The figure above shows a capacitated directed network and two cuts C_1 and C_2. The number on each arc is its capacity.

(b) State the values of the cuts C_1 and C_2.

(3)

Given that one of these two cuts is a minimum cut,

(c) find a maximum flow pattern by inspection, and show it on the diagram below.



(3)

(d) Find a second minimum cut for this network.

(1)

In order to increase the flow through the network it is decided to add an arc of capacity 100 joining *D* **either** to *E* **or** to *G*.

(e) State, with a reason, which of these arcs should be added, and the value of the increased flow.

(2)

(Total 11 marks)