Solutions

1. To maximise, subtract all entries from $n \geq 278$ e.g. M1

$$\begin{bmatrix} 11 & 6 & 2 & 17 \\ 14 & 7 & 0 & 15 \\ 11 & 5 & 3 & 15 \\ 17 & 9 & 4 & 21 \end{bmatrix}$$

A1 2

Reduce rows

$$\begin{bmatrix} 9 & 4 & 0 & 15 \\ 14 & 7 & 0 & 15 \\ 8 & 2 & 0 & 12 \\ 13 & 5 & 0 & 17 \end{bmatrix}$$

then columns

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 6 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 5 \end{bmatrix}$$

M1 A1ft A1ft 3

Min element = 1

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 5 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}$$

M1 A1ft A1 3

Min element = 1

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 4 & 1 & 0 & 3 \end{bmatrix}$$

or

Min element = 2

$$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

M1 A1ft A1ft 3

then min element 1

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 4 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$$

optimal

So A – H M1
H A1 2
B – P or
S
C – S
I
D – I
P
(both £1077)

[13]

2. e.g.

Stage	State	Action	Dest	Value
1 (Sept)	2	2	0	200 + 200 = 400 *
	1	3	0	200 + 100 = 300 *
	0	4	0	200 = 200 *
2 (Aug)	2	5	2	200 + 200 + 500 + 400 = 1300
		4	1	200 + 200 + 300 = 700
		3	0	200 + 200 + 200 = 600 *
	1	5	1	200 + 100 + 500 + 300 = 1100
		4	0	200 + 100 + 200 = 500 *
	0	5	0	200 + 500 + 200 = 900 *
3 (Jul)	2	5	0	200 + 200 + 500 + 900 = 1800 *
4 (Jun)	2	3	2	200 + 200 + 1800 = 2200 *
	1	4	2	200 + 100 + 1800 = 2100 *
	0	5	2	200 + 500 + 1800 = 2500 *
5 (May)	0	5	2	200 + 500 + 2200 = 2900
	1	4	2	200 + 2100 = 2300 *
	0	5	2	200 + 2500 = 2700 *

Month	May	June	July	August	September	
production schedule	4	4	5	5	4	M1 A1ft

Cost £2300 A1ft 3

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3. Let x_{ij} be the number of units transported from i to j , in 1000 litres
where $i \in \{F, G, H\}$ and $j \in \{S, T, U\}$ B2, 1, 0 2

Minimise $C = 23\, x_{fs} + 31x_{fft} + 46x_{fu} +$ B1
$35\, x_{gs} + 38x_{gt} + 51\, x_{gu} +$ B1 2
$41\, x_{hs} + 50x_{ht} + 63x_{hu}$

Unbalanced

Subject to $x_{fs} + x_{ft} + x_{fu} \le 540$
$x_{gs} + x_{gt} + x_{gu} \le 789$ M1
$x_{hs} + x_{ht} + x_{hu} \le 673$ A1
$x_{fs} + x_{gs} + x_{hs} \le 257$ }
$x_{ft} + x_{gt} + x_{ht} \le 348$ } accept = here A1 3
$x_{fu} + x_{gu} + x_{hu} \le 412$ }

$x_{ij} \ge 0$ B1 1

Accepted introduction of a dummy demand methods.

[8]

4.	(a)	Adds zero for costs in third column	B1	
		Adds 14 as the demand value	B1	2
	(b)	The total supply is greater than the total demand	B2, 1, 0	2
	(c)	The solution would otherwise be degenerate	B2	1
	(d)			

		10	15	0
		J	K	L
0	A		8	1
0	B			13
–6	C	9	3	

$I_{AJ} = 12 – 0 – 10 = 2$ M1 A1

$I_{BJ} = 8 – 0 – 10 = –2$ A1

$I_{BK} = 17 – 0 – 15 = 2$ A1

$I_{CL} = 0 + 6 – 0 = 6$ 4

	J	K	L
A		$8-\theta$	$1+\theta$
B	θ		$13-\theta$
C	$9-\theta$	$3+\theta$	

$\theta = 8$

Entering square BJ M1

Exiting square AK A1ft 2

		8	13	0
		J	K	L
0	A			9
0	B	8		5
–4	C	1	11	

$I_{AJ} = 12 – 0 – 8 = 4$ M1 A1ft

$I_{AK} = 15 – 0 – 13 = 2$ A1ft

$I_{BK} = 17 – 0 – 13 = 4$ A1ft

$I_{CL} = 0 + 4 – 0 = 4$ A1 5

No negatives, so optimal

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5. (a) Row minimums {–2, –1, –4, –2} row maximum = –1 M1

Column maximums {1, 3, 3, 3} column minimum = 1 A1

Since 1 ≠ –1 not stable A1 3

(b) Row 2 dominates Row 3 B1

Column 1 dominates column 4 B1 2

(c) Let A play row R, with probability P_1, R_2 with probability P_2 and "R_3" with probability P_3. B1

$$\begin{pmatrix} -2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{pmatrix} \begin{matrix} \text{eg} \\ \rightarrow \\ +3 \end{matrix} \begin{pmatrix} 1 & 4 & 6 \\ 2 & 6 & 5 \\ 4 & 1 & 2 \end{pmatrix}$$ M1 2

e.g. maximise P = V M1 A1

subject to $V – p_1 – 2p_2 – 4p_3 \leq 0$ A4ft, 3ft, 2ft, 1ft, 0 6

$V – 4p_1 – 6p_2 – p_3 \leq 0$

$V – 6p_1 – 5p_2 – 2p_3 \leq 0$

$p_1 + p_2 - p_3 \le 1$

$V, p_1, p_2, p_3 \ge 0$

OR

e.g. Let $x_i = \frac{p_i}{v}$ $\therefore$ $\frac{1}{v} = x_1 + x_2 + x_3$ M1

minimise $P = x_1 + x_2 + x_3$ A1

subject to $x_1 + 2x_2 + 4x_3 \ge 1$

$4x_1 + 6x_2 + x_3 \ge 1$ A4ft 3ft 2ft 1ft 0 6

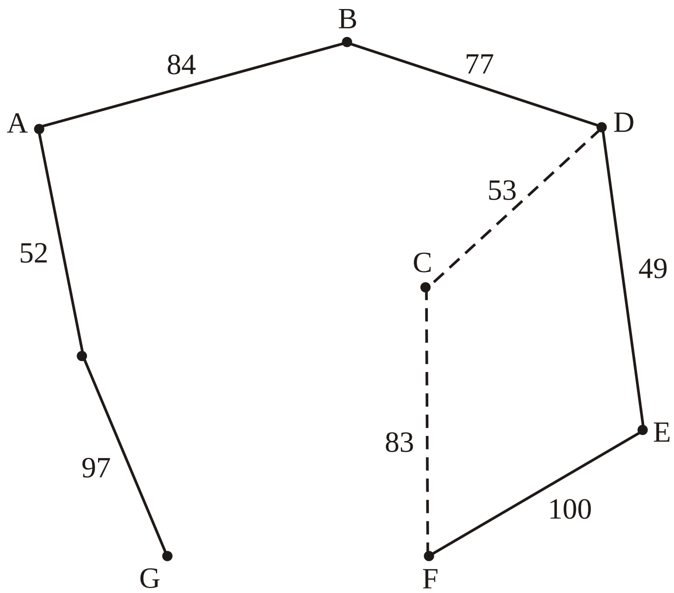
$6x_1 + 5x_1 + 2x_3 \ge 1$

$x_1 + x_2 + x_3 \ge 0$

+ other equivalent methods.

[13]

6. (a)



R.M.S.T

e.g. AH, AB, BD, DE M1

HG, EF using prim A1

length of R M S T = 459

$\therefore$ lower bound = 459 + 53 + 83 = 595 km (deleting c) A1

$\therefore$ Best lower bound is 595 km, by deleting c M1 A1ft 5

(b) Adds 167 to AF and FA

137 to CH and HC B1, 3, 2, 1, 0 4

136 to DF and FD

145 to DG and GD

(c)

C	D	E	F	H	A	B	G	C	M1 A1
53	49	120	115	52	84	222	92		

$\therefore$ Best upper bound is 707 starting at F B1ft 4

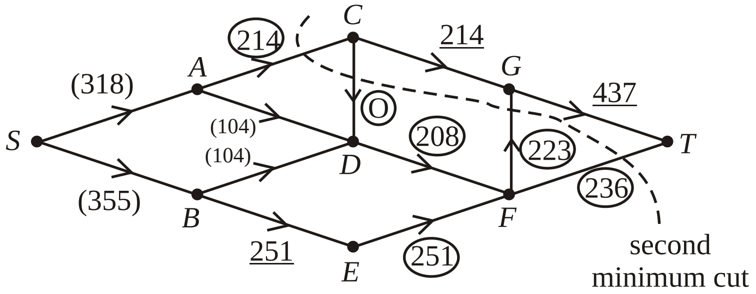
[13]

7. (a) (i) A cut is a division of the vertices of a flow network into 2 sets, one containing the source(s) and the other containing the sink(s). B1

(ii) A cut whose capacity is least B1 2

(b) C_1 = 1038, C_2 = 673 B1, B2, 0 3

(c) e.g.



M1A1A1 3

O = saturated

– = compulsory

(d) AC, CD, GF, FT B1 1

(e) DE would not allow any further flow into EF B1, 1, 0 2

DG would cross both minimum cuts – D can take extra flow, G can accept it. Flow increased by <u>8.6</u> to <u>759</u> (accept either number)

[11]