D2 June 2002 (adapted to include 2002 D1 flows and simplex questions)

1. **Figure 1**

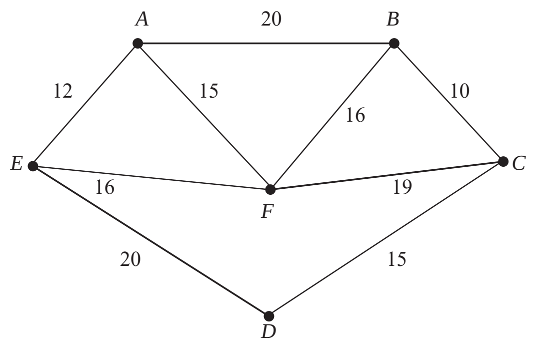


Figure 1 shows a network of roads connecting six villages *A*, *B*, *C*, *D*, *E* and *F*. The lengths of the roads are given in km.

(*a*) Complete the table in the answer booklet, in which the entries are the shortest distances between pairs of villages. You should do this by inspection.

(2)

The table can now be taken to represent a complete network.

(*b*) Use the nearest-neighbour algorithm, starting at *A*, on your completed table in part (*a*). Obtain an upper bound to the length of a tour in this complete network, which starts and finishes at *A* and visits every village exactly once.

(3)

(*c*) Interpret your answer in part (*b*) in terms of the original network of roads connecting the six villages.

(1)

(*d*) By choosing a different vertex as your starting point, use the nearest-neighbour algorithm to obtain a shorter tour than that found in part (*b*). State the tour and its length.

(2)

---

2. A two-person zero-sum game is represented by the following pay-off matrix for player *A*.

		B I	II	III	IV
A	I	–4	–5	–2	4
	II	–1	1	–1	2
	III	0	5	–2	–4
	IV	–1	3	–1	1

(*a*) Determine the play-safe strategy for each player.

(4)

(*b*) Verify that there is a stable solution and determine the saddle points.

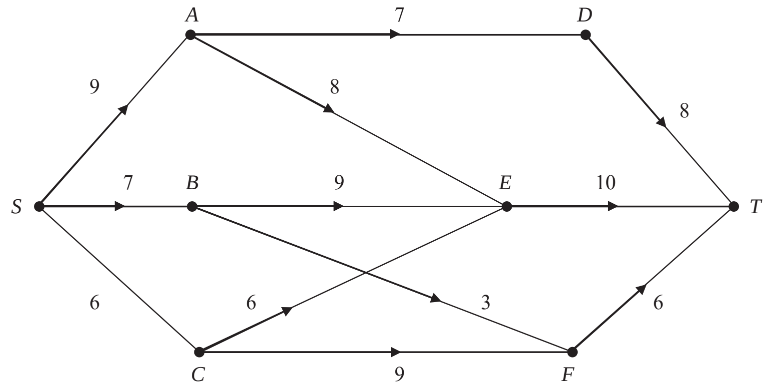
(3)

(*c*) State the value of the game to *B*.

(1)

---

3. **Figure 2**



The network in Fig. 2 shows possible routes that an aircraft can take from S to T. The numbers on the directed arcs give the amount of fuel used on that part of the route, in appropriate units. The airline wishes to choose the route for which the maximum amount of fuel used on any part of the route is as small as possible. This is the rninimax route.

(*a*) Complete the table in the answer booklet.

(8)

(*b*) Hence obtain the minimax route from S to T and state the maximum amount of fuel used on any part of this route.

(2)

---

4. Andrew (A) and Barbara (B) play a zero-sum game. This game is represented by the following pay-off matrix for Andrew.

$$A \overset{B}{\begin{pmatrix} 3 & 5 & 4 \\ 1 & 4 & 2 \\ 6 & 3 & 7 \end{pmatrix}}$$

(*a*) Explain why this matrix may be reduced to

$$\begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}.$$

(*b*) Hence find the best strategy for each player and the value of the game.

(8)

---

5. An engineering company has 4 machines available and 4 jobs to be completed. Each machine is to be assigned to one job. The time, in hours, required by each machine to complete each job is shown in the table below.

	Job 1	Job 2	Job 3	Job 4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

Use the Hungarian algorithm, *reducing rows first*, to obtain the allocation of machines to jobs which minimises the total time required. State this minimum time.

(11)

6. The table below shows the distances, in km, between six towns A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	85	110	175	108	100
B	85	–	38	175	160	93
C	110	38	–	148	156	73
D	175	175	148	–	110	84
E	108	160	156	110	–	92
F	100	93	73	84	92	–

(a) Starting from A, use Prim's algorithm to find a minimum connector and draw the minimum spanning tree. You must make your method clear by stating the order in which the arcs are selected.
(4)

(b) (i) Using your answer to part (a) obtain an initial upper hound for the solution of the travelling salesman problem.

(ii) Use a short cut to reduce the upper bound to a value less than 680.
(4)

(c) Starting by deleting F, find a lower bound for the solution of the travelling salesman problem.
(4)

---

7. A steel manufacturer has 3 factories F_1, F_2 and F_3 which can produce 35, 25 and 15 kilotonnes of steel per year, respectively. Three businesses B_1, B_2 and B_3 have annual requirements of 20, 25 and 30 kilotonnes respectively. The table below shows the cost C_{ij} in appropriate units, of transporting one kilotonne of steel from factory F_i to business B_j.

		Business		
		B_1	B_2	B_3
Factory	F_1	10	4	11
	F_2	12	5	8
	F_3	9	6	7

The manufacturer wishes to transport the steel to the businesses at minimum total cost.

(a) Write down the transportation pattern obtained by using the North-West corner rule.
(2)

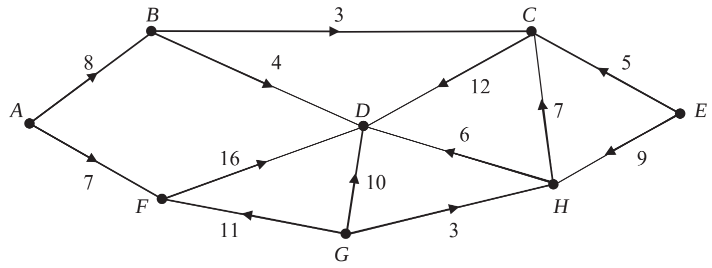
(b) Calculate all of the improvement indices I_{ij}, and hence show that this pattern is not optimal.
(5)

(c) Use the stepping-stone method to obtain an improved solution.
(3)

(d) Show that the transportation pattern obtained in part (c) is optimal and find its cost.

8.

Figure 4



The network in Fig. 4 models a drainage system. The number on each arc indicates the capacity of that arc, in litres per second.

(*a*) Write down the source vertices.

(2)

Figure 5

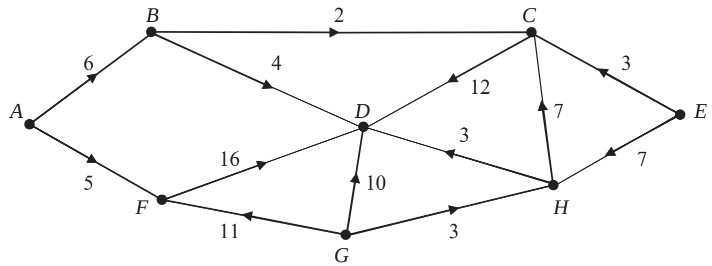


Figure 5 shows a feasible flow through the same network.

(*b*) State the value of the feasible flow shown in Fig. 5.

(1)

Taking the flow in Fig. 5 as your initial flow pattern,

(*c*) use the labelling procedure on Diagram 1 to find a maximum flow through this network. You should list each flow-augmenting route you use, together with its flow.

(6)

(*d*) Show the maximal flow on Diagram 2 and state its value.

(3)

(*e*) Prove that your flow is maximal.

(2)

---

9. T42 Co. Ltd produces three different blends of tea, Morning, Afternoon and Evening. The teas must be processed, blended and then packed for distribution. The table below shows the time taken, in hours, for each stage of the production of a tonne of tea. It also shows the profit, in hundreds of pounds, on each tonne.

	Processing	Blending	Packing	Profit (£100)
Morning blend	3	1	2	4
Afternoon blend	2	3	4	5
Evening blend	4	2	3	3

The total times available each week for processing, blending and packing are 35, 20 and 24 hours respectively. T42 Co. Ltd wishes to maximise the weekly profit.

Let x, y and z be the number of tonnes of Morning, Afternoon and Evening blend produced each week.

(*a*) Formulate the above situation as a linear programming problem, listing clearly the objective function, and the constraints as inequalities.

(4)

An initial Simplex tableau for the above situation is

Basic variable	x	y	z	r	s	t	Value
r	3	2	4	1	0	0	35
s	1	3	2	0	1	0	20
t	2	4	3	0	0	1	24
P	–4	–5	–3	0	0	0	0

(*b*) Solve this linear programming problem using the Simplex algorithm. Take the most negative number in the profit row to indicate the pivot column at each stage.

(11)

T42 Co. Ltd wishes to increase its profit further and is prepared to increase the time available for processing or blending or packing or any two of these three.

(*c*) Use your answer to part (*b*) to advise the company as to which stage(s) it should increase the time available.

(2)

10. While solving a maximizing linear programming problem, the following tableau was obtained.

Basic variable	x	y	z	r	s	t	Value
r	0	0	$1\frac{2}{3}$	1	0	$-\frac{1}{6}$	$\frac{2}{3}$
y	0	1	$3\frac{1}{3}$	0	1	$-\frac{1}{3}$	$\frac{1}{3}$
x	1	0	–3	0	–1	$\frac{1}{2}$	1
P	0	0	1	0	1	1	11

(*a*) Explain why this is an optimal tableau.

(1)

(*b*) Write down the optimal solution of this problem, stating the value of every variable.

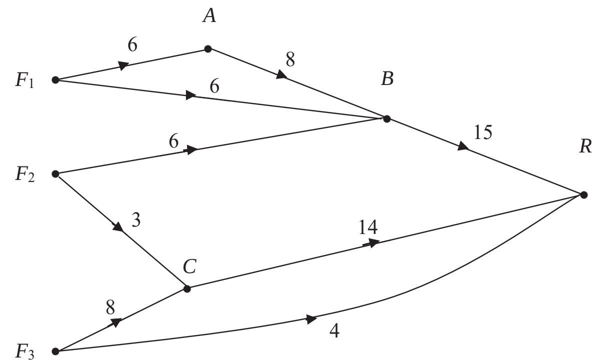
(3)

(*c*) Write down the profit equation from the tableau. Use it to explain why changing the value of any of the non-basic variables will decrease the value of *P*.

(2)

11. A company wishes to transport its products from 3 factories F_1, F_2 and F_3 to a single retail outlet *R*. The capacities of the possible routes, in van loads per day, are shown in Fig. 5.

Figure 5



(*a*) On Diagram 1 in the answer booklet add a supersource *S* to obtain a capacitated network with a single source and a single sink. State the minimum capacity of each arc you have added.

(2)

(*b*) (i) State the maximum flow along SF_1ABR and SF_3CR.

(ii) Show these maximum flows on Diagram 2 in the answer booklet, using numbers in circles.

(2)

Taking your answer to part (*b*)(ii) as the initial flow pattern,

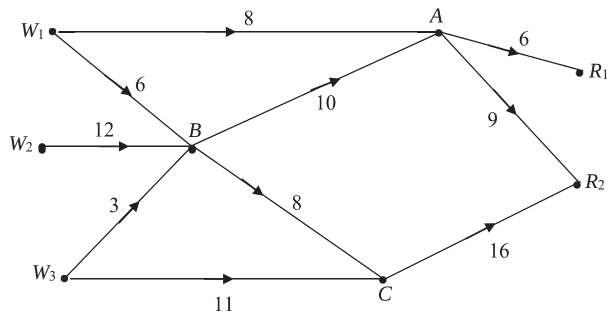
(*c*) (i) use the labelling procedure to find a maximum flow from *S* to *R*.
Your working should be shown on Diagram 3. List each flow-augmenting route you find together with its flow.

(ii) Prove that your final flow is maximal.

(7)

12.

Figure 2



A company has 3 warehouses W_1, W_2, and W_3. It needs to transport the goods stored there to 2 retail outlets R_1 and R_2. The capacities of the possible routes, in van loads per day, are shown in Fig 2. Warehouses W_1, W_2 and W_3 have 14, 12 and 14 van loads respectively available per day and retail outlets R_1 and R_2 can accept 6 and 25 van loads respectively per day.

(*a*) On Diagram 1 on the answer sheet add a supersource *W*, a supersink *R* and the appropriate directed arcs to obtain a single-source, single-sink capacitated network. State the minimum capacity of each arc you have added.

(3)

(*b*) State the maximum flow along

(i) $W \quad W_1 \quad A \quad R_1 \quad R$, (ii) $W \quad W_3 \quad C \quad R_2 \quad R$.

(2)

(*c*) Taking your answers to part (*b*) as the initial flow pattern, use the labelling procedure to obtain a maximum flow through the network from *W* to *R*. Show your working on Diagram 2. List each flow-augmenting route you use, together with its flow.

(5)

(*d*) From your final flow pattern, determine the number of van loads passing through *B* each day.

(1)

D2 2003 (adapted for new spec)

1. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays I	*B* plays II	*B* plays III
A plays I	–3	2	5
A plays II	4	–1	–4

(a) Write down the pay off matrix for player *B*.

(2)

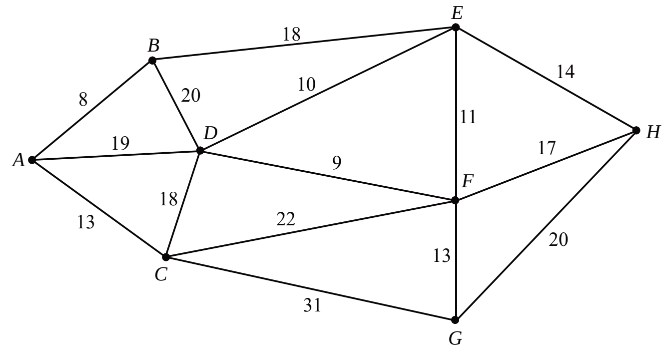
(b) Formulate the game as a linear programming problem for player *B*, writing the constraints as equalities and stating your variables clearly.

(4)

(Total 6 marks)

2. (a) Explain the difference between the classical and practical travelling salesman problems.

(2)



The network in the diagram above shows the distances, in kilometres, between eight McBurger restaurants. An inspector from head office wishes to visit each restaurant. His route should start and finish at *A*, visit each restaurant at least once and cover a minimum distance.

(b) Obtain a minimum spanning tree for the network using Kruskal's algorithm. You should draw your tree and state the order in which the arcs were added.

(3)

(c) Use your answer to part (b) to determine an initial upper bound for the length of the route.

(2)

(d) Starting from your initial upper bound and using an appropriate method, find an upper bound which is less than 135 km. State your tour.

(3)

(Total 10 marks)

3. Talkalot College holds an induction meeting for new students. The meeting consists of four talks: I (Welcome), II (Options and Facilities), III (Study Tips) and IV (Planning for Success). The four department heads, Clive, Julie, Nicky and Steve, deliver one of these talks each. The talks are delivered consecutively and there are no breaks between talks. The meeting starts at 10 a.m. and ends when all four talks have been delivered. The time, in minutes, each department head takes to deliver each talk is given in the table below.

	Talk I	Talk II	Talk III	Talk IV
Clive	12	34	28	16
Julie	13	32	36	12
Nicky	15	32	32	14
Steve	11	33	36	10

(a) Use the Hungarian algorithm to find the earliest time that the meeting could end. You must make your method clear and show

(i) the state of the table after each stage in the algorithm,

(ii) the final allocation.

(10)

(b) Modify the table so it could be used to find the latest time that the meeting could end.

(3)

(Total 13 marks)

4. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays I	*B* plays II	*B* plays III
A plays I	2	–1	3
A plays II	1	3	0
A plays III	0	1	–3

(a) Identify the play safe strategies for each player.

(4)

(b) Verify that there is no stable solution to this game.

(1)

(c) Explain why the pay-off matrix above may be reduced to

	B plays I	*B* plays II	*B* plays III
A plays I	2	–1	3
A plays II	1	3	0

(2)

(d) Find the best strategy for player *A*, and the value of the game.

(7)

(Total 14 marks)

5. The manager of a car hire firm has to arrange to move cars from three garages *A*, *B* and *C* to three airports *D*, *E* and *F* so that customers can collect them. The table below shows the transportation cost of moving one car from each garage to each airport. It also shows the number of cars available in each garage and the number of cars required at each airport. The total number of cars available is equal to the total number required.

	Airport *D*	Airport *E*	Airport *F*	Cars available
Garage *A*	£20	£40	£10	6
Garage *B*	£20	£30	£40	5
Garage *C*	£10	£20	£30	8
Cars required	6	9	4	

(a) Use the North-West corner rule to obtain a possible pattern of distribution and find its cost.

(3)

(b) Calculate shadow costs for this pattern and hence obtain improvement indices for each route.

(4)

(c) Use the stepping-stone method to obtain an optimal solution and state its cost.

(7)

(Total 14 marks)

6. Kris produces custom made racing cycles. She can produce up to four cycles each month, but if she wishes to produce more than three in any one month she has to hire additional help at a cost of £350 for that month. In any month when cycles are produced, the overhead costs are £200. A maximum of 3 cycles can be held in stock in any one month, at a cost of £40 per cycle per month. Cycles must be delivered at the end of the month. The order book for cycles is

Month	August	September	October	November
Number of cycles required	3	3	5	2

Disregarding the cost of parts and Kris' time,

(a) determine the total cost of storing 2 cycles and producing 4 cycles in a given month, making your calculations clear.

(2)

There is no stock at the beginning of August and Kris plans to have no stock after the November delivery.

(b) Use dynamic programming to determine the production schedule which minimises the costs, showing your working in the table below.

Stage	Demand	State	Action	Destination	Value
1 (Nov)	2	0 (in stock)	(make) 2	0	200
		1 (in stock)	(make) 1	0	240
		2 (in stock)	(make) 0	0	80
2 (Oct)	5	1	4	0	590 + 200 = 790
		2	3	0	
			4	1	

(13)

The fixed cost of parts is £600 per cycle and of Kris' time is £500 per month. She sells the cycles for £2000 each.

(c) Determine her total profit for the four month period.

(3)
(Total 18 marks)

7.

Figure 1

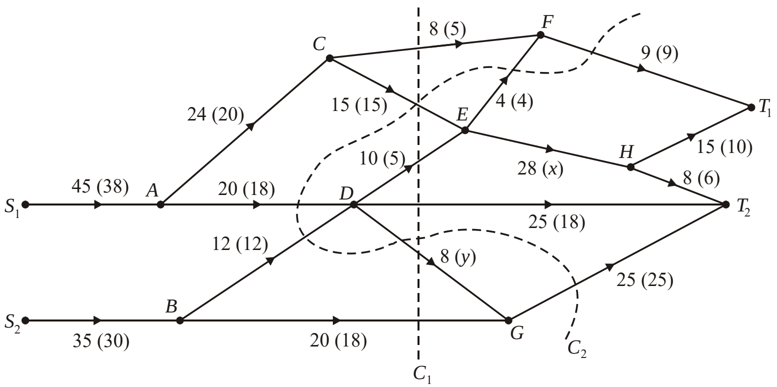
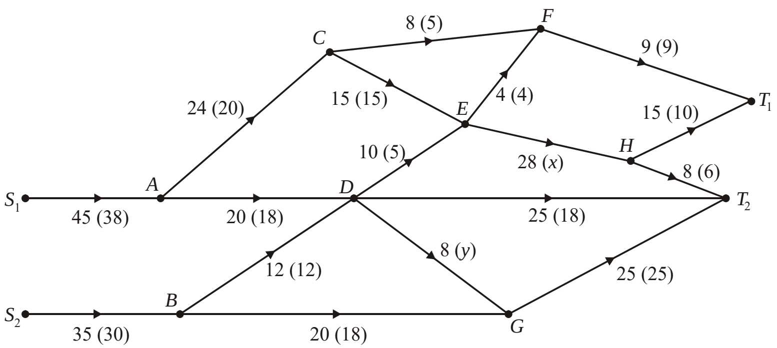


Figure 1 shows a capacitated, directed network. The unbracketed number on each arc indicates the capacity of that arc, and the numbers in brackets show a feasible flow of value 68 through the network.

(a) Add a supersource and a supersink, and arcs of appropriate capacity, to Diagram 1 below.

Diagram 1



(2)

(b) Find the values of x and y, explaining your method briefly.

(2)

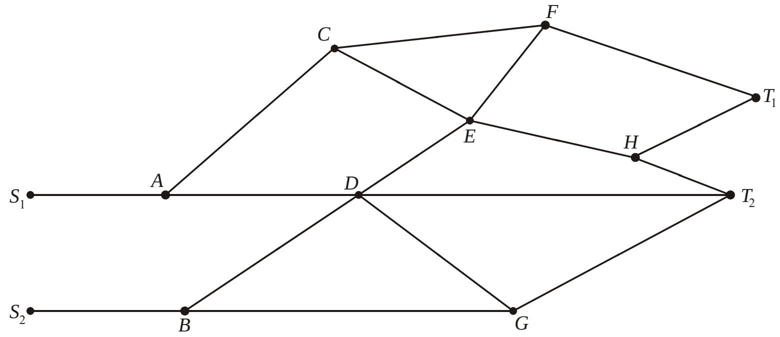
(c) Find the value of cuts C_1 and C_2.

(3)

Starting with the given feasible flow of 68,

(d) use the labelling procedure on Diagram 2 to find a maximal flow through this network. List each flow-augmenting route you use, together with its flow.

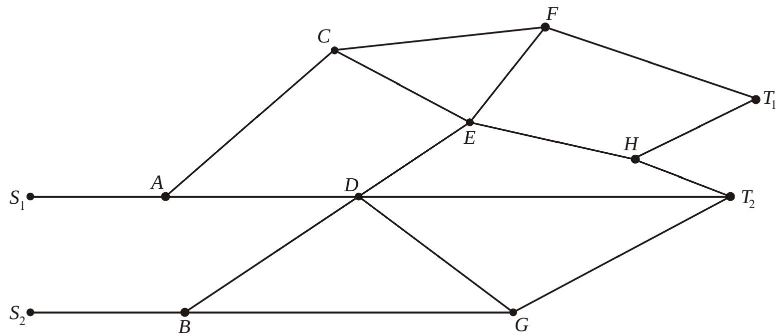
Diagram 2



(6)

(e) Show your maximal flow on Diagram 3 and state its value.

Diagram 3



(3)

(f) Prove that your flow is maximal.

(2)
(Total 18 marks)

8. The tableau below is the initial tableau for a maximising linear programming problem.

Basic variable	x	y	z	r	s	Value
r	2	3	4	1	0	8
s	3	3	1	0	1	10
P	–8	–9	–5	0	0	0

(a) For this problem $x \geq 0, y \geq 0, z \geq 0$. Write down the other two inequalities and the objective function.
(3)

(b) Solve this linear programming problem.

You may not need to use all of these tableaux.

b.v.	x	y	z	r	s	Value
P						

b.v.	x	y	z	r	s	Value
P						

-

b.v.	x	y	z	r	s	Value
P						

b.v.	x	y	z	r	s	Value
P						

(8)

(c) State the final value of P, the objective function, and of each of the variables.
(3)
(Total 14 marks)

D2 2004 (adapted for new spec)

1. In game theory explain what is meant by

(a) zero-sum game, (b) saddle point.

(Total 4 marks)

2. In a quiz there are four individual rounds, Art, Literature, Music and Science. A team consists of four people, Donna, Hannah, Kerwin and Thomas. Each of four rounds must be answered by a different team member. The table shows the number of points that each team member is likely to get on each individual round.

	Art	Literature	Music	Science
Donna	31	24	32	35
Hannah	16	10	19	22
Kerwin	19	14	20	21
Thomas	18	15	21	23

Use the Hungarian algorithm, reducing rows first, to obtain an allocation which maximises the total points likely to be scored in the four rounds. You must make your method clear and show the table after each stage.

(Total 9 marks)

3. The table shows the least distances, in km, between five towns, A, B, C, D and E.

	A	B	C	D	E
A	–	153	98	124	115
B	153	–	74	131	149
C	98	74	–	82	103
D	124	131	82	–	134
E	115	149	103	134	–

Nassim wishes to find an interval which contains the solution to the travelling salesman problem for this network.

(a) Making your method clear, find an initial upper bound starting at A and using

(i) the minimum spanning tree method,

(ii) the nearest neighbour algorithm.

(7)

(b) By deleting E, find a lower bound.
(4)

(c) Using your answers to parts (a) and (b), state the smallest interval that Nassim could correctly write down.
(1)
(Total 12 marks)

4. Emma and Freddie play a zero-sum game. This game is represented by the following pay-off matrix for Emma.

$$\begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$$

(a) Show that there is no stable solution.
(3)

(b) Find the best strategy for Emma and the value of the game to her.
(8)

(c) Write down the value of the game to Freddie and his pay-off matrix.
(3)
(Total 14 marks)

5. (a) Describe a practical problem that could be solved using the transportation algorithm.

(2)

A problem is to be solved using the transportation problem. The costs are shown in the table. The supply is from *A*, *B* and *C* and the demand is at *d* and *e*.

	d	*e*	Supply
A	5	3	45
B	4	6	35
C	2	4	40
Demand	50	60	

(b) Explain why it is necessary to add a third demand *f*.

(1)

(c) Use the north-west corner rule to obtain a possible pattern of distribution and find its cost.

	d	*e*	*f*	Supply
A	5	3		45
B	4	6		35
C	2	4		40
Demand	50	60		

(5)

(d) Calculate shadow costs and improvement indices for this pattern.

(5)

(e) Use the stepping-stone method once to obtain an improved solution and its cost.

(5)
(Total 16 marks)

6. Joan sells ice cream. She needs to decide which three shows to visit over a three-week period in the summer. She starts the three-week period at home and finishes at home. She will spend one week at each of the three shows she chooses travelling directly from one show to the next.

Table 1 gives the week in which each show is held. Table 2 gives the expected profits from visiting each show. Table 3 gives the cost of travel between shows.

Table 1

Week	1	2	3
Shows	A, B, C	D, E	F, G, H

Table 2

Show	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Expected Profit (£)	900	800	1000	1500	1300	500	700	600

Table 3

Travel costs (£)	A	B	C	D	E	F	G	H
Home	70	80	150			80	90	70
A				180	150			
B				140	120			
C				200	210			
D						200	160	120
E						170	100	110

It is decided to use dynamic programming to find a schedule that maximises the total expected profit, taking into account the travel costs.

(a) Define suitable stage, state and action variables.

(3)

(b) Determine the schedule that maximises the total profit. Show your working in a table.

(12)

(c) Advise Joan on the shows that she should visit and state her total expected profit.

(3) (Total 18 marks)

7.

Figure 1

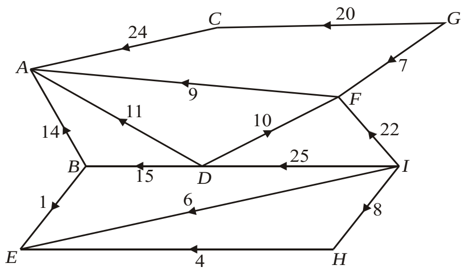


Figure 1 shows a capacitated directed network. The number on each arc is its capacity.

Figure 2

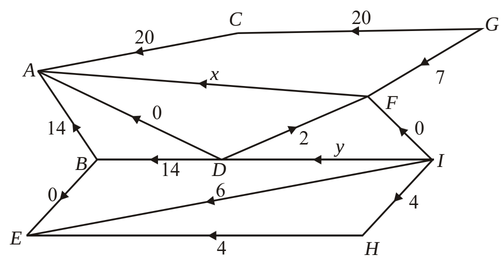


Figure 2 shows a feasible initial flow through the same network.

(a) Write down the values of the flow x and the flow y.

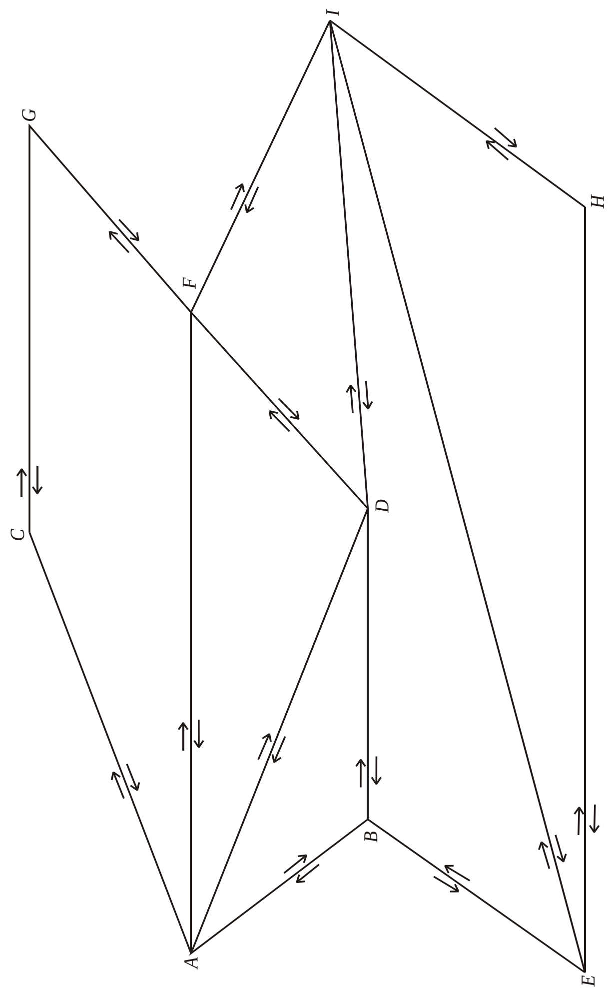
(2)

(b) Obtain the value of the initial flow through the network, and explain how you know it is not maximal.

(2)

(c) Use this initial flow and the labelling procedure on Diagram 1 below to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

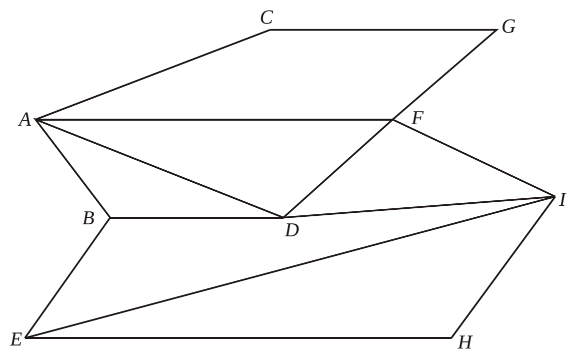
Diagram 1



(5)

d) Show your maximal flow pattern on Diagram 2.

Diagram 2



(2)

(e) Prove that your flow is maximal.

(2)
(Total 13 marks)

8. A three-variable linear programming problem in x, y and z is to be solved. The objective is to maximise the profit P. The following tableau was obtained.

Basic variable	x	y	Z	r	s	t	Value
s	3	0	2	0	1	$-\frac{2}{3}$	$\frac{2}{3}$
r	4	0	$\frac{7}{2}$	1	0	8	$\frac{9}{2}$
y	5	1	7	0	0	3	7
P	3	0	2	0	0	8	63

(a) State, giving your reason, whether this tableau represents the optimal solution.

(1)

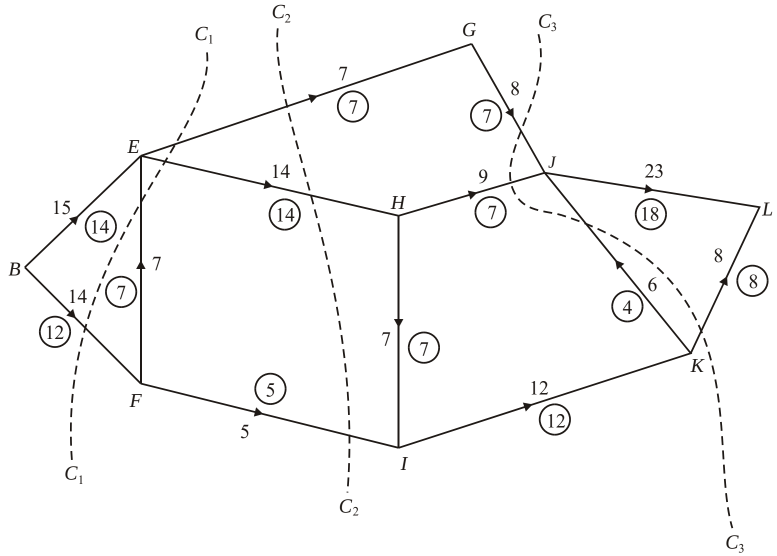
(b) State the values of every variable.

(3)

(c) Calculate the profit made on each unit of y.

(2)
(Total 6 marks)

9.



The diagram above shows a network of roads represented by arcs. The capacity of the road represented by that arc is shown on each arc. The numbers in circles represent a possible flow of 26 from *B* to *L*.

Three cuts C_1, C_2 and C_3 are shown on The diagram above.

(a) Find the capacity of each of the three cuts. **(3)**

(b) Verify that the flow of 26 is maximal. **(1)**

The government aims to maximise the possible flow from *B* to *L* by using one of two options.

Option 1: Build a new road from *E* to *J* with capacity 5.

or Option 2: Build a new road from *F* to *H* with capacity 3.

(c) By considering **both** options, explain which one meets the government's aim **(3)**

10. Flatland UK Ltd makes three types of carpet, the Lincoln, the Norfolk and the Suffolk. The carpets all require units of black, green and red wool.

For each roll of carpet,
the Lincoln requires 1 unit of black, 1 of green and 3 of red,
the Norfolk requires 1 unit of black, 2 of green and 2 of red,
and the Suffolk requires 2 units of black, 1 of green and 1 of red.

There are up to 30 units of black, 40 units of green and 50 units of red available each day.
Profits of £50, £80 and £60 are made on each roll of Lincoln, Norfolk and Suffolk respectively. Flatland UK Ltd wishes to maximise its profit.

Let the number of rolls of the Lincoln, Norfolk and Suffolk made daily be *x*, *y* and *z* respectively.

(a) Formulate the above situation as a linear programming problem, listing clearly the constraints as inequalities in their simplest form, and stating the objective function. **(4)**

This problem is to be solved using the Simplex algorithm. The most negative number in the profit row is taken to indicate the pivot column at each stage.

(b) Stating your row operations, show that after one complete iteration the tableau becomes

Basic variable	x	y	z	r	s	t	Value
r	$\frac{1}{2}$	0	$1\frac{1}{2}$	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	–1	1	10
P	–10	0	–20	0	40	0	1600

(4)

You may not need to use all of the tableaux.

Basic variable	x	y	z	r	s	t	Value	Row operations
r								
s								
t								
P								

-

Basic variable	x	y	z	r	s	t	Value	Row operations

-

(c) Explain the practical meaning of the value 10 in the top row.

(2)

(d) (i) Perform one further complete iteration of the Simplex algorithm.

Basic variable	x	y	z	r	s	t	Value	Row operations

Basic variable	x	y	z	r	s	t	Value	Row operations

(ii) State whether your current answer to part (d)(i) is optimal. Give a reason for your answer.

(iii) Interpret your current tableau, giving the value of each variable.

(8)
(Total 18 marks)

D2 2005 (adapted for new spec)

1. Freezy Co. has three factories *A*, *B* and *C*. It supplies freezers to three shops *D*, *E* and *F*. The table shows the transportation cost in pounds of moving one freezer from each factory to each outlet. It also shows the number of freezers available for delivery at each factory and the number of freezers required at each shop. The total number of freezers required is equal to the total number of freezers available.

	D	*E*	*F*	Available
A	21	24	16	24
B	18	23	17	32
C	15	19	25	14
Required	20	30	20	

(a) Use the north-west corner rule to find an initial solution.

(2)

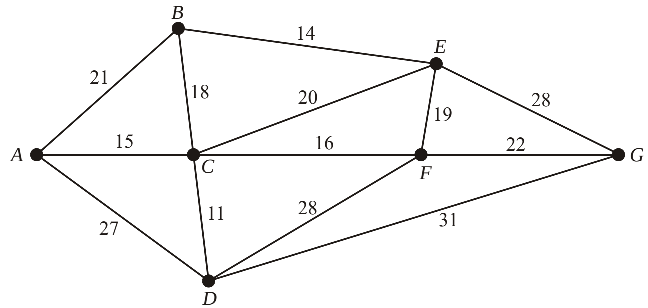
(b) Obtain improvement indices for each unused route.

(5)

(c) Use the stepping-stone method **once** to obtain a better solution and state its cost.

(4)
(Total 11 marks)

2.



The network in the diagram shows the distances, in km, of the cables between seven electricity relay stations *A*, *B*, *C*, *D*, *E*, *F* and *G*. An inspector needs to visit each relay station. He wishes to travel a minimum distance, and his route must start and finish at the same station.

By deleting *C*, a lower bound for the length of the route is found to be 129 km.

(a) Find another lower bound for the length of the route by deleting *F*. State which is the better lower bound of the two.

(5)

(b) By inspection, complete the table of least distances.

(2)

The table can now be taken to represent a complete network.

(c) Using the nearest-neighbour algorithm, starting at *F*, obtain an upper bound to the length of the route. State your route.

(4) (Total 11 marks)

3. Three warehouses *W*, *X* and *Y* supply televisions to three supermarkets *J*, *K* and *L*. The table gives the cost, in pounds, of transporting a television from each warehouse to each supermarket. The warehouses have stocks of 34, 57 and 25 televisions respectively, and the supermarkets require 20, 56 and 40 televisions respectively. The total cost of transporting the televisions is to be minimised.

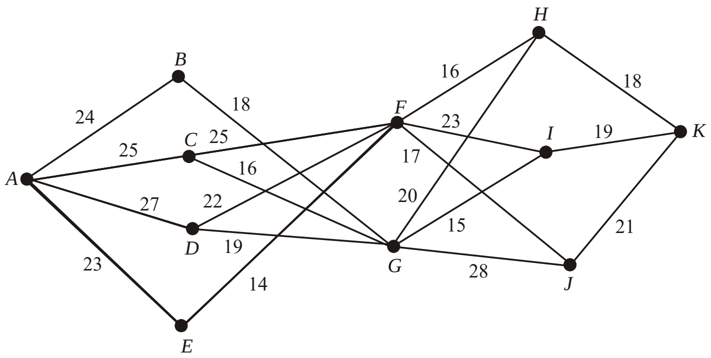
	J	*K*	*L*
W	3	6	3
X	5	8	4
Y	2	5	7

Formulate this transportation problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 7 marks)

4. (a) Explain what is meant by a maximin route in dynamic programming, and give an example of a situation that would require a maximin solution.

(3)



A maximin route is to be found through the network shown in the diagram.

(b) Complete the table in the answer book, and hence find a maximin route.

(9)

(c) List **all** other maximin routes through the network.

(2)

(Total 14 marks)

5. Four salesperson *A*, *B*, *C* and *D* are to be sent to visit four companies 1, 2, 3 and 4. Each salesperson will visit exactly one company, and all companies will be visited.

Previous sales figures show that each salesperson will make sales of different values, depending on the company that they visit. These values (in £10 000s) are shown in the table below.

	1	2	3	4
Ann	26	30	30	30
Brenda	30	23	26	29
Connor	30	25	27	24
Dave	30	27	25	21

(a) Use the Hungarian algorithm to obtain an allocation that **maximises** the sales. You must make your method clear and show the table after each stage.

(11)

(b) State the value of the maximum sales.

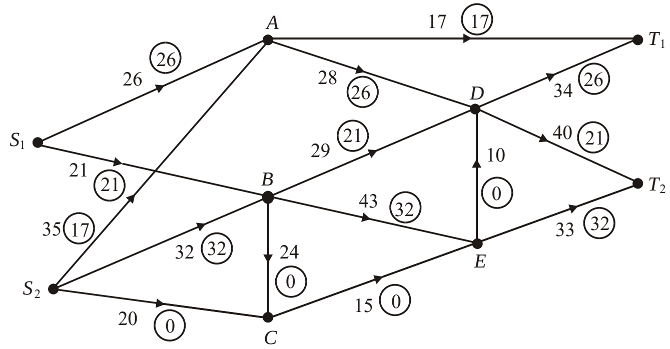
(2)

(c) Show that there is a second allocation that maximises the sales.

(2)

(Total 15 marks)

6.



This figure shows a capacitated directed network. The number on each arc is its capacity. The numbers in circles show a feasible flow through the network. **Take this as the initial flow.**

(a) On Diagram 1 **and** Diagram 2 in the answer book, add a supersource *S* and a supersink *T*. On Diagram 1 show the minimum capacities of the arcs you have added.

(2)

Diagram 2 in the answer book shows the first stage of the labelling procedure for the given initial flow.

(b) Complete the initial labelling procedure in Diagram 2.

(2)

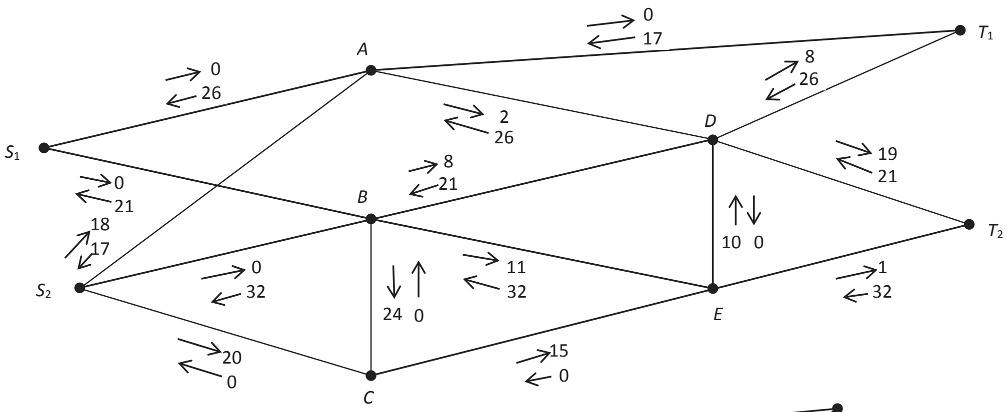
(c) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow, and state the maximal flow.

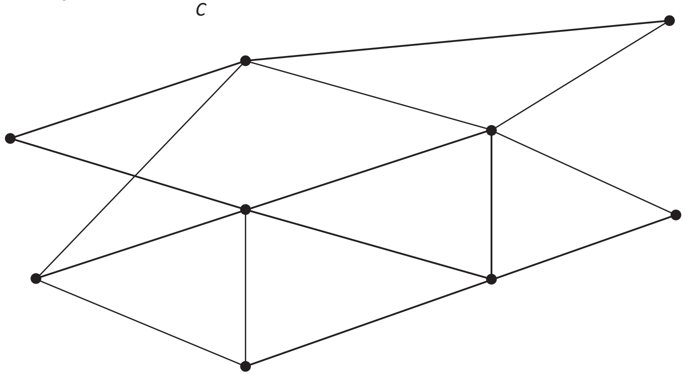
(6)

(d) Show a maximal flow pattern on Diagram 3.
(2)

(e) Prove that your flow is maximal.
(2)

(f) Describe briefly a situation for which this network could be a suitable model.
(2)
(Total 16 marks)





7. (a) Explain briefly what is meant by a **zero-sum** game.
(1)

A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	I	II	III
I	5	2	3
II	3	5	4

(b) Verify that there is no stable solution to this game.
(3)

(c) Find the best strategy for player *A* and the value of the game to her.
(8)

(d) Formulate the game as a linear programming problem for player *B*. Write the constraints as inequalities and define your variables clearly.
(5)
(Total 17 marks)

8. Polly has a bird food stall at the local market. Each week she makes and sells three types of packs *A*, *B* and *C*.

Pack *A* contains 4 kg of bird seed, 2 suet blocks and 1 kg of peanuts.

Pack *B* contains 5 kg of bird seed, 1 suet block and 2 kg of peanuts.

Pack *C* contains 10 kg of bird seed, 4 suet blocks and 3 kg of peanuts.

Each week Polly has 140 kg of bird seed, 60 suet blocks and 60 kg of peanuts available for the packs.

The profit made on each pack of *A*, *B* and *C* sold is £3.50, £3.50 and £6.50 respectively. Polly sells every pack on her stall and wishes to maximise her profit, *P* pence.

Let *x*, *y* and *z* be the numbers of packs *A*, *B* and *C* sold each week.

An initial Simplex tableau for the above situation is

Basic variable	x	y	z	r	s	t	Value
r	4	5	10	1	0	0	140
s	2	1	4	0	1	0	60
t	1	2	3	0	0	1	60
P	–350	–350	–650	0	0	0	0

(a) Explain the meaning of the variables *r*, *s* and *t* in the context of this question.
(2)

(b) Perform one complete iteration of the Simplex algorithm to form a new tableau *T*. Take the most negative number in the profit row to indicate the pivotal column.
(5)

(c) State the value of every variable as given by tableau *T*.
(3)

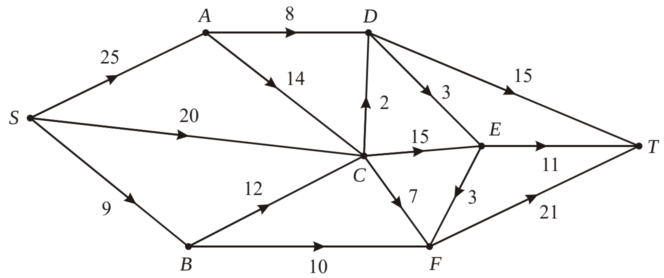
(d) Write down the profit equation given by tableau *T*.
(2)

(e) Use your profit equation to explain why tableau *T* is not optimal.
(1)

Taking the most negative number in the profit row to indicate the pivotal column,

(f) identify clearly the location of the next pivotal element.
(2)
(Total 15 marks)

9.



This diagram shows a capacitated directed network. The number on each arc is its capacity.

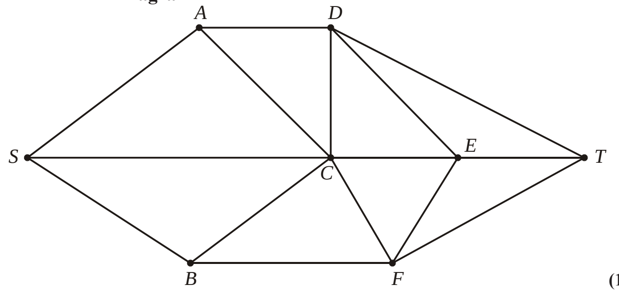
(a) State the maximum flow along

(i) *SADT*, (ii) *SCET*, (iii) *SBFT*.

(2)

(b) Show these maximum flows on Diagram 1 below.

Diagram 1

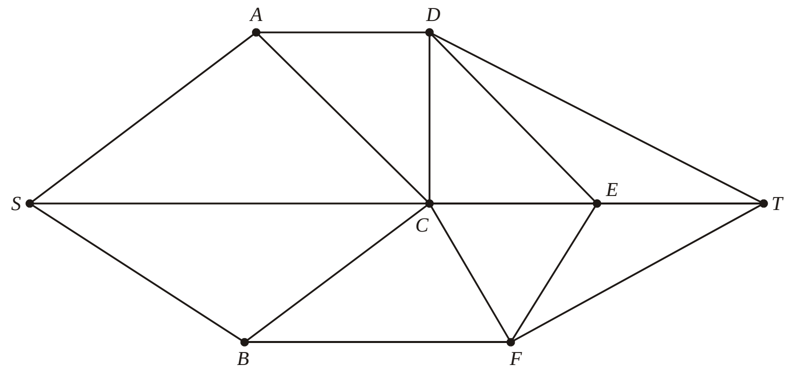


(1)

Take your answer to part (*b*) as the initial flow pattern.

(c) (i) Use the labelling procedure to find a maximum flow from *S* to *T*. Your working should be shown on Diagram 2 below. List each flow-augmenting route you use, together with its flow.

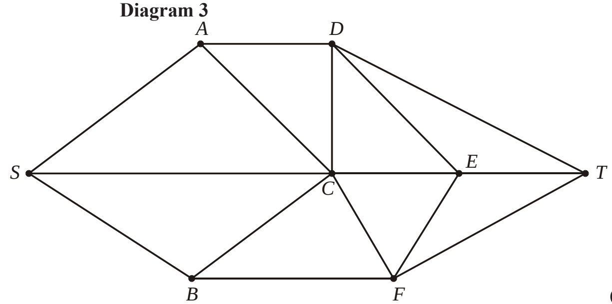
Diagram 2



(5)

(ii) Draw your final flow pattern on Diagram 3 below.

Diagram 3



(2)

(iii) Prove that your flow is maximal.

(3)

(d) Give an example of a practical situation that could have been modelled by the original network.

(1)

(Total 14 marks)

D2 Jan 2006 (adapted for new spec)

1. A theme park has four sites, A, B, C and D, on which to put kiosks. Each kiosk will sell a different type of refreshment. The income from each kiosk depends upon what it sells and where it is located. The table below shows the expected daily income, in pounds, from each kiosk at each site.

	Hot dogs and beef burgers (H)	Ice cream (I)	Popcorn, candyfloss and drinks (P)	Snacks and hot drinks (S)
Site A	267	272	276	261
Site B	264	271	278	263
Site C	267	273	275	263
Site D	261	269	274	257

Reducing rows first, use the Hungarian algorithm to determine a site for each kiosk in order to maximise the total income. State the site for each kiosk and the total expected income. You must make your method clear and show the table after each stage.

(Total 13 marks)

2. An engineering firm makes motors. They can make up to five in any one month, but if they make more than four they have to hire additional premises at a cost of £500 per month. They can store up to two motors for £100 per motor per month. The overhead costs are £200 in any month in which work is done.
Motors are delivered to buyers at the end of each month. There are no motors in stock at the beginning of May and there should be none in stock after the September delivery.

The order book for motors is:

Month	May	June	July	August	September
Number of motors	3	3	7	5	4

Use dynamic programming to determine the production schedule that minimises the costs, showing your working in the table provided below.

Stage (month)	State (Number in store at start of month)	Action (Number made in month)	Destination (Number in store at end of month)	Value (cost)

Production schedule

Month	**May**	**June**	**July**	**August**	**September**
Number to be made					

Total cost: ....

(Total 12 marks)

3. Three depots, F, G and H, supply petrol to three service stations, S, T and U. The table gives the cost, in pounds, of transporting 1000 litres of petrol from each depot to each service station.

	S	T	U
F	23	31	46
G	35	38	51
H	41	50	63

F, G and H have stocks of 540 000, 789 000 and 673 000 litres respectively.

S, T and U require 257 000, 348 000 and 412 000 litres respectively. The total cost of transporting the petrol is to be minimised.

Formulate this problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(Total 8 marks)

4. The following minimising transportation problem is to be solved.

	J	K	Supply
A	12	15	9
B	8	17	13
C	4	9	12
Demand	9	11	

(a) Complete the table below.

	J	K	L	Supply
A	12	15		9
B	8	17		13
C	4	9		12
Demand	9	11		34

(2)

(b) Explain why an extra demand column was added to the table above.

(2)

A possible north-west corner solution is:

	J	K	L
A	9	0	
B		11	2
C			12

(c) Explain why it was necessary to place a zero in the first row of the second column.

(1)

After three iterations of the stepping-stone method the table becomes:

	J	K	L
A		8	1
B			13
C	9	3	

(d) Taking the most negative improvement index as the entering square for the stepping stone method, solve the transportation problem. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal.

(11) (Total 16 marks)

5. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	–2	1	3	–1
A plays 2	–1	3	2	1
A plays 3	–4	2	0	–1
A plays 4	1	–2	–1	3

(a) Verify that there is no stable solution to this game.

(3)

(b) Explain why the 4 × 4 game above may be reduced to the following 3 × 3 game.

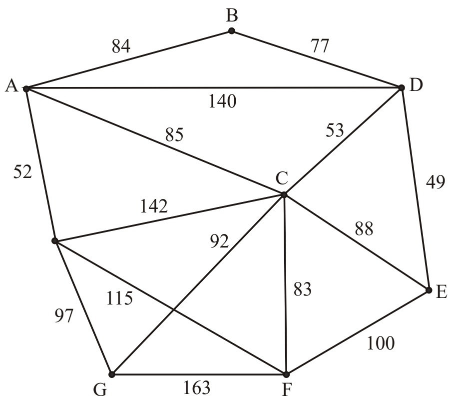
–2	1	3
–1	3	2
1	–2	–1

(2)

(c) Formulate the 3 × 3 game as a linear programming problem for player A. Write the constraints as inequalities. Define your variables clearly.

(8)
(Total 13 marks)

6. The network in the figure above, shows the distances in km, along the roads between eight towns, A, B, C, D, E, F, G and H. Keith has a shop in each town and needs to visit each one. He wishes to travel a minimum distance and his route should start and finish at A.



By deleting D, a lower bound for the length of the route was found to be 586 km.
By deleting F, a lower bound for the length of the route was found to be 590 km.

(a) By deleting C, find another lower bound for the length of the route. State which is the best lower bound of the three, giving a reason for your answer.

(5)

(b) By inspection complete the table of least distances.

(4)

	A	B	C	D	E	F	G	H
A	-	84	85	138	173		149	52
B	84	-	130	77	126	213	222	136
C	85	130	-	53	88	83	92	
D	138	77	53	-	49			190
E	173	126	88	49	-	100	180	215
F		213	83		100	-	163	115
G	149	222	92		180	163	-	97
H	52	136		190	215	115	97	-

The table can now be taken to represent a complete network.

The nearest neighbour algorithm was used to obtain upper bounds for the length of the route: Starting at D, an upper bound for the length of the route was found to be 838 km.
Starting at F, an upper bound for the length of the route was found to be 707 km.

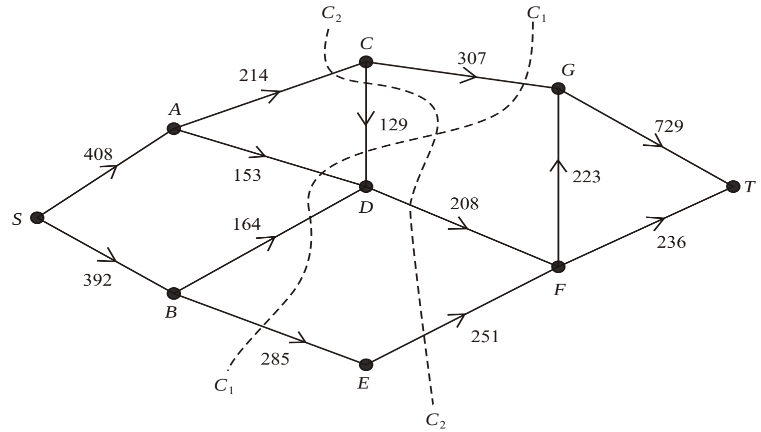
(c) Starting at C, use the nearest neighbour algorithm to obtain another upper bound for the length of the route. State which is the best upper bound of the three, giving a reason for your answer.

(4) (Total 13 marks)

7. (a) Define the terms (i) cut, (ii) minimum cut,

as applied to a directed network flow.

(2)



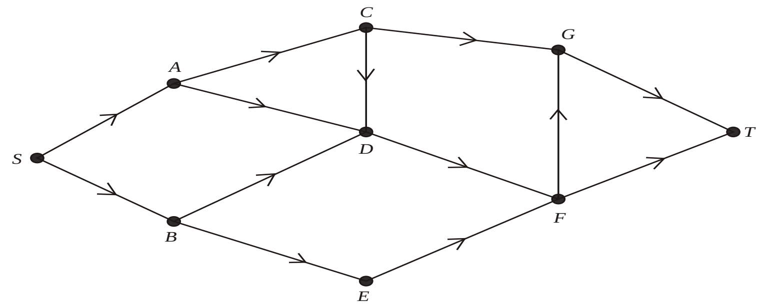
The figure above shows a capacitated directed network and two cuts C_1 and C_2. The number on each arc is its capacity.

(b) State the values of the cuts C_1 and C_2.

(3)

Given that one of these two cuts is a minimum cut,

(c) find a maximum flow pattern by inspection, and show it on the diagram below.



(3)

(d) Find a second minimum cut for this network.

(1)

In order to increase the flow through the network it is decided to add an arc of capacity 100 joining *D* **either** to *E* **or** to *G*.

(e) State, with a reason, which of these arcs should be added, and the value of the increased flow.

(2)
(Total 11 marks)

D2 June 2006 (adapted for new spec)

1. (a) State Bellman's principle of optimality.

(1)

(b) Explain what is meant by a minimax route.

(1)

(c) Describe a practical problem that would require a minimax route as its solution.

(2)
(Total 4 marks)

2. Three workers, *P*, *Q* and *R*, are to be assigned to three tasks, 1, 2 and 3. Each worker is to be assigned to one task and each task must be assigned to one worker. The cost, in hundreds of pounds, of using each worker for each task is given in the table below. The cost is to be minimised.

Cost (in £100s)	Task 1	Task 2	Task 3
Worker *P*	8	7	3
Worker *Q*	9	5	6
Worker *R*	10	4	4

Formulate the above situation as a linear programming problem, defining the decision variables and making the objective and constraints clear.

(Total 7 marks)

3. A college wants to offer five full-day activities with a different activity each day from Monday to Friday. The sports hall will only be used for these activities. Each evening the caretaker will prepare the hall by putting away the equipment from the previous activity and setting up the hall for the activity next day. On Friday evening he will put away the equipment used that day and set up the hall for the following Monday.

The 5 activities offered are Badminton (*B*), Cricket nets (*C*), Dancing (*D*), Football coaching (*F*) and Tennis (*T*). Each will be on the same day from week to week.

The college decides to offer the activities in the order that minimises the total time the caretaker has to spend preparing the hall each week.

The hall is initially set up for Badminton on Monday.

The table below shows the time, in minutes, it will take the caretaker to put away the equipment from one activity and set up the hall for the next.

		To				
	Time	*B*	*C*	*D*	*F*	*T*
From	*B*	–	108	150	64	100
	C	108	–	54	104	60
	D	150	54	–	150	102
	F	64	104	150	–	68
	T	100	60	102	68	–

(a) Explain why this problem is equivalent to the travelling salesman problem.

(2)

A possible ordering of activities is

Monday	Tuesday	Wednesday	Thursday	Friday
B	*C*	*D*	*F*	*T*

(b) Find the total time taken by the caretaker each week using this ordering.

(2)

(c) Starting with Badminton on Monday, use a suitable algorithm to find an ordering that reduces the total time spent each week to less than 7 hours.

(3)

(d) By deleting *B*, use a suitable algorithm to find a lower bound for the time taken each week. Make your method clear.

(4)
(Total 11 marks)

4. During the school holidays four building tasks, rebuilding a wall (*W*), repairing the roof (*R*), repainting the hall (*H*) and relaying the playground (*P*), need to be carried out at a Junior School.

Four builders, *A*, *B*, *C* and *D* will be hired for these tasks. Each builder must be assigned to one task. Builder *B* is not able to rebuild the wall and therefore cannot be assigned to this task.

The cost, in thousands of pounds, of using each builder for each task is given in the table below.

Cost	*H*	*P*	*R*	*W*
A	3	5	11	9
B	3	7	8	–
C	2	5	10	7
D	8	3	7	6

(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that minimises the total cost. State the allocation and its total cost. You must make your method clear and show the table after each stage.

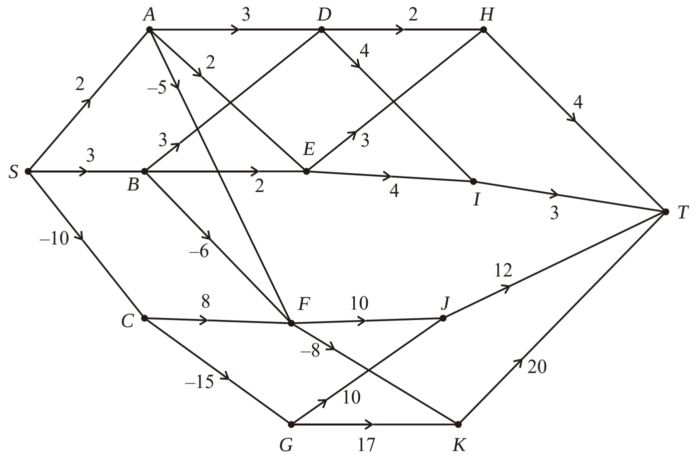
(9)

(b) State, with a reason, whether this allocation is unique.

(2)
(Total 11 marks)

5. Victor owns some kiosks selling ice cream, hot dogs and soft drinks.

The network below shows the choices of action and the profits, in thousands of pounds, they generate over the next four years. The negative numbers indicate losses due to the purchases of new kiosks.



Use a suitable algorithm to determine the sequence of actions so that the profit over the four years is maximised and state this maximum profit.

(Total 12 marks)

6. (a) Explain briefly the circumstances under which a **degenerate** feasible solution may occur to a transportation problem.

(2)

(b) Explain why a dummy location may be needed when solving a transportation problem.

(1)

The table below shows the cost of transporting one unit of stock from each of three supply points *A*, *B* and *C* to each of two demand points 1 and 2. It also shows the stock held at each supply point and the stock required at each demand point.

	1	2	Supply
A	62	47	15
B	61	48	12
C	68	58	17
Demand	16	11	

(c) Complete the table below to show a possible initial feasible solution generated by the north-west corner method.

	1	2	3
A			
B			0
C			

(1)

(d) Use the stepping-stone method to obtain an optimal solution and state its cost. You should make your method clear by stating shadow costs, improvement indices, stepping-stone route, and the entering and exiting squares at each stage.

(10)
(Total 14 marks)

7. A two person zero-sum game is represented by the following pay-off matrix for player *A*.

	B plays 1	*B* plays 2	*B* plays 3
A plays 1	5	7	2
A plays 2	3	8	4
A plays 3	6	4	9

(a) Formulate the game as a linear programming problem for player *A*, writing the constraints as equalities and clearly defining your variables.

(5)

(b) Explain why it is necessary to use the simplex algorithm to solve this game theory problem.

(1)

(c) Write down an initial simplex tableau making your variables clear.

(2)

(d) Perform two complete iterations of the simplex algorithm, indicating your pivots and stating the row operations that you use.

(8)
(Total 16 marks)

8. The tableau below is the initial tableau for a maximising linear programming problem.

Basic variable	x	y	z	r	s	t	Value
r	7	10	10	1	0	0	3600
s	6	9	12	0	1	0	3600
t	2	3	4	0	0	1	2400
P	–35	–55	–60	0	0	0	0

(a) Write down the four equations represented in the initial tableau above.

(4)

(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. State the row operations that you use.

(9)

(c) State the values of the objective function and each variable.

(3)
(Total 16 marks)

b.v.	x	y	z	r	s	t	Value	Row Operations

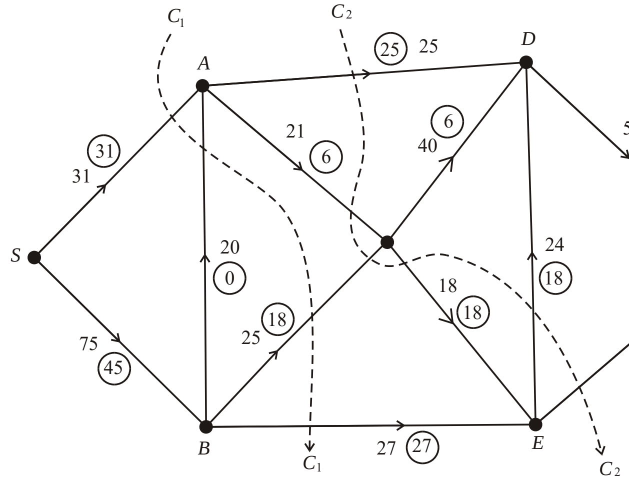
b.v.	x	y	z	r	s	t	Value	Row Operations

b.v.	x	y	z	r	s	t	Value	Row Operations

b.v.	x	y	z	r	s	t	Value	Row Operations

b.v.	x	y	z	r	s	t	Value	Row Operations

9.

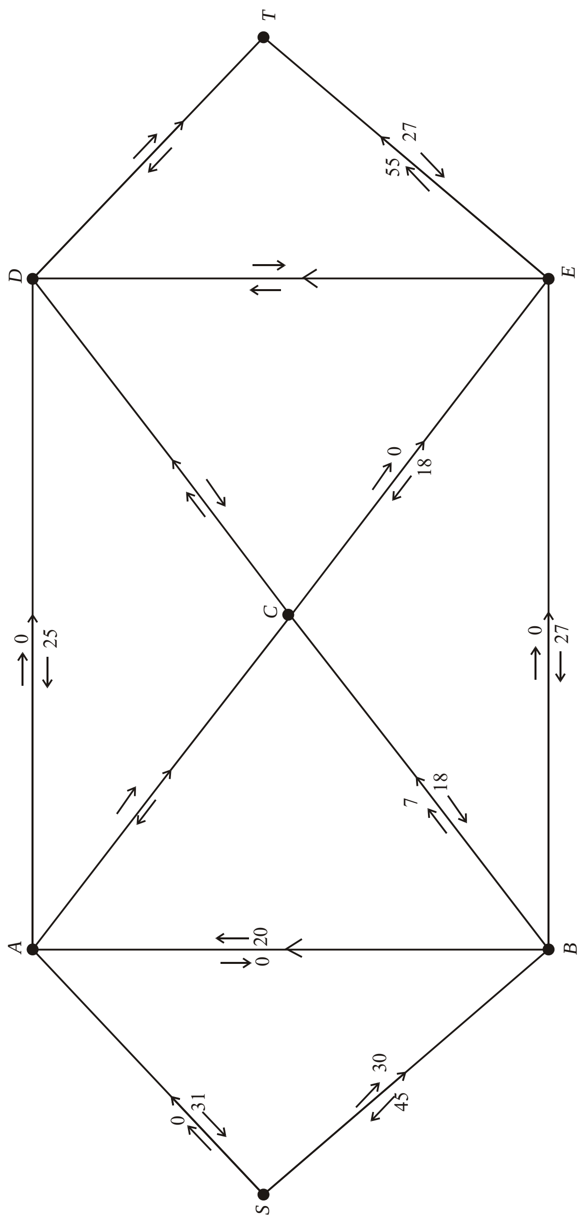


The figure above shows a capacitated, directed network. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from S to T.

Two cuts C_1 and C_2 are shown on the figure.

(a) Write down the capacity of each of the two cuts and the value of the initial flow. **(3)**

(b) Complete the initialisation of the labelling procedure on the diagram below by entering values along arcs AC, CD, DE and DT.

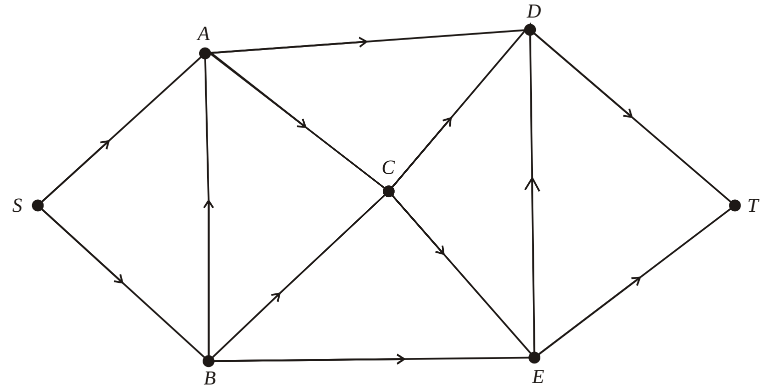


(2)

(c) Hence use the labelling procedure to find a maximal flow through the network. You must list each flow-augmenting path you use, together with its flow.

(5)

(d) Show your maximal flow pattern on the diagram below.



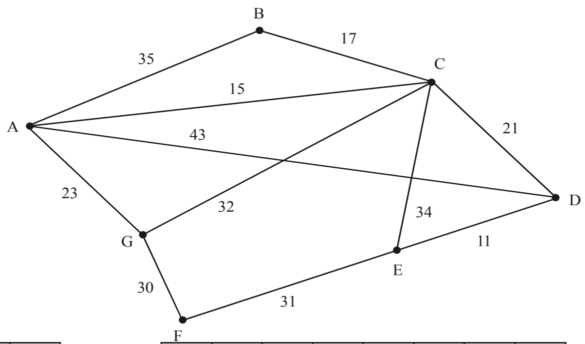
(2)

(e) Prove that your flow is maximal.

(2)
(Total 14 marks)

D2 2007 (adapted for new spec)

1. The network above shows the distances, in miles, between seven gift shops, *A*, *B*, *C*, *D*, *E*, *F* and *G*.



The area manager needs to visit each shop. She will start and finish at shop A and wishes to minimise the total distance travelled.

(a) By inspection, complete the two copies of the table of least distances below.

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–		15	36		53	23
B		–	17	38	49	80	49
C	15	17	–	21		62	32
D	36	38	21	–	11	42	
E		49		11	–	31	61
F	53	80	62	42	31	–	30
G	23	49	32		61	30	–

(4)

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–		15	36		53	23
B		–	17	38	49	80	49
C	15	17	–	21		62	32
D	36	38	21	–	11	42	
E		49		11	–	31	61
F	53	80	62	42	31	–	30
G	23	49	32		61	30	–

(b) Starting at A, and making your method clear, find an upper bound for the route length, using the nearest neighbour algorithm.

(3)

(c) By deleting A, and all of its arcs, find a lower bound for the route length.

(4) (Total 11 marks)

2. Denis (D) and Hilary (H) play a two-person zero-sum game represented by the following pay-off matrix for Denis.

	H plays 1	H plays 2	H plays 3
D plays 1	2	–1	3
D plays 2	–3	4	–4

(a) Show that there is no stable solution to this game.

(3)

(b) Find the best strategy for Denis and the value of the game to him.

(10) (Total 13 marks)

3. To raise money for charity it is decided to hold a Teddy Bear making competition. Teams of four compete against each other to make 20 Teddy Bears as quickly as possible.

There are four stages: first *cutting*, then *stitching*, then *filling* and finally *dressing*.

Each team member can only work on one stage during the competition. As soon as a stage is completed on each Teddy Bear the work is passed immediately to the next team member.

The table shows the time, in seconds, taken to complete each stage of the work on one Teddy Bear by the members *A*, *B*, *C* and *D* of one of the teams.

	cutting	*stitching*	*filling*	*dressing*
A	66	101	85	36
B	66	98	74	38
C	63	97	71	34
D	67	102	78	35

(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that minimises the time taken by this team to produce one Teddy Bear. You must make your method clear and show the table after each iteration.

(9)

(b) State the minimum time it will take this team to produce one Teddy Bear.

(1)

Using the allocation found in (a),

(c) calculate the minimum total time this team will take to complete 20 Teddy Bears. You should make your reasoning clear and state your answer in minutes and seconds.

(3)
(Total 13 marks)

4. A group of students and teachers from a performing arts college are attending the Glasenburgh drama festival. All of the group want to see an innovative modern production of the play 'The Decision is Final'. Unfortunately there are not enough seats left for them all to see the same performance.

	Adult	Student
Performance 1	£5.00	£4.50
Performance 2	£4.20	£3.80
Performance 3	£4.60	£4.00

There are three performances of the play, 1, 2, and 3. There are two types of ticket, Adult and Student. Student tickets will be purchased for the students and Adult tickets for the teachers.

The table below shows the price of tickets for each performance of the play. There are 18 teachers and 200 students requiring tickets. There are 94, 65 and 80 seats available for performances 1, 2, and 3 espectively.

(a) Complete the table below.

	Adult	Student	Dummy	Seats available
Performance 1	£5.00	£4.50		
Performance 2	£4.20	£3.80		
Performance 3	£4.60	£4.00		
Tickets needed				

(2)

(b) Explain why a dummy column was added to the table above.

(1)

(c) Use the north-west corner method to obtain a possible solution.

(1)

(d) Taking the most negative improvement index to indicate the entering square, use the stepping stone method **once** to obtain an improved solution. You must make your shadow costs and improvement indices clear.

(6)

After a further iteration the table becomes:

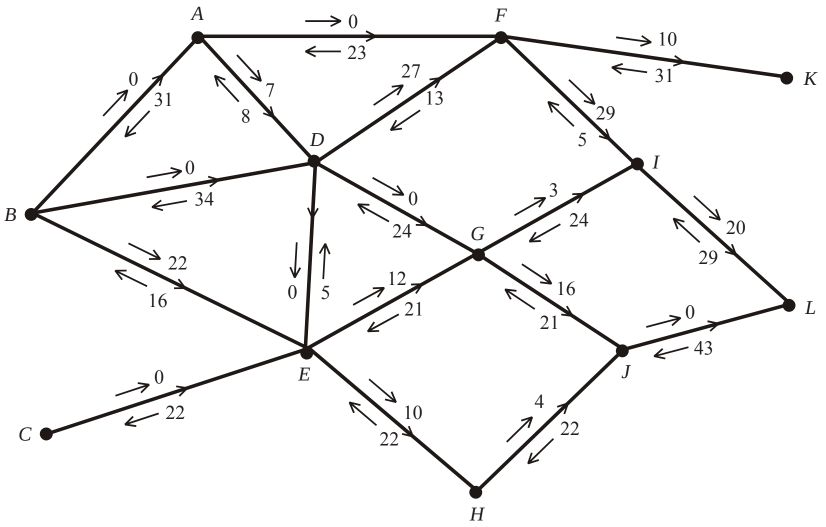
	Adult	Student	Dummy
Performance 1		73	21
Performance 2	18	47	
Performance 3		80	

(e) Demonstrate that this solution gives the minimum cost, and find its value.

(6)

(Total 16 marks)

5. **Figure 1**



In solving a network flow problem using the labelling procedure, the diagram in Figure 1 was created.
The arrow on each arc indicates the direction of the flow along that arc.
The arrows above and below each arc show the direction and value of the flow as indicated by the labelling procedure.

(a) Add a supersource S, a supersink T and appropriate arcs to the diagram above, and complete the labelling procedure for these arcs.

(3)

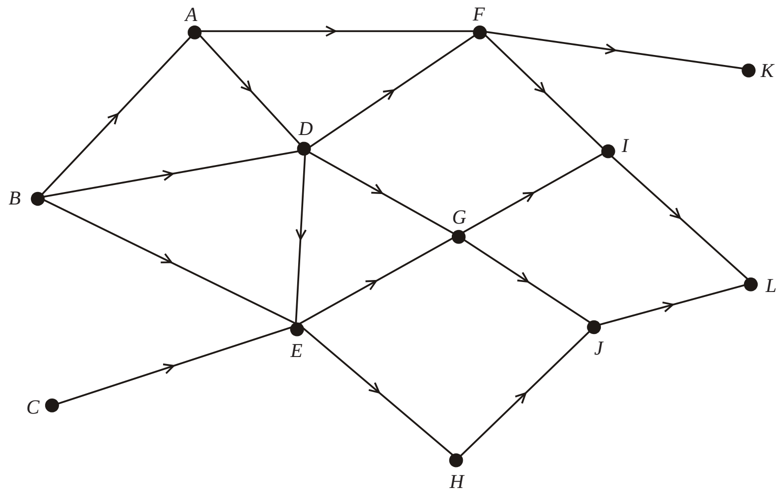
(b) Write down the value of the initial flow shown in Figure 1.

(1)

(c) Use Figure 2 below, the initial flow and the labelling procedure to find the maximal flow of 124 through this network. List each flow-augmenting path you use, together with its flow.

(5)

(d) Show your flow on the diagram below and state its value.



(3)

(e) Prove that your flow is maximal.

(2)

(Total 14 marks)

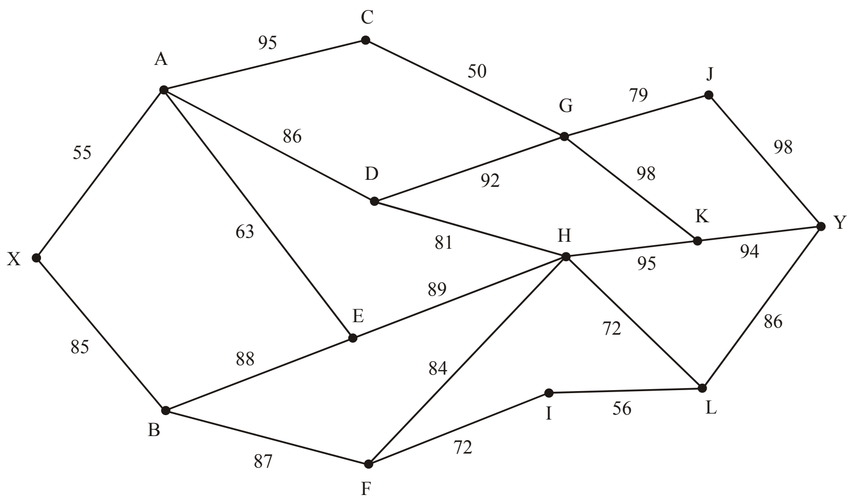
6. Anna (A) and Roland (R) play a two-person zero-sum game which is represented by the following pay-off matrix for Anna.

	R plays 1	R plays 2	R plays 3
A plays 1	6	–2	–3
A plays 2	–3	1	2
A plays 3	5	4	–1

Formulate the game as a linear programming problem **for player R**. Write the constraints as inequalities. Define your variables clearly.

(Total 8 marks)

7.



Agent Goodie has successfully recovered the stolen plans from Evil Doctor Fiendish and needs to take them from Evil Doctor Fiendish's secret headquarters at X to safety at Y. To do this he must swim through a network of underwater tunnels. Agent Goodie has no breathing apparatus, but knows that there are twelve points, *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J*, *K* and *L*, at which there are air pockets where he can take a breath.

The network is modelled above, and the number on each arc gives the time, in seconds, it takes Agent Goodie to swim from one air pocket to the next.

Agent Goodie needs to find a route through this network that minimises the longest time between successive air pockets.

(a) Use dynamic programming to complete the table below and hence find a suitable route for Agent Goodie.

(12)

Unfortunately, just as Agent Goodie is about to start his journey, tunnel XA becomes blocked.

(b) Find an optimal route for Agent Goodie avoiding tunnel XA.

(2)
(Total 14 marks)

8. The tableau below is the initial tableau for a linear programming problem in *x*, *y* and *z*. The objective is to maximise the profit, *P*.

basic variable	*x*	*y*	*z*	*r*	*s*	*t*	Value
r	12	4	5	1	0	0	246
s	9	6	3	0	1	0	153
t	5	2	–2	0	0	1	171
P	–2	–4	–3	0	0	0	0

Using the information in the tableau, write down

(a) the objective function,

(2)

(b) the three constraints as inequalities with integer coefficients.

(3)

Taking the most negative number in the profit row to indicate the pivot column at each stage,

(c) solve this linear programming problem. Make your method clear by stating the row operations you use.

b.v.	x	y	z	r	s	t	Value	Row operations

b.v.	x	y	z	r	s	t	Value	Row operations

b.v.	x	y	z	r	s	t	Value	Row operations

b.v.	x	y	z	r	s	t	Value	Row operations

(9)

(d) State the final values of the objective function and each variable.

(3)

(e) One of the constraints is not at capacity. Explain how it can be identified.

(1)
(Total 18 marks)

9.

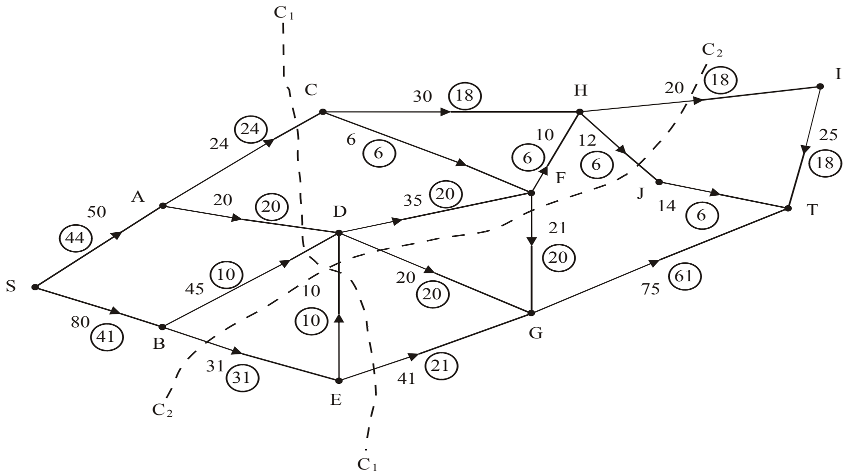


Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) State the capacities of cuts C_1 and C_2.

(2)

Figure 2 shows the labelling procedure applied to the above network.

(c) Using Figure 2, increase the flow by a further 19 units. You **must** list each flow-augmenting path you use, together with its flow.

(5)

(d) Prove that the flow is now maximal.

(2)
(Total 10 marks)

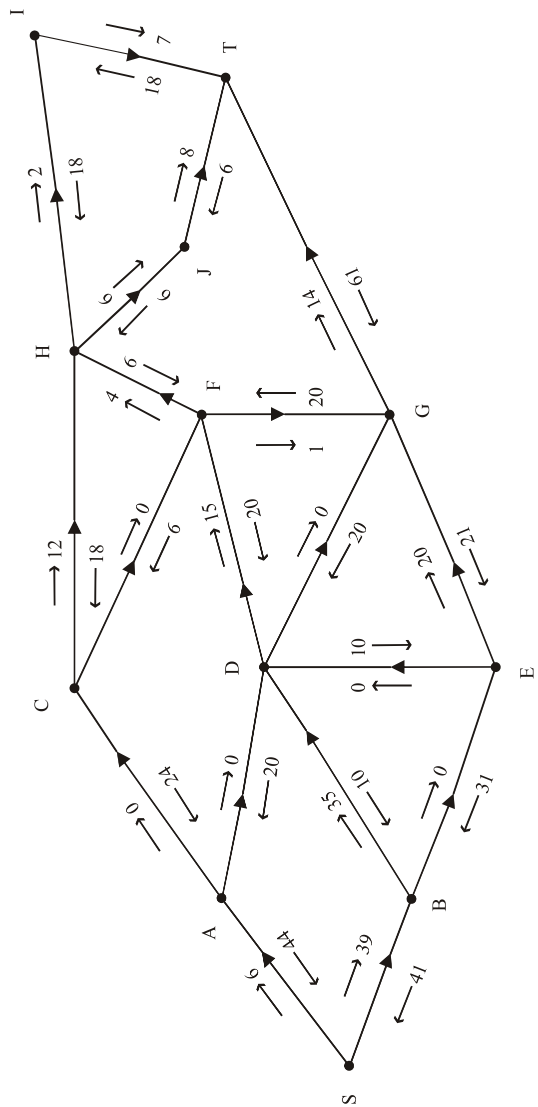
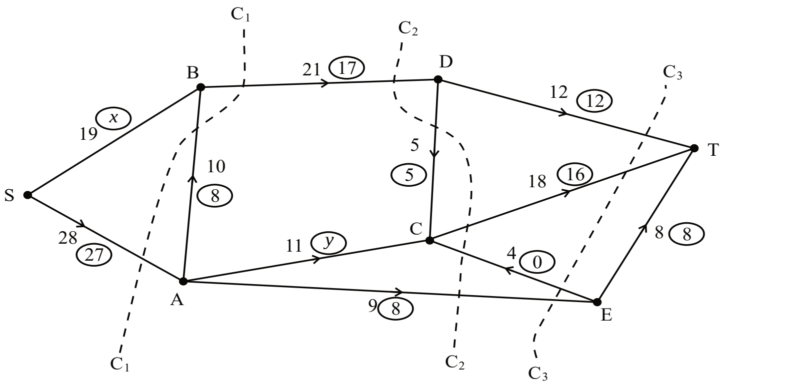


Figure 2

D2 2008 (adapted for new spec)

1.



The diagram above shows a capacitated, directed network of pipes. The number on each arc represents the capacity of that pipe. The numbers in circles represent a feasible flow.

(a) State the values of *x* and *y*. **(2)**

(b) List the saturated arcs. **(2)**

(c) State the value of the feasible flow. **(1)**

(d) State the capacities of the cuts C_1, C_2, and C_3. **(3)**

(e) By inspection, find a flow-augmenting route to increase the flow by one unit. You must state your route. **(1)**

(f) Prove that the new flow is maximal. **(2) (Total 11 marks)**

2. Explain what is meant, in a network, by (a) a walk, (b) a tour. **(2) (2) (Total 4 marks)**

3. Jameson cars are made in two factories A and B. Sales have been made at the two main showrooms in London and Edinburgh. Cars are to be transported from the factories to the showrooms. The table below shows the cost, in pounds, of transporting one car from each factory to each showroom. It also shows the number of cars available at each factory and the number required at each showroom.

	London (L)	Edinburgh (E)	Supply
A	80	70	55
B	60	50	45
Demand	35	60	

It is decided to use the transportation algorithm to obtain a minimal cost solution.

(a) Explain why it is necessary to add a dummy demand point. **(2)**

(b) Complete the table below.

	L	E	Dummy	Supply
A	80	70		55
B	60	50		45
Demand	35	60		100

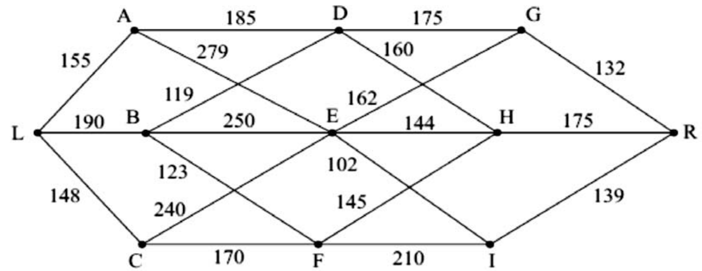
(2)

(c) Use the north-west corner rule to obtain a possible pattern of distribution. **(1)**

(d) Taking the most negative improvement index to indicate the entering square, use the stepping-stone method to obtain an optimal solution. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal. **(7)**

(e) State the cost of your optimal solution. **(1) (Total 13 marks)**

4. (a) Explain the difference between a maximin route and a minimax route in dynamic programming. **(2)**



A maximin route from L to R is to be found through the staged network shown above.

(b) Use dynamic programming to complete a table below and hence find a maximin route. **(10) (Total 12 marks)**

5. (a) In game theory, explain the circumstances under which column (*x*) dominates column (*y*) in a two-person zero-sum game. **(2)**

Liz and Mark play a zero-sum game. This game is represented by the following pay-off matrix for Liz.

	Mark plays 1	Mark plays 2	Mark plays 3
Liz plays 1	5	3	2
Liz plays 2	4	5	6
Liz plays 3	6	4	3

(b) Verify that there is no stable solution to this game. **(3)**

(c) Find the best strategy for Liz and the value of the game to her. **(9)**

The game now changes so that when Liz plays 1 and Mark plays 3 the pay-off to Liz changes from 2 to 4. All other pay-offs for this zero-sum game remain the same.

(d) Explain why a graphical approach is no longer possible and briefly describe the method Liz should use to determine her best strategy. **(2) (Total 16 marks)**

6. Four salespersons, Joe, Min-Seong, Olivia and Robert, are to attend four business fairs, *A*, *B*, *C* and *D*. Each salesperson must attend just one fair and each fair must be attended by just one salesperson. The expected sales, in thousands of pounds, that each salesperson would make at each fair is shown in the table below.

	A	*B*	*C*	*D*
Joe	48	49	42	42
Min-Seong	53	49	51	50
Olivia	51	53	48	48
Robert	47	50	46	43

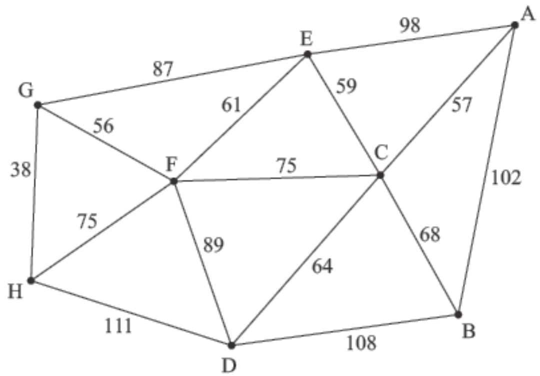
(a) Use the Hungarian algorithm, reducing rows first, to obtain an allocation that maximises the total expected sales from the four salespersons. You must make your method clear and show the table after each stage.

(10)

(b) State all possible optimal allocations and the optimal total value.

(4)(Total 14 marks)

7.



The network in the diagram above shows the distances, in km, between eight weather data collection points. Starting and finishing at A, Alice needs to visit each collection point at least once, in a minimum distance.

(a) Obtain a minimum spanning tree for the network using Kruskal's algorithm, stating the order in which you select the arcs.

(2)

(b) Use your answer to part (a) to determine an initial upper bound for the length of the route.

(1)

(c) Starting from your initial upper bound use short cuts to find an upper bound, which is below 630km. State the corresponding route.

(4)

(d) Use the nearest neighbour algorithm starting at B to find a second upper bound for the length of the route.

(3)

(e) By deleting C, and all of its arcs, find a lower bound for the length of the route.

(4)

(f) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(2) (Total 16 marks)

8. The tableau below is the initial tableau for a maximising linear programming problem in x, y and z.

Basic variable	x	y	z	r	s	t	Value
r	4	$\frac{7}{3}$	$\frac{5}{2}$	1	0	0	64
s	1	3	0	0	1	0	16
t	4	2	2	0	0	1	60
P	–5	$-\frac{7}{2}$	–4	0	0	0	0

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, perform two complete iterations of the simplex algorithm. State the row operations you use.
You may not need to use all of these tableaux.

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

(9)

(b) Explain how you know that your solution is not optimal.

(1) (Total 10 marks)

Paper Reference(s)
6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Monday 1 June 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this question paper is 75.
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
M34284A

W850/R6690/57570 5/5/4/3

Turn over

edexcel advancing learning, changing lives

Write your answers in the D2 answer book for this paper.

1. A company, Kleenitquick, has developed a new stain remover. To promote sales, three salespersons, Jess, Matt and Rachel, will be assigned to three of four department stores 1, 2, 3 and 4, to demonstrate the stain remover. Each salesperson can only be assigned to one department store.

The table below shows the cost, in pounds, of assigning each salesperson to each department store.

	1	**2**	**3**	**4**
Jess	15	11	14	12
Matt	13	8	17	13
Rachel	14	9	13	15

(a) Explain why a dummy row needs to be added to the table.
(1)

(b) Complete Table 1 in the answer book.
(1)

(c) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the cost of assigning salespersons to department stores. You must make your method clear and show the table after each iteration.
(6)

(d) Find the minimum cost.
(1)

(Total 9 marks)

2. (a) Explain the difference between the classical and the practical travelling salesperson problems.

(2)

The table below shows the distances, in km, between six data collection points, A, B, C, D, E, and F.

	A	**B**	**C**	**D**	**E**	**F**
A	-	77	34	56	67	21
B	77	-	58	58	36	74
C	34	58	-	73	70	42
D	56	58	73	-	68	38
E	67	36	70	68	-	71
F	21	74	42	38	71	-

Rachel must visit each collection point. She will start and finish at A and wishes to minimise the total distance travelled.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound. Make your method clear.

(3)

Starting at B, a second upper bound of 293 km was found.

(c) State the better upper bound of these two, giving a reason for your answer.

(1)

By deleting A, a lower bound was found to be 245 km.

(d) By deleting B, find a second lower bound. Make your method clear.

(4)

(e) State the better lower bound of these two, giving a reason for your answer.

(1)

(f) Taking your answers to (c) and (e), use inequalities to write down an interval that must contain the length of Rachel's optimal route.

(1)

(Total 12 marks)

3. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–5	6	–3
A plays 2	1	–4	13
A plays 3	–2	3	–1

(a) Verify that there is no stable solution to this game.

(3)

(b) Reduce the game so that player B has a choice of only two actions.

(1)

(c) Write down the reduced pay-off matrix **for player B**.

(2)

(d) Find the best strategy for player B and the value of the game to player B.

(7)

(Total 13 marks)

4.

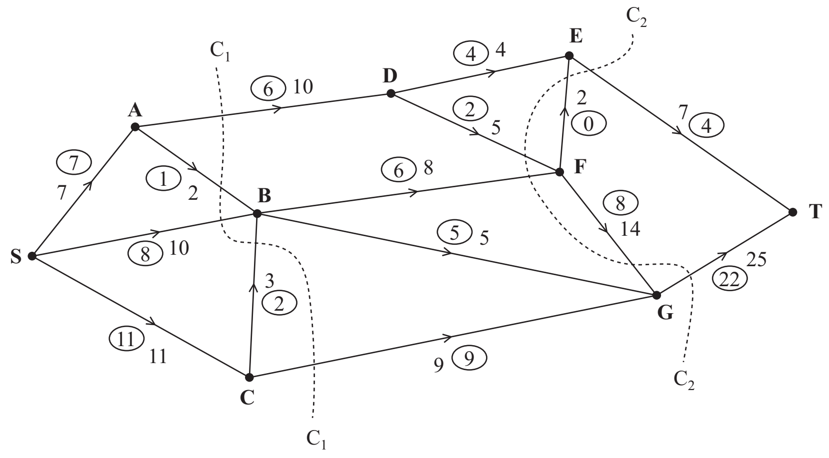


Figure 1

Figure 1 shows a capacitated network. The capacity of each arc is shown on the arc. The numbers in circles represent an initial flow from S to T.

Two cuts C_1 and C_2 are shown in Figure 1.

(a) Find the capacity of each of the two cuts.

(2)

(b) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

(3)

(Total 5 marks)

5. While solving a maximising linear programming problem, the following tableau was obtained.

Basic Variable	x	y	z	r	s	t	value
z	$\frac{1}{4}$	$-\frac{1}{4}$	1	$\frac{1}{4}$	0	0	2
s	$\frac{5}{4}$	$\frac{7}{4}$	0	$-\frac{3}{4}$	1	0	4
t	3	$\frac{5}{2}$	0	$-\frac{1}{2}$	0	1	2
P	−2	−4	0	$\frac{5}{4}$	0	0	10

(a) Write down the values of x, y and z as indicated by this tableau.

(2)

(b) Write down the profit equation from the tableau.

(2)

(Total 4 marks)

6. The table below shows the cost, in pounds, of transporting one unit of stock from each of three supply points, X, Y and Z to three demand points, A, B and C. It also shows the stock held at each supply point and the stock required at each demand point.

	A	**B**	**C**	Supply
X	17	8	7	22
Y	16	12	15	17
Z	6	10	9	15
Demand	16	15	23	

(a) This is a **balanced problem**. Explain what this means.
(1)

(b) Use the north west corner method to obtain a possible solution.
(1)

(c) Taking ZA as the entering cell, use the stepping-stone method to find an improved solution. Make your route clear and state your exiting cell.
(3)

(d) Perform one more iteration of the stepping-stone method to find a further improved solution. You must make your shadow costs, improvement indices, entering cell, exiting cell and route clear.
(6)

(e) State the cost of the solution you found in part (d).
(1)

(Total 12 marks)

7. Minty has £250 000 to allocate to three investment schemes. She will allocate the money to these schemes in units of £50 000. The net income generated by each scheme, in £1000s, is given in the table below.

	£0	**£50 000**	**£100 000**	**£150 000**	**£200 000**	**£250 000**
Scheme 1	0	60	120	180	240	300
Scheme 2	0	65	125	190	235	280
Scheme 3	0	55	110	170	230	300

Minty wishes to maximise the net income. She decides to use dynamic programming to determine the optimal allocation, and starts the table shown in your answer book.

(a) Complete the table in the answer book to determine the amount Minty should allocate to each scheme in order to maximise the income. State the maximum income and the amount that should be allocated to each scheme.
(10)

(b) For this problem give the meaning of the table headings
(i) Stage,
(ii) State,
(iii) Action.
(3)

(Total 13 marks)

8. Laura (L) and Sam (S) play a two-person zero-sum game which is represented by the following pay-off matrix for Laura.

	S plays 1	S plays 2	S plays 3
L plays 1	–2	8	–1
L plays 2	7	4	–3
L plays 3	1	–5	4

Formulate the game as a linear programming problem for Laura, writing the constraints as inequalities. Define your variables clearly.

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Friday 11 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this question paper is 75.
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
N35400A

N 3 5 4 0 0 A

W850/R6690/57570 4/5/5/5/3/

Turn over

edexcel
advancing learning, changing lives

Write your answers in the D2 answer book for this paper.

1. The table below shows the least costs, in pounds, of travelling between six cities, A, B, C, D, E and F.

	A	B	C	D	E	F
A	-	36	18	28	24	22
B	36	-	54	22	20	27
C	18	54	-	42	27	24
D	28	22	42	-	20	30
E	24	20	27	20	-	13
F	22	27	24	30	13	-

Vicky must visit each city at least once. She will start and finish at A and wishes to minimise the total cost.

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network.
(2)

(b) Use your answer to part (a) to help you calculate an initial upper bound for the length of Vicky's route.
(1)

(c) Show that there are two nearest neighbour routes that start from A. You must make your routes and their lengths clear.
(3)

(d) State the best upper bound from your answers to (b) and (c).
(1)

(e) Starting by deleting A, and all of its arcs, find a lower bound for the route length.
(4)

(Total 11 marks)

2. A team of four workers, Harry, Jess, Louis and Saul, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to one task and each task must be done by just one worker.

Jess cannot be assigned to task 4.

The amount, in pounds, that each person would earn while assigned to each task is shown in the table below.

	1	2	3	4
Harry	18	24	22	17
Jess	20	25	19	-
Louis	25	24	27	22
Saul	19	26	23	14

(a) **Reducing rows first**, use the Hungarian algorithm to obtain an allocation that maximises the total amount earned by the team. You must make your method clear and show the table after each stage.
(8)

(b) State who should be assigned to each task and the total amount earned by the team.
(2)

(Total 10 marks)

3. The table below shows the cost of transporting one block of staging from each of two supply points, X and Y, to each of four concert venues, A, B, C and D. It also shows the number of blocks held at each supply point and the number of blocks required at each concert venue. A minimal cost solution is required.

	A	B	C	D	Supply
X	28	20	19	16	53
Y	15	12	14	17	47
Demand	18	31	22	29	

(a) Use the north-west corner method to obtain a possible solution.
(1)

(b) Taking the most negative improvement index to indicate the entering square, use the stepping stone method **twice** to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, routes, entering cells and exiting cells.
(9)

(c) Is your current solution optimal? Give a reason for your answer.
(1)

(Total 11 marks)

4.

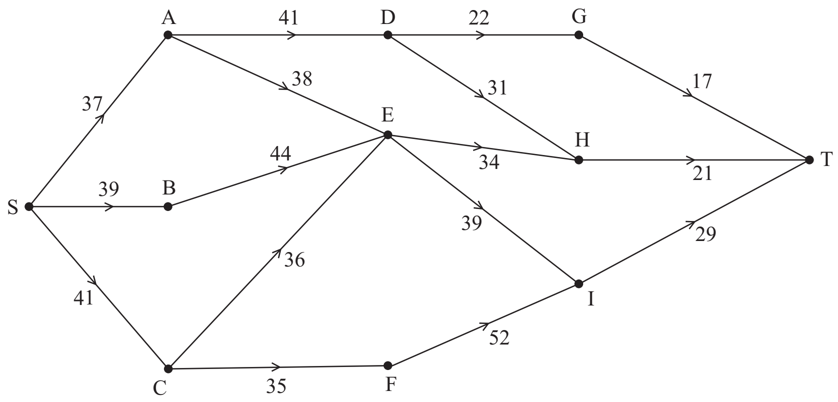


Figure 1

Figure 1 represents the maintenance choices a council can make and their costs, in £1000s, over the next four years.

The council wishes to minimise the greatest annual cost of maintenance.

(a) Use dynamic programming to find a minimax route from S to T. **(9)**

(b) State your route and the greatest annual cost incurred by the council. **(2)**

(c) Calculate the average annual cost to the council. **(2)**

(Total 13 marks)

5.

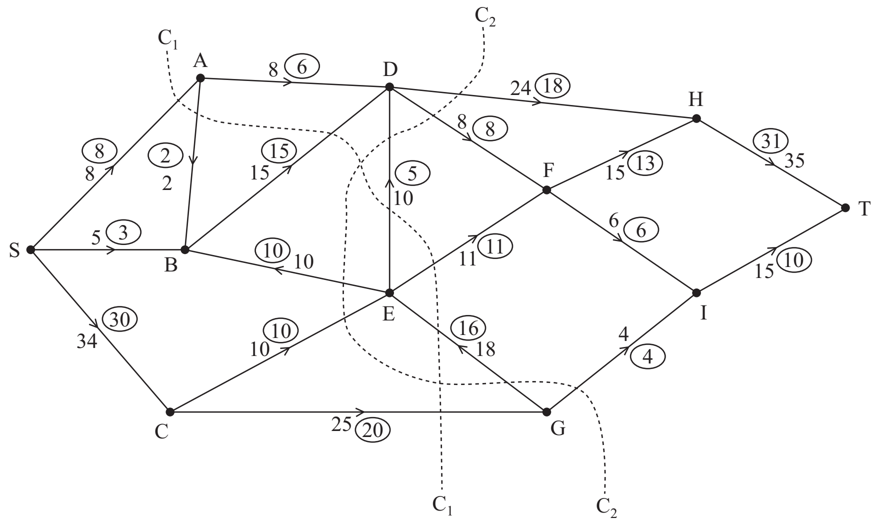


Figure 2

Figure 2 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow. **(1)**

(b) State the capacities of cuts C_1 and C_2. **(2)**

(c) By entering values along DH, FH, FI and IT, complete the labelling procedure on Diagram 1 in the answer book. **(2)**

(d) Using Diagram 1, increase the flow by a further 4 units. You **must** list each flow-augmenting route you use, together with its flow. **(3)**

(e) Prove that the flow is now maximal. **(2)**

(Total 10 marks)

6. The tableau below is the initial tableau for a linear programming problem in *x*, *y* and *z*. The objective is to maximise the profit, *P*.

Basic Variable	x	y	z	r	s	t	Value
r	0	1	2	1	0	0	24
s	2	1	4	0	1	0	28
t	–1	$\frac{1}{2}$	3	0	0	1	22
P	–1	–2	–6	0	0	0	0

(a) Write down the profit equation represented in the initial tableau.
(1)

(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.
(9)

(c) State the final value of the objective function and of each variable.
(3)

(Total 13 marks)

---

7. A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–4	5	1
A plays 2	3	–1	–2
A plays 3	–3	0	2

Formulate the game as a linear programming problem for player A. Write the constraints as inequalities and define your variables.
(7)

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Monday 13 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 12 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P35419A

W850/R6690/57570 5/5/3/3/3

Turn over

Write your answers in the D2 answer book for this paper.

1.

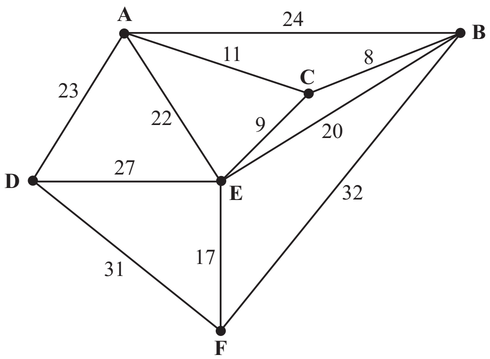


Figure 1

The network in Figure 1 shows the distances, in km, between six towns, A, B, C, D, E and F. Mabintou needs to visit each town. She will start and finish at A and wishes to minimise the total distance travelled.

(a) By inspection, complete the two copies of the table of least distances in your answer book. **(3)**

(b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Mabintou's route. Write down the route which gives this upper bound. **(3)**

(c) Starting by deleting A, and all of its arcs, find a lower bound for the route length. **(4)**

(Total 10 marks)

2. The table below shows the cost of transporting one unit of stock from each of four supply points, 1, 2, 3 and 4, to each of three demand points, A, B and C. It also shows the stock held at each supply point and the stock required at each demand point. A minimal cost solution is required.

	A	B	C	Supply
1	31	29	32	20
2	22	33	27	22
3	25	27	32	20
4	23	26	38	38
Demand	35	25	30	

(a) Add a dummy demand point and appropriate values to Table 1 in the answer book. **(1)**

Table 2 shows an initial solution given by the north-west corner method.
Table 3 shows some of the improvement indices for this solution.

	A	B	C	D
1	20			
2	15	7		
3		18	2	
4			28	10

Table 2

	A	B	C	D
1		–13		–9
2			–11	
3				
4	1	–7		

Table 3

(b) Calculate the shadow costs and the missing improvement indices and enter them into Table 3 in the **answer book**. **(4)**

(c) Taking the most negative improvement index to indicate the entering square, use the stepping-stone method once to obtain an improved solution. You must make your route clear and state your entering cell and exiting cell. **(4)**

(Total 9 marks)

3. A three-variable linear programming problem in x, y and z is to be solved. The objective is to maximise the profit, P.
The following tableau is obtained.

Basic variable	x	y	z	r	s	t	Value
r	$-\frac{1}{2}$	0	2	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	5
t	$\frac{1}{2}$	0	1	0	$-\frac{1}{4}$	1	4
P	–7	0	1	0	4	0	320

(a) Write down the profit equation represented in the tableau.
(2)

(b) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.
(5)

(c) State the value of the objective function and of each variable.
(3)

(Total 10 marks)

4. Laura and Sam play a zero-sum game. This game is represented by the following pay-off matrix for Laura.

	S plays 1	S plays 2	S plays 3
L plays 1	–4	–1	1
L plays 2	3	–1	–2
L plays 3	–3	0	2

Find the best strategy for Laura and the value of the game to her.

(Total 9 marks)

5.

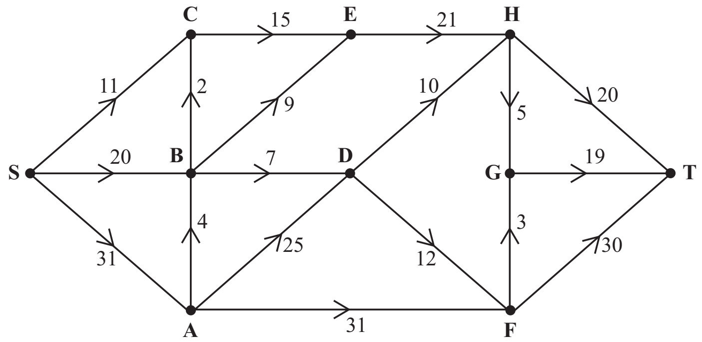


Figure 2

Figure 2 shows a capacitated directed network. The number on each arc is its capacity.

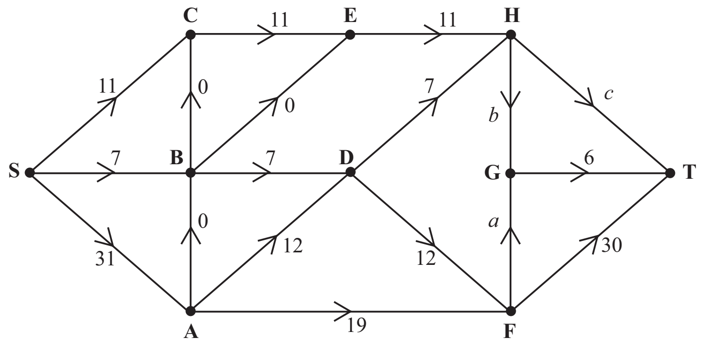


Figure 3

Figure 3 shows an initial flow through the same network.

(a) State the values of flows a, b and c, and the value of the initial flow. **(4)**

(b) By entering values along HG, HT and FG, complete the labelling procedure on Diagram 1 in the answer book. **(2)**

(c) Find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. **(4)**

(d) State the value of the maximum flow through the network. **(1)**

(e) Show your maximum flow on Diagram 2 in the answer book. **(2)**

(f) Prove that your flow is maximal. **(2)**

(Total 15 marks)

6. Three workers, P, Q and R, are to be assigned to three tasks, A, B and C. Each worker must be assigned to just one task and each task must be assigned to just one worker.
Table 1 shows the cost of using each worker for each task. The total cost is to be minimised.

	Task A	Task B	Task C
Worker P	27	31	25
Worker Q	26	30	34
Worker R	35	29	32

Table 1

(a) Formulate the above situation as a linear programming problem. You must define your decision variables and make the objective and constraints clear.
You are not required to solve the problem.
(7)

Table 2 shows the profit gained by using each worker for each task. The total profit is to be maximised.

	Task A	Task B	Task C
Worker P	33	37	31
Worker Q	32	36	40
Worker R	41	35	38

Table 2

(b) Modify Table 2 in the answer book so that the Hungarian Algorithm could be used to find the maximum total profit. You are not required to solve the problem.
(2)

(Total 9 marks)

7. Patrick is to take orders for his company's products.
He will visit four countries over the next four weeks.
He will visit just one country each week.
He will leave from his office in London and will only return there after visiting the four countries.
He will travel directly from one country to the next.
He wishes to determine a schedule of four countries to visit.

Table 1 shows the countries he could visit in each week.

Week	Week 1	Week 2	Week 3	Week 4
Possible countries	A or B	C, D or E	F or G	H or I

Table 1

Table 2 shows the value of the orders, in £100s, he expects to take in each country.

Country	A	B	C	D	E	F	G	H	I
Value of expected orders in £100s	22	17	42	41	39	29	27	36	38

Table 2

Table 3 shows the cost, in £100s, of travelling between the various countries.

Travel costs in £100s	A	B	C	D	E	F	G	H	I
London	5	3						5	4
A			5	4	2				
B			4	4	3				
C						6	5		
D						6	3		
E						4	4		
F								6	7
G								5	6

Table 3

The expected income is the value of the expected orders minus the cost of travel.

It is decided to use dynamic programming to find a schedule that maximises the total expected income for these four weeks.

(a) Complete the table in the answer book to determine the optimal expected income.
(11)

(b) State Patrick's two optimal schedules.
(2)

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Thursday 31 May 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P38284A

W850/R6690/57570 5/4/4

Turn over

Write your answers in the D2 answer book for this paper.

1. Five workers, A, B, C, D and E, are to be assigned to five tasks, 1, 2, 3, 4 and 5. Each worker is to be assigned to one task and each task must be assigned to one worker.

The cost, in pounds, of assigning each person to each task is shown in the table below. The cost is to be minimised.

	1	2	3	4	5
A	129	127	122	134	135
B	127	125	123	131	132
C	142	131	121	140	139
D	127	127	122	131	136
E	141	134	129	144	143

(a) **Reducing rows first**, use the Hungarian algorithm to obtain an allocation that minimises the cost. You must make your method clear and show the table after each stage.
(8)

(b) Find the minimum cost.
(1)

(Total 9 marks)

2. The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	-	16	25	21	12	15
B	16	-	24	22	21	12
C	25	24	-	18	30	27
D	21	22	18	-	15	12
E	12	21	30	15	-	18
F	15	12	27	12	18	-

Toby must visit each town at least once. He will start and finish at A and wishes to minimise the total distance.

(a) Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of Toby's route.

(3)

(b) Starting by deleting A, and all of its arcs, find a lower bound for the route length.

(4)

(Total 7 marks)

3. The table below shows the cost, in pounds, of transporting one tonne of concrete from each of three supply depots, A, B and C, to each of four building sites, D, E, F and G. It also shows the number of tonnes that can be supplied from each depot and the number of tonnes required at each building site. A minimum cost solution is required.

	D	E	F	G	Supply
A	17	19	21	20	18
B	21	20	19	22	23
C	18	17	16	21	29
Demand	15	24	18	13	

The north-west corner method gives the following possible solution.

	D	E	F	G	Supply
A	15	3			18
B		21	2		23
C			16	13	29
Demand	15	24	18	13	

Taking AG as the first entering cell,

(a) use the stepping stone method **twice** to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, routes, entering cells and exiting cells.

(8)

(b) Determine whether your current solution is optimal. Justify your answer.

(4)

(Total 12 marks)

4. The tableau below is the initial tableau for a maximising linear programming problem in x, y and z which is to be solved.

Basic variable	x	y	z	r	s	t	Value
r	5	$\frac{1}{2}$	0	1	0	0	5
s	1	–2	4	0	1	0	3
t	8	4	6	0	0	1	6
P	–5	–7	–4	0	0	0	0

(a) Starting by increasing y, perform one complete iteration of the simplex algorithm, to obtain tableau T. State the row operations you use.
(5)

(b) Write down the profit equation given by tableau T.
(2)

(c) Use the profit equation from part (b) to explain why tableau T is optimal.
(1)

(Total 8 marks)

5. Agent Goodie is planning to break into Evil Doctor Fiendish's secret base.

He uses game theory to determine whether to approach the base from air, sea or land.

Evil Doctor Fiendish decides each day which of three possible plans he should use to protect his base.

Agent Goodie evaluates the situation. He assigns numbers, negative indicating he fails in his mission, positive indicating success, to create a pay-off matrix. The numbers range from –3 (he fails in his mission and is captured) to 5 (he successfully achieves his mission and escapes uninjured) and the pay-off matrix is shown below.

	Fiendish uses plan 1	Fiendish uses plan 2	Fiendish uses plan 3
Air	0	4	5
Sea	2	–3	1
Land	–2	3	–2

(a) Reduce the game so that Agent Goodie has only two choices, explaining your reasoning.
(1)

(b) Use game theory to determine Agent Goodie's best strategy.
(7)

(c) Find the value of the game to Agent Goodie.
(1)

(Total 9 marks)

6.

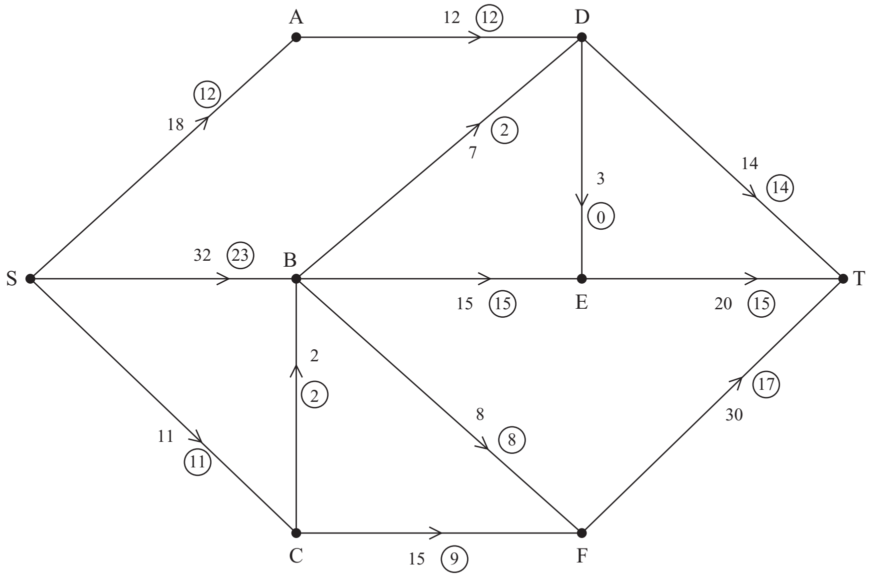


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) Complete the initialisation of the labelling procedure on Diagram 1 in the answer book by entering values along SB, BD, CF and FT.

(2)

(c) Hence use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(d) Draw a maximal flow pattern on Diagram 2 in your answer book.

(2)

(e) Prove that your flow is maximal.

(2)

(Total 11 marks)

7. Four workers, A, B, C and D, are to be assigned to four tasks, P, Q, R and S. Each worker is to be assigned to exactly one task and each task must be assigned to just one worker. The cost, in pounds, of using each worker for each task is given in the table below. The total cost is to be minimised.

	P	Q	R	S
A	23	41	34	44
B	21	45	33	42
C	26	43	31	40
D	20	47	35	46

Formulate the above situation as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.

(Total 7 marks)

8. A company makes industrial robots.
They can make up to four robots in any one month, but if they make more than three they will have to hire additional labour at a cost of £400 per month.
They can store up to two robots at a cost of £150 per robot per month.
The overhead costs are £300 in any month in which work is done.

Robots are delivered to buyers at the end of each month. There are no robots in stock at the beginning of January and there should be none in stock after the April delivery.

The order book for robots is

Month	January	February	March	April
Number of robots required	2	2	3	4

Use dynamic programming to determine the production schedule which minimises the costs, showing your working in the table provided in the answer book.

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6690/01R
Edexcel GCE
Decision Mathematics D2
Advanced/Advanced Subsidiary
Thursday 6 June 2013 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 12 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P42963RA
P 4 2 9 6 3 R A
W850/R6690/57570 5/5/5/e2/

Turn over

PEARSON

Write your answers in the D2 answer book for this paper.

1. Four workers, Chris (C), James (J), Katie (K) and Nicky (N), are to be allocated to four tasks, 1, 2, 3 and 4. Each worker is to be allocated to one task and each task must be allocated to one worker.

The profit, in pounds, resulting from allocating each worker to each task, is shown in the table below. The profit is to be maximised.

	1	2	3	4
Chris	127	116	111	113
James	225	208	205	208
Katie	130	113	112	114
Nicky	228	212	203	210

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total profit. You must make your method clear and show the table after each stage.

(8)

(b) State which worker should be allocated to each task and the resulting total profit made.

(2)

(Total 10 marks)

2. The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(1)

(Total 8 marks)

3. Table 1 below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to four demand points 1, 2, 3 and 4. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

	1	2	3	4	Supply
A	22	36	19	37	35
B	29	35	30	36	15
C	24	32	25	41	20
D	23	30	23	38	30
Demand	30	20	30	20	

Table 1

Table 2 shows an initial solution given by the north-west corner method.
Table 3 shows some of the improvement indices for this solution.

	1	2	3	4
A	30	5		
B		15	0	
C			20	
D			10	20

Table 2

	1	2	3	4
A	x	x		
B		x	x	
C	8	2	x	1
D	9	2	x	x

Table 3

(a) Explain why a zero has been placed in cell B3 in Table 2.

(1)

(b) Calculate the shadow costs and the missing improvement indices and enter them into Table 3 in your answer book.

(4)

(c) Taking the most negative improvement index to indicate the entering cell, state the stepping-stone route that should be used to obtain the next solution. You must state your entering cell and exiting cell.

(3)

(Total 8 marks)

4. Robin (R) and Steve (S) play a two-person zero-sum game which is represented by the following pay-off matrix for Robin.

	S plays 1	S plays 2	S plays 3
R plays 1	2	1	3
R plays 2	1	–1	2
R plays 3	–1	3	–3

Find the best strategy for Robin and the value of the game to him.

(Total 9 marks)

5. A three-variable linear programming problem in x, y and z is to be solved. The objective is to maximise the profit, P.
The following tableau is obtained.

Basic variable	x	y	z	r	s	t	Value
r	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	0	$-\frac{1}{2}$	10
s	$1\frac{1}{2}$	$2\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	5
z	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	5
P	–5	–10	0	0	0	20	220

(a) Starting by increasing y, perform one complete iteration of the Simplex algorithm, to obtain a new tableau, T. State the row operations you use.
(5)

(b) Write down the profit equation given by T.
(1)

(c) Use the profit equation from part (b) to explain why T is optimal.
(2)

(Total 8 marks)

6.

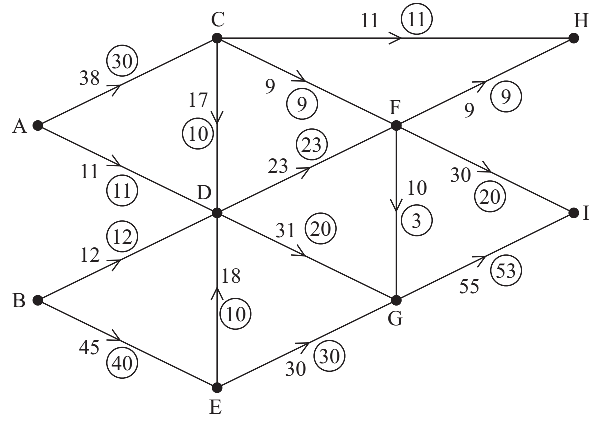


Figure 1

Figure 1 shows a capacitated directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.
(1)

(b) On Diagram 1 and Diagram 2 in the answer book, add a supersource S and a supersink T. On Diagram 1, show the minimum capacities of the arcs you have added.
(2)

(c) Complete the initialisation of the labelling procedure on Diagram 2 in the answer book by entering values on the arcs to S and T and on arcs CD, DE, DG, FG, FI and GI.
(3)

(d) Find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.
(3)

(e) Show your maximum flow on Diagram 3 in the answer book.
(2)

(f) Prove that your flow is maximal.
(2)

(Total 13 marks)

7. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	1	–3	2
A plays 2	–2	3	–1
A plays 3	5	–1	0

Formulate the game as a linear programming problem for player A. Write the constraints as inequalities. Define your variables clearly.

(Total 7 marks)

8. A factory can process up to five units of carrots each month.
Each unit can be sold fresh or frozen or canned.
The profits, in £100s, for the number of units sold, are shown in the table.
The total monthly profit is to be maximised.

Number of units	0	1	2	3	4	5
Fresh	0	45	85	120	150	175
Frozen	0	45	70	100	120	130
Canned	0	35	75	125	155	195

Use dynamic programming to determine how many of the five units should be sold fresh, frozen and canned in order to maximise the monthly profit. State the maximum monthly profit.

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Thursday 6 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P41818RA

W850/R6690/57570 5/5/5/5/5/e2/

Turn over

PEARSON

Write your answers in the D2 answer book for this paper.

1.

	A	B	C	D	E
A	–	15	19	25	20
B	15	–	15	15	25
C	19	15	–	22	11
D	25	15	22	–	18
E	20	25	11	18	–

The table shows the least distances, in km, between five hiding places, A, B, C, D and E.

Agent Goodie has to leave a secret message in each of the hiding places. He will start and finish at A, and wishes to minimise the total distance travelled.

(a) Use Prim's algorithm to find a minimum spanning tree for this network. Make your order of arc selection clear. **(2)**

(b) Use your answer to part (a) to determine an initial upper bound for the length of Agent Goodie's route. **(1)**

(c) Show that there are two nearest neighbour routes which start from A. State these routes and their lengths. **(3)**

(d) State the better upper bound from your answers to (b) and (c). **(1)**

(e) Starting by deleting B, and all of its arcs, find a lower bound for the length of Agent Goodie's route. **(4)**

(f) Consider your answers to (d) and (e) and hence state an optimal route. **(1)**

(Total 12 marks)

2. The table shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of three demand points, 1, 2 and 3. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

	1	2	3	Supply
A	10	11	20	18
B	15	7	13	14
C	24	15	12	21
D	9	21	18	12
Demand	27	18	20	

(a) Use the north-west corner method to obtain an initial solution.

(1)

(b) Taking D1 as the entering cell, use the stepping stone method to find an improved solution. Make your route clear.

(2)

(c) Perform one further iteration of the stepping stone method to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, route, entering cell and exiting cell.

(4)

(d) Determine whether your current solution is optimal, giving a reason for your answer.

(3)

(Total 10 marks)

3.

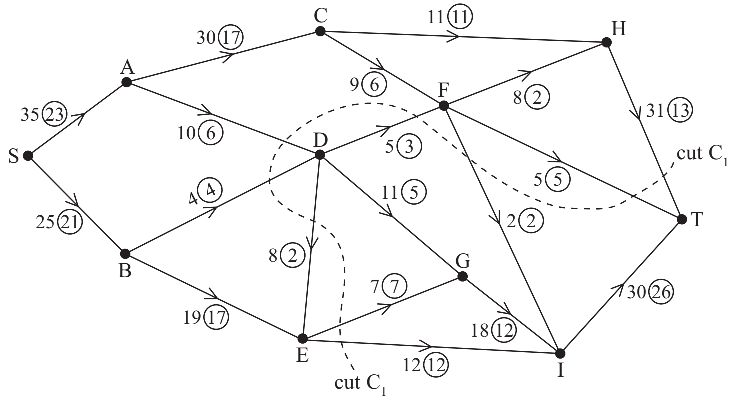


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) State the capacity of cut C_1.

(1)

The labelling procedure has been used and the result drawn on Diagram 1 in the answer book.

(c) Use Diagram 1 to find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(d) Draw a maximum flow pattern on Diagram 2 in your answer book.

(2)

(e) Prove that the flow shown in (d) is maximal.

(2)

(Total 10 marks)

4. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	5	4	–6
A plays 2	–1	–2	3
A plays 3	1	–1	2

(a) Reduce the game so that player B has only two possible actions.
(1)

(b) Write down the reduced pay-off matrix **for player B**.
(2)

(c) Find the best strategy for player B and the value of the game to him.
(8)

(Total 11 marks)

5. In solving a three-variable maximising linear programming problem, the following tableau was obtained after the first iteration.

Basic variable	x	y	z	r	s	t	Value
r	–1	2	0	1	0	1	8
s	–1	3	0	0	1	1	22
z	–2	1	1	0	0	1	11
P	2	–5	0	0	0	$\frac{1}{2}$	15

(a) State which variable was increased first, giving a reason for your answer.
(1)

(b) Solve this linear programming problem. Make your method clear by stating the row operations you use.
(8)

(c) State the final value of the objective function and the final values of each variable.
(2)

(Total 11 marks)

6. Three workers, Harriet, Jason and Katherine, are to be assigned to three tasks, 1, 2 and 3. Each worker must be assigned to just one task and each task must be done by just one worker.

The amount each person would earn, in pounds, while assigned to each task is shown in the table below.

	Task 1	Task 2	Task 3
Harriet	251	243	257
Jason	244	247	255
Katherine	249	252	246

The total income is to be maximised.

(a) Modify the table so it can be used to find the maximum income.
(1)

(b) Formulate the above situation as a linear programming problem. You must define your decision variables and make your objective function and constraints clear.
(7)

(Total 8 marks)

7. Nigel has a business renting out his fleet of bicycles to tourists.

At the start of each year Nigel must decide on one of two actions:

- Keep his fleet of bicycles, incurring maintenance costs.
- Replace his fleet of bicycles.

The cost of keeping the fleet of bicycles,
the cost of replacing the fleet of bicycles
and the annual income are dependent on the age of the fleet of bicycles.
Table 1 shows these amounts, in £1000s.

Age of fleet of bicycles	new	1 year old	2 years old	3 years old	4 years old
Cost of keeping (£1000s)	0	1	2	3	8
Cost of replacing (£1000s)	–	7	8	9	10
Income (£1000s)	11	8	5	2	0

Table 1

Nigel has a new fleet of bicycles now and wishes to maximise his total profit over the next four years.

He is planning to sell his business at the end of the fourth year.

The amount Nigel will receive will depend on the age of his fleet of bicycles.

These amounts, in £1000s, are shown in Table 2.

Age of fleet of bicycles at end of 4th year	1 year old	2 years old	3 years old	4 years old
Amount received at end of 4th year (£1000s)	6	4	2	1

Table 2

Complete the table in the answer book to determine Nigel's best strategy to maximise his total profit over the next four years. You must state the action he should take each year (keep or replace) and his total profit.

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6690/01R

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Tuesday 24 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P43158A

W850/R6690/57570 5/5/5/1/

Turn over

PEARSON

Write your answers in the D2 answer book for this paper.

1. Four bakeries, A, B, C and D, supply bread to four supermarkets, P, Q, R and S. The table gives the cost, in pounds, of transporting one lorry load of bread from each bakery to each supermarket. It also shows the number of lorry loads of bread at each bakery and the number of lorry loads of bread required at each supermarket. The total cost of transportation is to be minimised.

	P	Q	R	S	Supply
A	28	32	33	27	13
B	31	29	26	31	4
C	30	26	29	32	12
D	25	30	28	34	11
Demand	11	10	11	8	

(a) Use the north-west corner method to obtain a possible solution.
(1)

A partly completed table of improvement indices is given in Table 1 in the answer book.

(b) Complete Table 1.
(4)

(c) Taking the most negative improvement index to indicate the entering cell, use the stepping-stone method **once** to obtain an improved solution. You must make your route clear and state your entering cell and exiting cell.
(4)

(d) State the cost of your improved solution.
(1)

(Total 10 marks)

2. (a) Explain the difference between the classical and the practical travelling salesperson problem.
(2)

	A	B	C	D	E	F
A	–	65	48	15	30	40
B	65	–	50	51	35	26
C	48	50	–	37	20	34
D	15	51	37	–	17	25
E	30	35	20	17	–	14
F	40	26	34	25	14	–

The table above shows the least distances, in km, between six towns, A, B, C, D, E and F. Keith needs to visit each town, starting and finishing at A, and wishes to minimise the total distance he will travel.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound. You must state your route and its length.
(3)

(c) Starting by deleting A, and all of its arcs, find a lower bound for the route length.
(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.
(2)

(Total 10 marks)

3. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3
A plays 1	–2	2	–3
A plays 2	1	1	–1
A plays 3	2	–1	1

(a) Starting by reducing player B's options, find the best strategy for player B.
(9)

(b) State the value of the game to player B.
(1)

(Total 10 marks)

4. The tableau below is the initial tableau for a three-variable linear programming problem in x, y and z. The objective is to maximise the profit, P.

Basic Variable	x	y	z	r	s	t	Value
r	4	3	$\frac{5}{2}$	1	0	0	50
s	1	2	1	0	1	0	30
t	0	5	1	0	0	1	80
P	–25	–40	–35	0	0	0	0

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, perform **two** complete iterations of the simplex algorithm to obtain tableau T. Make your method clear by stating the row operations you use.
(9)

(b) Write down the profit equation given by T.
(1)

(c) Use your answer to (b) to determine whether T is optimal, justifying your answer.
(2)

(Total 12 marks)

5.

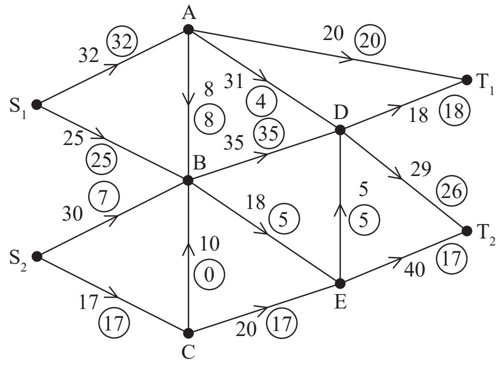


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) (i) Add a supersource, S, and a supersink, T, and corresponding arcs to Diagrams 1 and 2, in the answer book.

(ii) Enter the flow value and appropriate capacity on each of the arcs you have added to Diagram 1.

(3)

(b) Complete the initialisation of the labelling procedure on Diagram 2 by entering values along the new arcs from S and T, and along AB, AD and DT_2.

(2)

(c) Hence use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(d) Draw a maximal flow pattern on Diagram 3 in the answer book.

(2)

(e) Prove that your flow is maximal.

(2)

(Total 13 marks)

6. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker.

Worker C cannot do task 4 and worker D cannot do task 1.

The cost of assigning each worker to each task is shown in the table below.
The total cost is to be minimised.

	1	2	3	4
A	29	15	32	30
B	34	26	40	32
C	28	27	35	–
D	–	21	33	31

Formulate the above situation as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.

(Total 7 marks)

7. Susie has hired a team of four workers who can make three types of toy. The total number of toys the team can produce will depend on which toys they make, and on how many workers are assigned to make each type of toy.

The table shows how many of each toy would be made if different numbers of workers were assigned to make them. Each worker is to be assigned to make just one type of toy and all four workers are to be assigned. Susie wishes to maximise the total number of toys produced.

		Number of workers				
		0	1	2	3	4
TOYS	Bicycle	0	80	170	260	350
	Dolls House	0	95	165	245	335
	Train Set	0	100	180	260	340

(a) Use dynamic programming to determine the allocation of workers which maximises the total number of toys made. You should show your working in the table provided in the answer book. **(12)**

(b) State the maximum total number of toys produced by this team. **(1)**

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Tuesday 24 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P43157A
W850/R6690/57570 5/5/5/5/5/

Turn over

Write your answers in the D2 answer book for this paper.

1. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker.

Worker A cannot do task 4 and worker B cannot do task 2.

The amount, in pounds, that each worker would earn if assigned to the tasks, is shown in the table below.

	1	2	3	4
A	19	16	23	–
B	24	–	30	23
C	18	17	25	18
D	24	24	26	24

Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total earnings. You must make your method clear and show the table after each stage.

(Total 10 marks)

2. The table shows the least times, in seconds, that it takes a robot to travel between six points in an automated warehouse. These six points are an entrance, A, and five storage bins, B, C, D, E and F. The robot will start at A, visit each bin, and return to A. The total time taken for the robot's route is to be minimised.

	A	B	C	D	E	F
A	–	90	130	85	35	125
B	90	–	80	100	83	88
C	130	80	–	108	106	105
D	85	100	108	–	110	88
E	35	83	106	110	–	75
F	125	88	105	88	75	–

(a) Show that there are two nearest neighbour routes that start from A. You must make the routes and their lengths clear.

(4)

(b) Starting by deleting F, and all of its arcs, find a lower bound for the time taken for the robot's route.

(3)

(c) Use your results to write down the smallest interval which you are confident contains the optimal time for the robot's route.

(3)

(Total 10 marks)

3. The tableau below is the initial tableau for a three-variable linear programming problem in x, y and z. The objective is to maximise the profit, P.

Basic Variable	x	y	z	r	s	t	Value
r	5	3	$-\frac{1}{2}$	1	0	0	2500
s	3	2	1	0	1	0	1650
t	$\frac{1}{2}$	–1	2	0	0	1	800
P	–40	–50	–35	0	0	0	0

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, solve this linear programming problem. Make your method clear by stating the row operations you use.

(10)

(b) State the final values of the objective function and each variable.

(2)

(Total 12 marks)

4. A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	2	–1	1	–3
A plays 2	–3	2	–2	1

(a) Verify that there is no stable solution to this game.

(2)

(b) Find the best strategy for player A.

(9)

(Total 11 marks)

5.

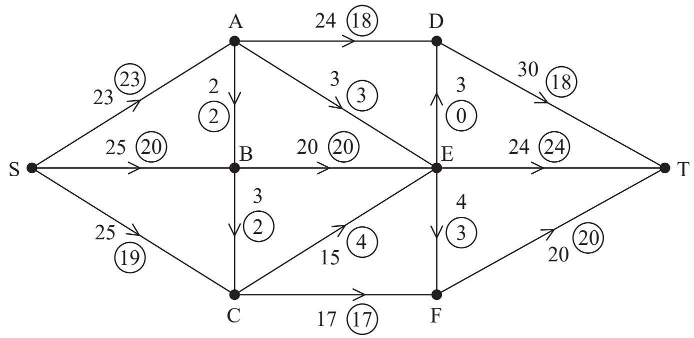


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) Complete the initialisation of the labelling procedure on Diagram 1 in the answer book by entering values along SC, AB, CE, DE and DT.

(2)

(c) Hence use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(d) Draw a maximal flow pattern on Diagram 2 in the answer book.

(2)

(e) Prove that your flow is maximal.

(2)

(Total 11 marks)

6. Three warehouses, P, Q and R, supply washing machines to four retailers, A, B, C and D. The table gives the cost, in pounds, of transporting a washing machine from each warehouse to each retailer. It also shows the number of washing machines held at each warehouse and the number of washing machines required by each retailer. The total cost of transportation is to be minimised.

	A	B	C	D	Supply
P	11	22	13	17	25
Q	21	8	19	14	27
R	15	10	9	12	28
Demand	18	16	20	26	

Formulate this transportation problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.
You do not need to solve this problem.

(Total 7 marks)

7. A company assembles microlight aircraft.
They can assemble up to four aircraft in any one month, but if they assemble more than three they will have to hire additional space at a cost of £1000 per month.
They can store up to two aircraft at a cost of £500 each per month.
The overhead costs are £2000 in any month in which work is done.

Aircraft are delivered at the end of each month. There are no aircraft in stock at the beginning of March and there should be none in stock at the end of July.
The order book for aircraft is

Month	March	April	May	June	July
Number ordered	3	4	2	4	3

Use dynamic programming to determine the production schedule which minimises the costs. Show your working in the table provided in the answer book and state the minimum production cost.

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Wednesday 24 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Nil	D2 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 8 pages in this question paper. The answer book has 20 pages. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P44853A

P 4 4 8 5 3 A

W850/R6690/57570 5/5/1/1/1/1/

Turn over

PEARSON

Write your answers in the D2 answer book for this paper.

1. The tableau below is the initial tableau for a linear programming problem in *x*, *y* and *z*. The objective is to maximise the profit, *P*.

Basic variable	*x*	*y*	*z*	*r*	*s*	*t*	Value
r	2	–4	1	1	0	0	15
s	4	2	–8	0	1	0	20
t	1	–1	4	0	0	1	8
P	–3	2	7	0	0	0	0

(a) Perform **one** iteration of the Simplex algorithm to obtain a new tableau, *T*. State the row operations you use.
(5)

(b) Write down the profit equation given by *T* and state the current values of the slack variables.
(2)

(Total 7 marks)

2. Rani and Greg play a zero-sum game. The pay-off matrix shows the number of points that Rani scores for each combination of strategies.

	Greg plays 1	Greg plays 2	Greg plays 3
Rani plays 1	–3	1	2
Rani plays 2	0	2	1
Rani plays 3	2	4	–5

(a) Explain what the term ‘zero-sum game’ means.
(1)

(b) State the number of points that Greg scores if he plays his strategy 3 and Rani plays her strategy 3.
(1)

(c) Verify that there is no stable solution to this game.
(3)

(d) Reduce the game so that Greg has only two possible strategies. Write down the reduced pay-off matrix for Greg.
(3)

(e) Find the best strategy for Greg and the value of the game to him.
(8)

(Total 16 marks)

3.

	A	B	C	D	E	F	G
A	–	x	41	43	38	21	30
B	x	–	27	38	19	29	51
C	41	27	–	24	37	35	40
D	43	38	24	–	44	52	25
E	38	19	37	44	–	20	28
F	21	29	35	52	20	–	49
G	30	51	40	25	28	49	–

The network represented by the table shows the least distances, in km, between seven theatres, A, B, C, D, E, F and G.

Jasmine needs to visit each theatre at least once starting and finishing at A. She wishes to minimise the total distance she travels. The least distance between A and B, is x km, where $21 < x < 27$

(a) Using Prim’s algorithm, starting at A, obtain a minimum spanning tree for the network. You should list the arcs in the order in which you consider them.
(2)

(b) Use your answer to (a) to determine an initial upper bound for the length of Jasmine’s route.
(1)

(c) Use the nearest neighbour algorithm, starting at A, to find a second upper bound for the length of the route.
(2)

The nearest neighbour algorithm starting at F gives a route of F – E – B – A – G – D – C – F.

(d) State which of these two nearest neighbour routes gives the better upper bound. Give a reason for your answer.
(2)

Starting by deleting A, and all of its arcs, a lower bound of 159 km for the length of the route is found.

(e) Find x, making your method clear.
(3)

(f) Write down the smallest interval that you can be confident contains the optimal length of Jasmine’s route. Give your answer as an inequality.
(2)

(Total 12 marks)

4.

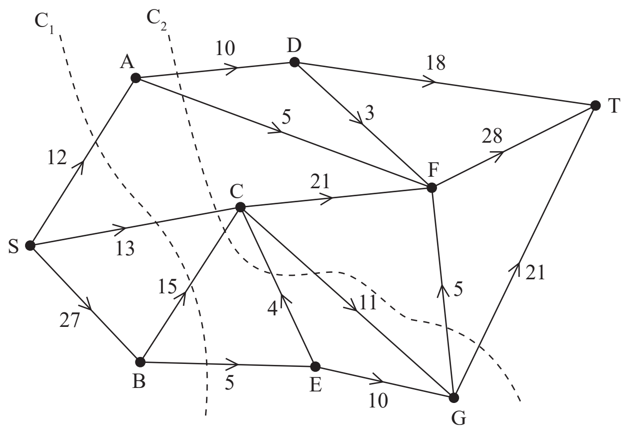


Figure 1

Figure 1 shows a capacitated network. The capacity of each arc is shown on the arc. Two cuts C_1 and C_2 are shown.

(a) Find the capacity of each of the two cuts.
(2)

Given that one of these two cuts is a minimum cut,

(b) write down the maximum possible flow through the network.
(1)

Given that the network now has a maximal flow from S to T,

(c) determine the flow along arc SB.
(1)

(d) Explain why arcs GF and GT cannot both be saturated.
(1)

Given that arcs EC, AD and DF are saturated and that there is no flow along arc GF,

(e) determine a maximum flow pattern for this network and draw it on Diagram 1 in the answer book. You do not need to use the labelling procedure to determine the maximum flow.
(2)

(Total 7 marks)

5. The table shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of three sales points, P, Q and R. It also shows the stock held at each supply point and the amount required at each sales point. A minimum cost solution is required.

	P	Q	R	Supply
A	20	5	13	74
B	7	15	8	58
C	9	14	21	63
D	22	16	10	85
Demand	145	57	78	

The north-west corner method gives the following initial solution.

	P	Q	R	Supply
A	74			74
B	58			58
C	13	50		63
D		7	78	85
Demand	145	57	78	

(a) Taking AQ as the entering cell, use the stepping stone method to find an improved solution. Make your route clear.
(2)

(b) Perform one further iteration of the stepping stone method to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, route, entering cell and exiting cell.
(4)

(c) Determine whether your current solution is optimal. Justify your answer.
(3)

(d) State the cost of the solution you found in (b).
(1)

(e) Formulate this problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.
(7)

(Total 17 marks)

6.

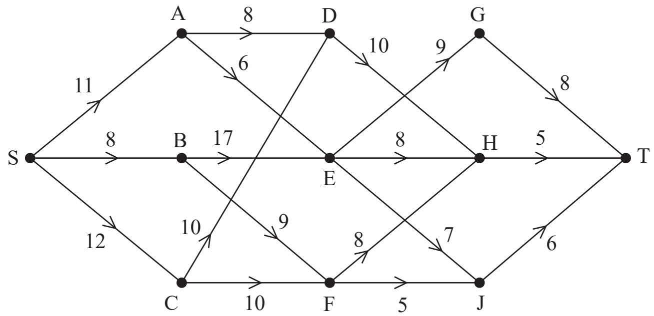


Figure 2

The staged, directed network in Figure 2 represents a series of roads connecting 11 towns, S, A, B, C, D, E, F, G, H, J and T. The number on each arc shows the weight limit, in tonnes, for the corresponding road. Janet needs to drive a truck from S to T, passing through exactly three other towns. She needs to find the maximum weight of the truck that she can use.

(a) Write down the type of dynamic programming problem that Janet needs to solve.
(1)

(b) Use dynamic programming to complete the table in the answer book.
(10)

(c) Hence find the maximum weight of the truck Janet can use.
(1)

(d) Write down the route that Janet should take.
(1)

Janet intends to ask for the weight limit to be increased on one of the three roads leading directly into T. Janet wishes to maximise the weight of her truck.

(e) (i) Determine which of the three roads she should choose and its new minimum weight limit.

(ii) Write down the maximum weight of the truck she would be able to use and the new route she would take.
(3)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END