Edexcel Maths D2

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2003-2015

Solutions

1. (a)

	A(I)	A(II)
B(I)	3	–4
B(II)	–2	1
B(III)	–5	4

B2, 1, 0 2

(b) e.g. matrix becomes

	A(I)	A(II)
B(I)	9	2
B(II)	4	7
B(III)	1	10

M1

Defines variables (–including non-zero constants) B1

e.g. maximise $P = V$

subject to $v - 9q_1 - 4q_2 - q_3 + r = 0$

$v - 2q_1 - 7q_2 - 10q_3 + s = 0$

$q_1 + q_2 + q_3 + t = 1$

OR

e.g. minimise $P = x_1 + x_2 + x_3$ where $x_i = \frac{q_i}{v}$

subject to $9x_1 + 4x_2 - x_3 + r = 1$

$2x_1 - 7x_2 - 10x_3 + s = 1$ A2 ft, 1 ft, 0 4

OR

e.g. maximise $P = V$

$v - 8q_1 - 3q_2 + R = 0$

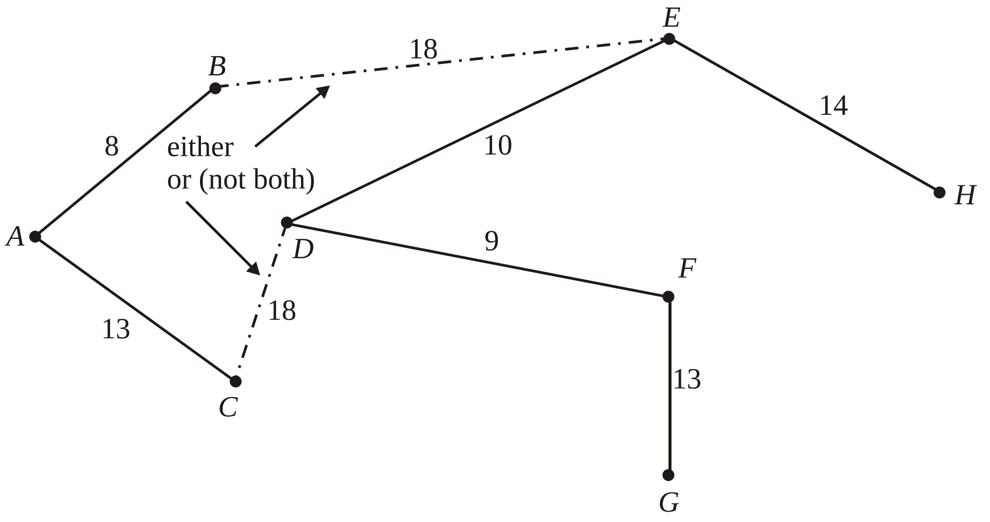
$v - 8q_1 - 3q_2 + S = 0$

[6]

2. (a) In the *practical* TSP each vertex must be visited *at least once* B1

In the *classical* TSP each vertex must be visited *exactly once* B1 2

(b) AB, DF, DE, (reject EF), $\begin{Bmatrix} FG \\ AC \end{Bmatrix} EH \begin{Bmatrix} DC \\ \text{or} \\ BE \end{Bmatrix}$ M1 A1



B1 3

(c) Initial upper bound = 2 × 85 = 170 km M1 A1 2

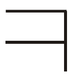
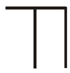
(d) e.g. when *CD* is part of the tree
use *GH* (saving 26) and *BD* (saving 19) giving new u. b.
of 125 km M1
Tour *A B D E H G F D C A* A1 3

(or e.g. when *BE* is part of the tree
use *CG* (saving 40) giving new upper bound of 130 km;
Tour *A B E H E D F G C A*)

[10]

3. (a) (i) Either *rows then columns* giving

	I	II	III	IV			I	II	III	IV		
C	0	22	16	4		*C*	0	4	0	4		
J	1	20	24	0	then	*J*	1	2	8	0	M1, A1, A1	3
N	1	18	18	0		*N*	1	0	2	0		
S	1	23	26	0		*S*	1	5	10	0		

3 lines only needed  (or ) least element 1 so

	I	II	III	IV			I	II	III	IV		
C	0	4	0	5		*C*	0	5	0	5		
J	0	1	7	0	or	*J*	0	2	7	0	M1, A1, A1	3
N	1	0	2	1		*N*	0	0	1	0		
S	0	4	9	0		*S*	0	5	9	0		

Alternative

(a) (i) or *columns then rows* giving

	I	II	III	IV		
C	1	2	0	6	(then no change)	
J	2	0	8	2		M1, A1
N	4	0	4	4		
S	0	1	8	0		

3 lines only needed  and *either* row 1 *or* column 3

if row 1: least uncovered 2

	I	II	III	IV
C	1	4	0	6
J	0	0	6	0
N	2	0	2	2
S	0	3	8	0

if column 3: least uncovered 1

	I	II	III	IV
C	0	2	0	5
J	1	0	8	1
N	3	0	4	3
S	0	2	9	0

Then least uncovered 1 M1 A1 M1 A1 6

	I	II	III	IV
C	0	3	0	5
J	0	0	7	0
N	2	0	3	2
S	0	3	9	0

(ii) *C* – III, *J* – I or IV, *N* – II, *S* – IV or I M1 A1
83 minutes ∴ 11.23 a.m. M1 A1 4

(b) Subtracting all entries from some $n \geq 36$ (stated) M1
e.g. subtractions from 36

	I	II	III	IV
C	24	2	8	20
J	23	4	0	24
N	21	4	4	22
S	25	3	0	26

M1, A2,1,0 3

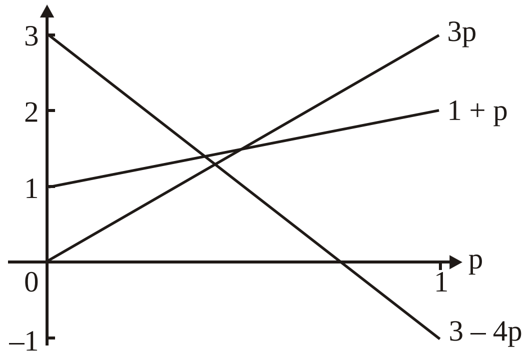
[13]

4. (a) Player *A*: row minimums are –1, 0, –3 so maximin choice is play II M1 A1
Player *B*: column maximums are 2, 3, 3 so minimax choice is play I M1 A1 4

(b) Since *A*'s maximin (0) ≠ *B*'s minimax (2) there is no stable solution B1 1

(c) For player *A* row II dominates row III, so *A* will *now* play III B2, 1, 0 2

(d) Let *A* play I with probability p and II with probability $(1 - p)$
If *B* plays I, *A*'s expected winnings are $2p + (1 - p) = 1 + p$
If *B* plays II, *A*'s expected winnings are $-p + 3(1 - p) = 3 - 4p$ M1, A2, 1, 0 3
If *B* plays III, *A*'s expected winnings are $3p$



M1

$3 - 4p = 3p \Rightarrow \underline{p = \frac{3}{7}}$ A1

A should play I with probability $\frac{3}{7}$

A should play II with probability $\frac{4}{7}$ A1

and never play III

The value of the game is $\underline{\frac{9}{7} \text{ to } A}$ A1 ft 4

[14]

5. (a) e.g.

	D	E	F
A	6		
B	0	5	
C		4	4

<u>or</u>

	D	E	F
A	6	0	
B		5	
C		4	4

M1 A1

<u>cost £470</u> A1 3

(b)

$S_A = 0, S_B = 0, S_C = -10$ | $S_A = 0, S_B = -10, S_C = -20$

$D_D = 20, D_E = 30, D_F = 40$ | $D_D = 20, D_E = 40, D_F = 50$ M1 A1

$I_{AE} = 40 - 30 = 10$ | $I_{AF} = 10 - 50 = -40$

$I_{AF} = 10 - 40 = -30$ | $I_{BD} = 20 - 10 = 10$

$I_{BF} = 40 - 40 = 0$ | $I_{BF} = 40 - 40 = 0$

$I_{CD} = 10 - 10 = 0$ | $I_{CD} = 10 - 0 = 10$ M1 A1 4

Choose *AF* as entering route

$AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow BE(-) \rightarrow BD(+) \rightarrow AD(-)$ | $AF(+) \rightarrow CF(-) \rightarrow CE(+) \rightarrow AE(-)$

Exiting route *CF* $\theta = 4$ | Exiting route *AE* $\theta = 0$ M1 A1 ft

	D	E	F
A	2		4
B	4	1	
C		8	

	D	E	F
A	6		0
B		5	
C		4	4

A1 3

$S_A = 0, S_B = 0, S_C = -10$

$D_D = 20, D_E = 30, D_F = 10$

$I_{AE} = 10, I_{BF} = 30,$

$I_{CD} = 0, I_{CF} = 30$

$\therefore$ optimal, cost £350

$S_A = 0, S_B = 30, S_C = 20$

$D_D = 20, D_E = 0, D_F = 10$

$I_{AE} = 40, I_{BD} = -30,$

$I_{BF} = 20, I_{CD} = -30$ M1 A1 A1

$CD(+) \rightarrow AD(-) \rightarrow AF(+) \rightarrow CF(-)$

$\theta = 4$

	D	E	F
A	2		4
B		5	
C	4	4	

$S_A = 0$, $S_B = 0$, $S_C = -10$
$D_D = 20$, $D_E = 30$, $D_F = 10$
$I_{AE} = 10$, $I_{BD} = 0$, $I_{BF} = 30$, $I_{CF} = 30$
$\therefore$ optimal, cost £350 A1 7

[14]

6. (a) Total cost = 2 × 40 + 350 + 200 = £630 M1 A1 2

(b)

Stage	Demand	State	Action	Destination	Value		
(2) Oct	(5)	(1)	(4)	(0)	(590 + 200 = 790)		
		(2)	(3) (4)	(0) (1)	280 + 200 = 480 630 + 240 = 870	M1 A1	
		(3)	(2) 3 4	0 1 2	320 + 200 = 520 320 + 240 = 560 670 + 80 = 750	M1 A1	4
3 Sept	3	0	4	1	550 + 790 = 1340	M1 A1	
		1	3 4	1 2	240 + 790 = 1030 590 + 480 = 1070	M1 A1 ft	
4 Aug	3	0	3 4	0 1	200 + 1340 = 1540 550 + 1030 = 1580	M1 A1 ft	6

Month	August	September	October	November
Make	3	4	4	2

M1 A1

cost = £1540 A1 ft 3

(c) Profit per cycle = 13 × 1400 = 18 200
Cost of Kim's time = £2000
Cost of production = £1540 B1

$\therefore$ Total profit = 18 200 – 3540 M1
= <u>£14 660</u> A1 ft 3

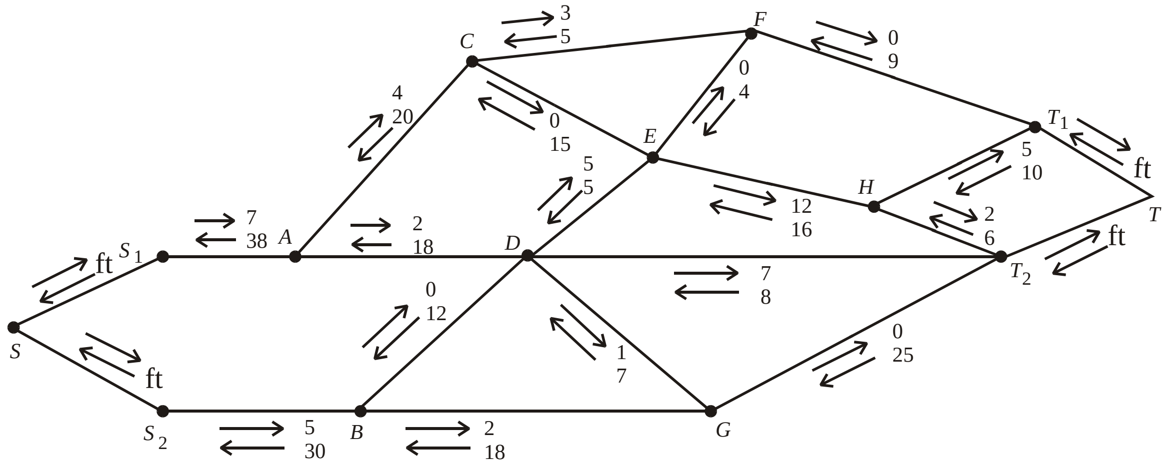
[18]

7. (a) Adds S and T and arcs M1
$SS_1 \geq 45$, $SS_2 \geq 35$, $T_1T \geq 24$, $T_2T \geq 58$ A1 2

(b) Using conservation of flow through vertices $x = 16$ and $y = 7$ B1 B1 2

(c) $C_1 = 86$, $C_2 = 81$ B1 B2 3

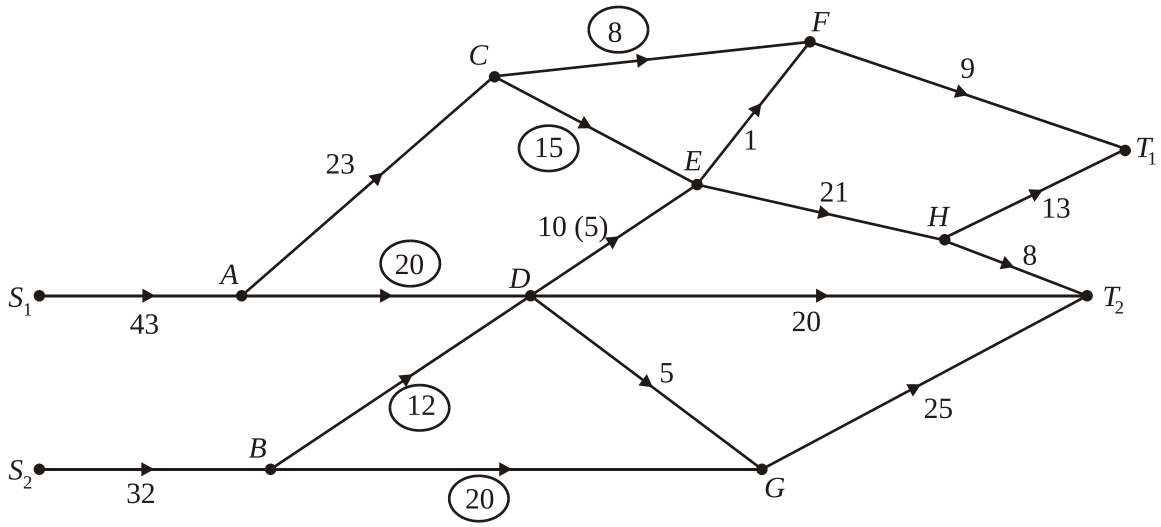
(d)



M1 A1
dM1

e.g. $S\,S_1\,A\,D\,E\,H\,T_2\,T$ – 2 A1
$S\,S_1\,A\,C\,F\,E\,H\,T_1\,T$ – 3 A1
$S\,S_2\,B\,G\,D\,T_2\,T$ – 2 A1 6

(e) e.g.:



M1 A1
Flow 75 A1 3

(f) Max flow – min cut theorem cut through dM1
CF, CE, AD, BD, BG (value 75) A1 2

[18]

8. (a) $2x + 3y + 4z \leq 8$ B1

$3x + 3y + z \leq 10$ B1

$P = 8x + 9y + 5z$ B1 3

(b)

↓ (column y)

b.v	x	y	z	r	s	Value
r	2	(3)	4	1	0	8
s	3	3	1	0	1	10
P	–8	–9	–5	0	0	0

↓ (column x)

b.v	x	y	z	r	s	Value	
y	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{\ }{3}$	$R_1 \div 3$
s	(1)	0	–3	–1	1	2	$R_2 - 3R_1$
P	–2	0	7	3	0	24	$R_3 + 9R_1$

b.v	x	y	z	r	s	Value			
								M1	
y	0	1	$\frac{10}{3}$	1	$-\frac{2}{3}$	$\frac{4}{3}$	$R_1 - \frac{2}{3} R_2$	A1	
x	1	0	–3	–1	1	2		M1	
P	0	0	1	1	2	28	$R_3 + 2R_2$	A1	8

(c) $P = 28$ M1

$x = 2$, $y = \frac{4}{3}$ A1

$z = 0$, $r = 0$, $s = 0$ A1 3

[14]

1. (a) A game in which the gain to one player is equal to the loss of the other — B2, 1, 0 — 2

(b) If there is a <u>stable solution</u>(s) a_{ij} in a game, the <u>location</u> of this stable solution is called the saddle point. It is the point(s) where row maximum = column maximum. — B2, 1, 0 — 2

[4]

2. Subtract all terms from some $n \geq 35$, e.g.35

4	11	3	0
19	25	16	13
16	21	15	14
17	20	14	12

M1
A1 — 2

Reducing rows then columns

2	4	2	0
4	5	2	0
0	0	0	0
3	1	1	0

B1

Minimum uncovered 1

1	3	1	0
3	4	1	0
0	0	0	1
2	0	0	0

M1
A1 ft — 3

Minimum uncovered 1

0	2	0	0
2	3	0	0
0	0	0	2
2	0	0	1

M1
A1 ft

e.g. matching	*D - A*		*A*		*M*		*S*	A1 ft
	H – S	*or*	*S*	*or*	*S*	*or*	*M*	
	K - M		*L*		*A*		*A*	
	T - L		*M*		*L*		*L*	A1 4

Total 88 points

[9]

3. (a) (i) Minimum connector using Prim: *AC*, *CB*, *CD*, *CE* — M1 A1

Length = 98 + 74 + 82 + 103 = 357 — {1, 3, 2, 4, 5}

So upper bound = 2 × 357 = 714 — M1 A1 — 4

(ii) *A* (98) *C* (74) *B* (131) *D* (134) *E* (115) *A* — M1 A1

Length = 98 + 74 + 131 + 134 + 115 = 552 — A1 — 3

(b) Residual minimum connector is *AC*, *CB*, *CD* — M1

Length 254 — A1

Lower bound = 254 + 103 + 115 = 472 — M1 A1 — 4

(c) 472 ≤ solution ≤ 552 — B1 ft — 1

[12]

4. (a)

$$\begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix} \quad \begin{matrix} \text{row min} \\ -4 \\ -2 \leftarrow \max \end{matrix}$$

Col. max 2 1 3 — M1 A1

↑ min (1)

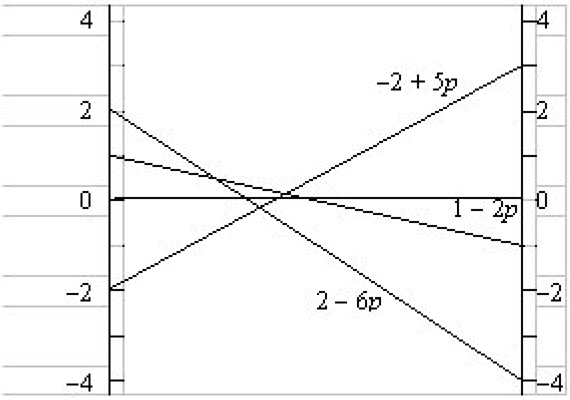
$-2 \neq 1 \therefore$ not stable — A1 — 3

(b) Let Emma play R_1 with probability p

If Freddie plays C_1, Emma's winnings are $-4p + 2(1 - p) = 2 - 6p$

C_2, Emmas winnings are $-p + 1(1 - p) = 1 - 2p$ — M1 A1

C_3, Emma's winnings are $3p - 2(1 - p) = -2 + 5p$ — A1 — 3



M1 A1 ft — 2

Need intersection of $2 - 6p$ and $-2 + 5p$ — M1

$2 - 6p = -2 + 5p$,

$4 = 11p$,

$p = \frac{4}{11}$ — A1

So Emma should play R_1 with probability $\frac{4}{11}$

R_2 with probability $\frac{7}{11}$ A1 ft 3

The value of the game is $-\frac{2}{11}$ to Emma

(c) Value to Freddie $\frac{2}{11}$, matrix $\begin{pmatrix} 4 & -2 \\ 1 & -1 \\ -3 & 2 \end{pmatrix}$ B1 ft B1, B1 3

[14]

5. (a) Idea of many supply and demand points and many units to be moved. Costs are variable and dependent upon the supply and demand points, need to minimise costs. Practical costs proportional to number of units B2, 1, 0 2

(b) Supply = 120 Demand = 110 so not balanced B1 1

(c) Adds 0, 0, 0, 10 to column *f* M1 A1

M1 A1

	d	*e*	*f*
A	45		
B	5	30	
C		30	10

Cost 545 B1 ft 5

(d) $R_1 = 0$ $R_2 = -1$ $R_3 = -3$

$k_1 = 5$ $k_2 = 7$ $k_3 = 3$ M1 A1

$Ae = 3 - 0 - 7 = -4$

$Af = 0 - 0 - 3 = -3$ M1 A1 ft

$Bf = 0 + 1 - 3 = -2$ A1 ft 5

$Cd = 2 + 3 - 5 = 0$

(e) $Ae^+ \rightarrow Be^- \rightarrow Bd^+ \rightarrow Ad^-$ send 30 M1 A1 ft

	d	*e*	*f*
A	15	30	
B	35		
C		30	10

depM1

A1 ft

Cost 425 A1 5

[18]

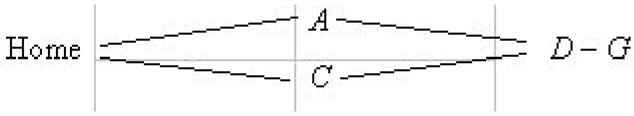
6. (a) Stage – Number of weeks to finish B1

State – Show being attended B1

Action – Next journey to undertake B1 3

(b) eg

Stage	State	Action	Value		
1	*F* *G* *H*	*F* – Home *G* – Home *H* – Home	500 – 80 = 420 * 700 – 90 = 610 * 600 – 70 = 530 *	M1 A1	
2	*D*	*DF* *DG* *DH*	1500 – 200 + 420 = 1720 1500 – 160 + 610 = 1950 * 1500 – 120 + 530 = 1910	M1 A1ft A1 ft	
	E	*EF* *EG* *EH*	1300 – 170 + 420 = 1550 1300 – 100 + 610 = 1810 * 1300 – 110 + 530 = 1720	A1	
3	A	*AD* *AE*	900 – 180 + 1950 = 2670 * 900 – 150 + 1810 = 2560	M1 A1 ft	
	B	*BD* *BE*	800 – 140 + 1950 = 2610 * 800 – 120 + 1810 = 2490	A1 ft	
	C	*CD* *CE*	1000 – 200 + 1950 = 2750 * 1000 – 210 + 1810 = 2600	A1	
4	Home	Home – *A* Home – *B* Home – *C*	–70 + 2670 = 2600 * –80 + 2610 = 2530 –150 + 2750 = 2600 *	M1 A1	12

(c) 

B2 ft 1 ft 0

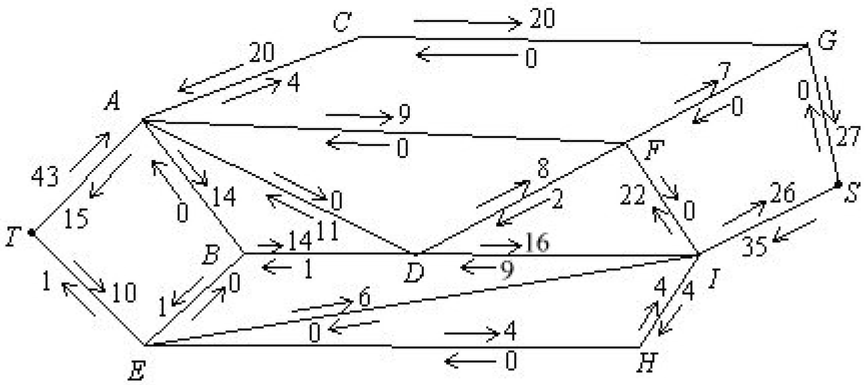
Total profit £2 600 B1 ft 3

[18]

7. (a) $x = 9$, $y = 16$ B1 B1 2

(b) Initial flow = 53 – Either finds a flow-augmenting route
or demonstrates not enough saturated arcs for a minimum cut B1 B1 2

(c)

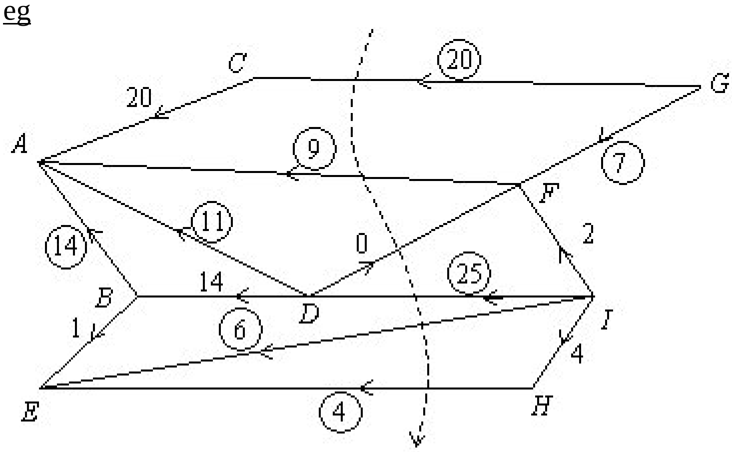


M1A1 2

e.g. *IDA* – 9 A1

IFDA – 2 A1
max flow – 64 B1 3

(d) eg M1 A1 2



(e) Max flow – min cut M1
Finds a cut *GC*, *AF*, *DF*, *DJ*, *EI*, *EH* value 64 A1 2
Note: must not use supersource or supersink arcs.

[13]

8. (a) Yes, there are no negative values in the profit row B1 1

(b) $p = 63, x = 0, y = 7, z = 0, r = \frac{9}{2}, s = \frac{2}{3}, t = 0$ M1, A1, A1, 3

(c) $\frac{63}{7} = 9$ M1, A1 2

[6]

9. (a) $C_1 = 7 + 14 + 0 + 14 = 35$ B1
$C_2 = 7 + 14 + 5 = 26$ B1
$C_3 = 8 + 9 + 6 + 8 = 31$ B1 3

(b) Either Min cut = Max flow and we have a flow of 26 and a cut of 26
or C2 is through saturated arcs B1 1

(c) Using EJ (capacity 5) e. g – will increase flow by 1– ie increase it to M1
27 since only one more unit can leave E. A1
- BEJL – 1

Using FH (capacity 3) e. g.– will increase flow by 2 – ie increase it
to 28 since only two more units can
leave F.
- BFHJL - 2
Thus choose option 2 add FH capacity 3. A1 3

[7]

10. (a) Maximise $P = 50x + 80y + 60z$ — B1

subject to $x + y + 2z \leq 30$

$x + 2y + z \leq 40$

$3x + 2y + z \leq 50$ — B3, 2, 1,0 — 4

where $x, y, z \geq 0$

(b) Initialising tableau — B1ft M1

bv	x	y	z	r	s	t	value
r	1	1	2	1	0	0	30
s	1	(2)	1	0	1	0	40
t	3	2	1	0	0	1	50
p	–50	–80	–60	0	0	0	0

chooses correct pivot, divides R_2 by 2 — A1 ft

states correct row operation $R_1 - R_2$, $R_3 - 2R_2$, $R_4 + 80R_2$, $R_2 \div 2$ — A1 — 4

(c) The solution found after one iteration has a stack of 10 units of black per day — B2, 1, 0 — 2

(d) (i)

bv	x	y	z	r	s	t	value
r	$\frac{1}{2}$	0	($\frac{3}{2}$)	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	–1	1	10
p	–10	0	–20	0	40	0	1600

(given)

bv	x	y	z	r	s	t	value
z	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	$6\frac{2}{3}$
y	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	$16\frac{2}{3}$
t	2	0	0	0	–1	1	10
p	$-3\frac{1}{3}$	0	0	$13\frac{1}{3}$	$33\frac{1}{3}$	0	$1733\frac{1}{3}$

$R_1 \div \frac{3}{2}$ — M1 A1

$R_2 - \frac{1}{2} R_1$

R_3 – no change — M1 A1 — 4

$R_4 + 20R$

(ii) not optimal, a negative value in profit row — B1ft

(iii) $x = 0$ $y = 16\frac{2}{3}$ $z = 6\frac{2}{3}$ — M1 A1ft

$p = £1733.33$ $r = 0, s = 0, t = 10$ — A1ft — 4

[18]

Solutions

1. (a)

	D	E	F
A	20	4	
B		26	6
C			14

M1
A1 2

(b) $S_A = 0$ $S_B = -1$ $S_C = 7$ M1
$D_P = 21$ $D_E = 24$ $D_F = 18$ A1

$I_{13} = I_{AF} = 16 - 0 - 18 = -2$
$I_{21} = I_{BD} = 18 + 1 - 21 = -2$ M1
$I_{31} = I_{CD} = 15 - 7 - 21 = -13$ (*) A1ft
$I_{32} = I_{CE} = 19 - 7 - 24 = -12$ A1ft 5

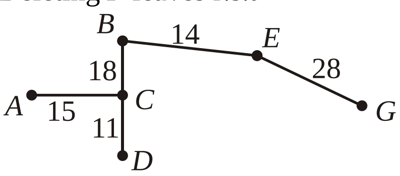
(c) eg $CD(+) \rightarrow AD(-) \rightarrow AE(+) \rightarrow BE(-) \rightarrow BF(+) \rightarrow CF(-)$ $\theta = 14$ M1 A1ft

	D	E	F
A	6	18	
B		12	20
C	14		

A1ft A1
cost £1384
4

[11]

2. (a) Deleting F leaves r.s.t



M1
r.s.t. length = <u>86</u> A1
s_0 lower bound = 86 + 16 + 19 = <u>121</u> M1 a1 4

$\therefore$ <u>best L.B is 129</u> by deleting C(ft from <u>choice</u>) B1 ft 1

(b) Add 33 to BF and FB B1
Add 31 to DE and ED B1 2

(c) Tour, visits each vertex, order correct using table of least distances. M1 A1
e.g. F C D A B E G F (actual route F C D C A B E G F)A1
upper bound of 138 km A1 4

[11]

3. Let x_{ij} be number of units transported from i to j

where $i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ — B1 — 1

warehouse — supermarket

objective minimise "C" = $3x_{WJ} + 6x_{WK} + 3x_{WL} +$ — B1

$5x_{XJ} + 8x_{XK} + 4x_{XL} +$ — B1 — 2

$2x_{YJ} + 5x_{YK} + 7x_{YL}$

subject to $x_{WJ} + x_{WK} + x_{UL} = 34$ — M1 A1

$x_{XJ} + x_{XK} + x_{XL} = 57$

$x_{YJ} + x_{YK} + x_{YL} = 25$

$x_{WJ} + x_{XY} + x_{YJ} = 20$ — A1 — 3

$x_{WK} + x_{XK} + x_{YK} = 56$

$x_{WL} + x_{XL} + x_{YL} = 40$

$x_{ij} \geq 0 \quad \forall \; i \in \{W, X, Y\}$ and $j \in \{J, K, L\}$ — B1 — 1

[7]

4. (a) The route from start to finish in which the arc of minimum length is as large as possible. — B2, 1, 0

e.g. must be pratical, involve choice of route, have are 'cuts'. — B1 — 3

(b)

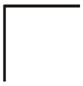
Stage	State	Action	Value		
1	H	HK	18(*)	M1 A1	2
	I	IK	19(*)		
	J	JK	21(*)		
2	F	FH	min (16,18) = 16		
		FI	min (23,19) = 19(*)	M1 A1 A1	3
		FJ	min (17,21) = 17		
	G	GH	min (20,18) = 18		
		GI	min (15,19) = 15		
		GJ	min (28,21) = 21(*)		
3	B	BG	min (18,21) = 18(*)		
	C	CF	min (25,19) = 19(*)	M1 A1ft	
		CG	min (16,21) = 16		
	D	DF	min (22,19) = 19(*)		
		DG	min (19,21) = 19(*)		
	E	EF	min(14,19) = 14(*)		
4	A	AB	min (24,18) = 18	A1ft	3
		AC	min (25,19) = 19(*)		
		AD	min (27,19) = 19(*)		
		AE	min (23,14) = 14		

(c) Routes *A C F I K*, *A D F I K*, *A D G J K* — A1ft A1ft A1ft — 3

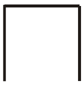
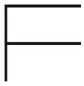
[14]

5. (a) To maximise, subtract all entries from $n \geq 30$ M1

e.g. $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$ A2,1,0 3



minimum uncovered element is 1: so $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ M1 A2ft1ft0 3

 <u>or</u>  M1

min. el. = 2 min. el = 2

$\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ A2ft1ft0 3

$A - 2 \quad B - 4 \quad C - 3 \quad D - 1$ M1 A1ft 2
$A - 3 \quad B - 4 \quad C - 1 \quad D - 2$

(b) £1160 000 B2,1,0 2

(c) Gives other solution M1 A1ft 2

[15]

6. (a) $SS_1 - 47$, $SS_2 - 87$, $T_1T - S_1$, $T_2T - T_3$ added to diagram 1 M1 A1 2

If all 4 nos. zero then M0

M1 4 arcs added correctly + 4 numbers given (diagram 1 only) condone lack of arrows

A1 c.a.o. (diagram 1 only) penalise arrow errors here

(b)

$SS_1 \rightarrow 0$, $SS_2 \rightarrow 38$, $T_1T \rightarrow 8$, $T_2T \rightarrow 20$ M1 A1 2

$\leftarrow 47$ $\leftarrow 49$ $\leftarrow 43$ $\leftarrow 53$

M1 4 arcs, 2 numbers and 2 arrows ⇄ *per arc*

A1 c.a.o.

(c) e.g. $S \; S_2 \; A \; D \; T_1 \; T$ — 2
$S \; S_2 \; C \; E \; T_2 \; T$ — 1 M1
$S \; S_2 \; C \; E \; D \; T_2 \; T$ — 10 A4,3,2,1,0
$S \; S_2 \; C \; E \; B \; D \; T_1 \; T$ — 4
Maximum flow — 113 B1 6

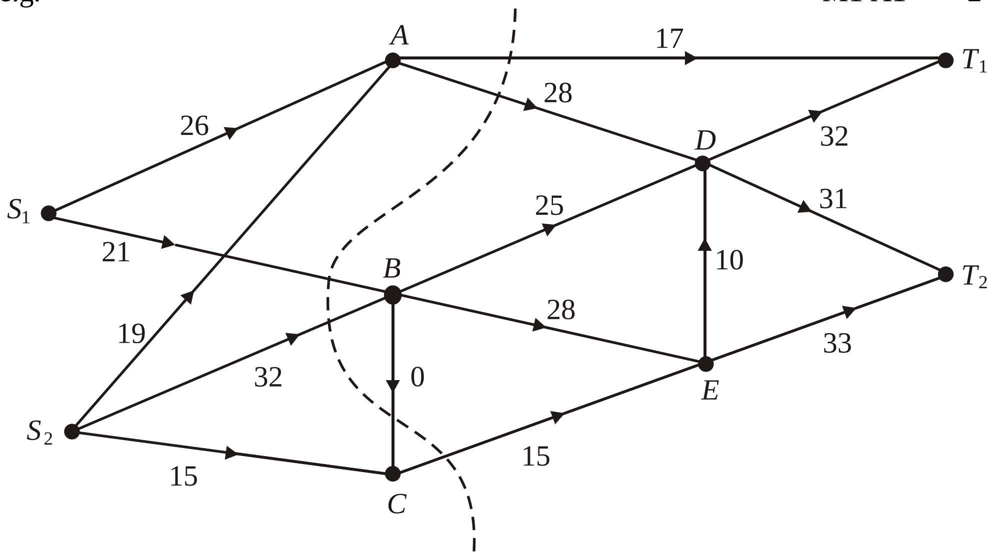
M1 2 correct routes + flows found (flow > 10 gets M0) (condone initial f.a. routes <u>only</u> if clearly repeated from new ones)

A4 all flows + routes to 15 more <u>or</u> flow increased above 17 more

A2 ≥ 3 flows + routes to 11 more <u>or</u>
A1 at least 2 flows + routes found to 5 more

B1 113 c.a.o.

(d) e.g. M1 A1 2



M1 consistent flow of 101(*), complete clear (doesn't need to ft from (c))

A1 correct flow of 113 including arrows

(e) Max flow – min cut theorem; cut AT_1, AD, S_1B, S_2B, BC, CE M1 A1 2

M1 flow of 113 + cut attempted + max flow – min cut theorem referred to (3 out of 4)

A1 c.a.o.

(f) Idea of a <u>directed</u> flow along arcs; from S to T; through a system/network; practical B2,1,0 2

B2 all 4 bits there

B1 2 out of 4 there

[16]

7. (a) A zero-sum game is one in which the sum of the gains for all players is zero. (o.e.) B1 1

(b)

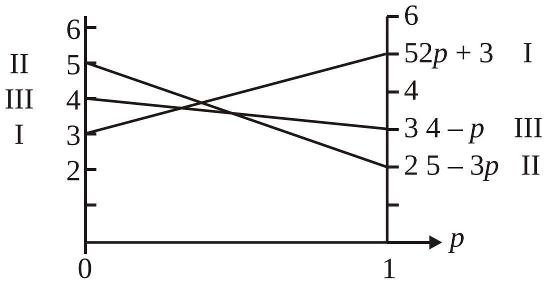
	I	II	III	
I	5	2	3	min 2
II	3	5	4	min 3 ← max
	max 5	5	4	
			↑ min	

M1 A1

Since 3 ≠ 4 not stable A1 3

(c) Let A play I with probability p
Let A play II with probability $(1 - p)$

If B plays I A's gains are $5p + 3(1 - p) = 2p + 3$
If B plays II A's gains are $2p + 5(1 - p) = 5 - 3p$ M1 A1 2
If B plays III A's gains are $3p + 4(1 - p) = 4 - p$



A2,1,0 2

Intersection of $2p + 3$ and $4 - p \Rightarrow p = \frac{1}{3}$ M1 A1ft 2

$\therefore$ A should play I $\frac{1}{3}$ of time and II $\frac{2}{3}$ of time; value (to A) = $3\frac{2}{3}$ A1ft A1ft 2

(d) Let B play I with probability q_1,
II with probability q_2 and
III with probability q_3 B1

e.g. Let $x_1 = \frac{q_1}{v}$ $x_2 = \frac{q_2}{v}$ $x_3 = \frac{q_3}{v}$ M1

Maximise $P = x_1 + x_2 = x_3$ A1

subject to $5x_1 + 2x_2 + 3x_3 \le 1$

$3x_1 + 5x_2 + 4x_3 \le 1$ A2,1,0 5

$x_1, x_2, x_3 \ge 0$

[17]

Alt 1

e.g. $\begin{bmatrix} -5 & -3 \\ -2 & -5 \\ -3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$

maximise P = V

subject to $v - q_1 - 4q_2 - 3q_3 \le 0$

$v - 3q_1 - q_2 - 2q_3 \le 0$ $q_1 + q_2 + q_3 \le 1$

$v, q_1, q_2, q_3 \ge 0$ or = 1

8. (a) r, s and t are unused amounts of bird seed (in kg), suet blocks and peanuts (in kg) that Polly has at the end of each week after shehas made up and sold her packs B2,1,0 2

B2 Ref to "unused" "bird seed, suet blocks & peanuts"

B1 Ref to "unused" <u>*or*</u> *bird seed etc* <u>*or*</u> *muddled explanation.*

"bad" gets B1 must engage with context

(b)

b.v.	x	y	z	r	s	t	value		
z	$\frac{2}{5}$	$\frac{1}{2}$	1	$\frac{1}{10}$	0	0	14	$R_1 \div 10$	M1 A1
s	$\frac{2}{5}$ (circled)	–1	0	$-\frac{2}{5}$	1	0	4	$R_2 - 4R_1$	M1

t	$-\frac{1}{5}$	$\frac{1}{2}$	0	$-\frac{3}{10}$	0	1	18	$R_3 - 3R_1$	A2ft, 1ft, 0	5
p	–90	–25	0	65	0	0	9100	$R_4 + 650R_1$		

M1 correct pivot

A1 pivot row correct c.a.o. incl.bv

M1ft correct row operations used (all 3) – at least 1 non zero or 1 term correct in each row.
Where row not ft ⇒ M0

A2ft non-pivoted rows correct; –1 each error ft on error in pivot choice only.
Penalise b.v once only

(c) $x = 0$ $y = 0$ $z = 14$ $r = 0$ $s = 4$ $t = 18$ $p = £91$ M1 A2ft, 1ft, 0 3

M1 3 variables stated – must have completed b.v. + value columns on tableau.
Any negatives M0

A1ft all 7 c.a.o. Need £91 ft but accept 9100

A1ft at least 4 c.a.o. (condone P = 9100ft)

(d) $p - 90x - 2\sqrt{y} + 65r = 9100$ (o.e.) M1 A1ft 2

M1ft P, (–)90x, (–)25y, 65r and 9100 (or 91) all present and one = sign

A1ft c.a.o. (o.e.)

(e) $p = 9100 + 90x + 25y - 65r$
So increasing x or y would increase the profit B1ft 3

B1ft stating that increasing x or y would increase profit, probably re-arranging profit equation. Generous.

(f) The $\frac{2}{5}$ in the x column and 2nd (s) row. B2ft, 1ft, 0 2

B2ft $\frac{2}{5}$ identified, x column and 2nd (s) row.

Accept ringed in last tableau

B1ft "bad" gets B1, if ft their "optional" tableau B1.

[15]

(b) *Notes*

1. Wrong pivot chosen in col 2 (–usually 4) M0 then for M1A2ft

(a)

b.v.	x	y	z	r	s	t	value	
r	–1	$2\frac{1}{2}$	0	1	$-2\frac{1}{2}$	0	–10	$R_1 - 10R_2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	15	$R_2 \div 4$
t	$-\frac{1}{2}$	$1\frac{1}{4}$ (circled)	0	0	$-\frac{3}{4}$	1	15	$R_3 - 3R_2$
p	–25	$-187\frac{1}{2}$	0	0	$162\frac{1}{2}$	0	9750	$R_4 + 650R_2$

(b)

b.v.	x	y	z	r	s	t	value	
r	$\frac{2}{3}$	$-1\frac{2}{3}$	0	1	0	$\frac{-10}{3}$	–60	$R_1 - 10R_3$
s	$\frac{2}{3}$	$-1\frac{2}{3}$	0	0	1	$\frac{-4}{3}$	–20	$R_2 - 4R_3$
z	$\frac{1}{3}$ (circled)	$\frac{2}{3}$	1	0	0	$\frac{1}{3}$	20	$R_3 \div 3$
p	$-133\frac{1}{3}$	$83\frac{1}{3}$	0	0	0	$216\frac{2}{3}$	13000	$R_4 + 650R_3$

2. *MISREADS* – use col x or col y (–2 A marks if earned)

(a)

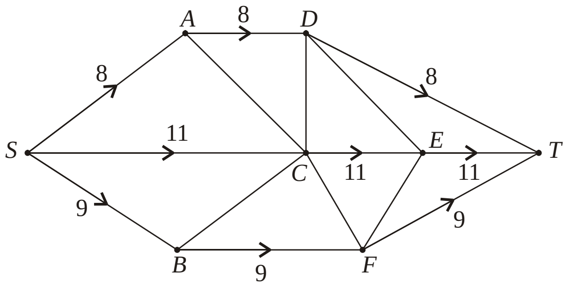
b.v.	x	y	z	r	s	t	value	
r	0	3 (circled)	2	1	–2	0	20	$R_1 - 4R_2$
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	30	$R_2 \div 2$
t	0	$1\frac{1}{2}$	1	0	$-\frac{1}{2}$	1	30	$R_3 - R_2$
p	0	–175	50	0	175	0	10500	$R_4 + 350R_2$

(b)

b.v.	x	y	z	r	s	t	value	
y	$\frac{4}{5}$	1	2	$\frac{1}{5}$	0	0	28	$R_1 - 5$
s	$1\frac{1}{5}$ (circled)	0	2	$-\frac{1}{5}$	1	0	32	$R_2 - R_1$
t	$-\frac{3}{5}$	0	–1	$-\frac{2}{5}$	0	1	4	$R_3 - 2R_1$
p	–70	0	50	70	0	0	9800	$R_4 + 350R_2$

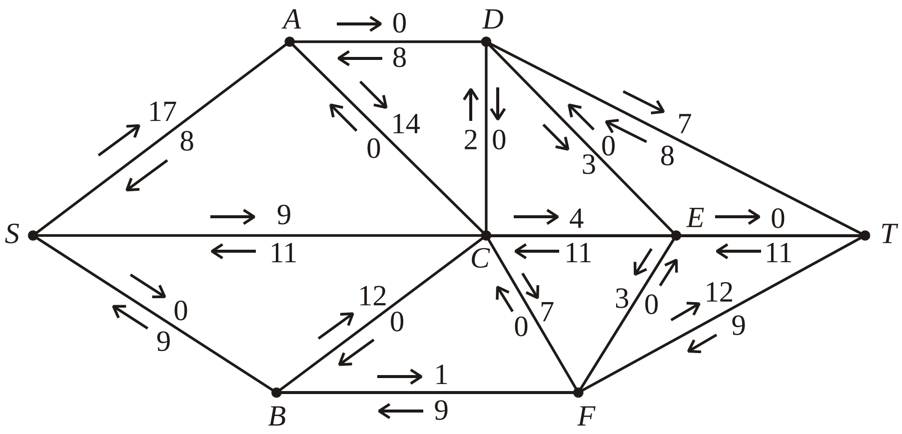
9. (a) SADT – 8 SCET – 11 SBFT – 9 B2, 1, 0

(b)



B1 3

(c) (i)



M1
A1 2

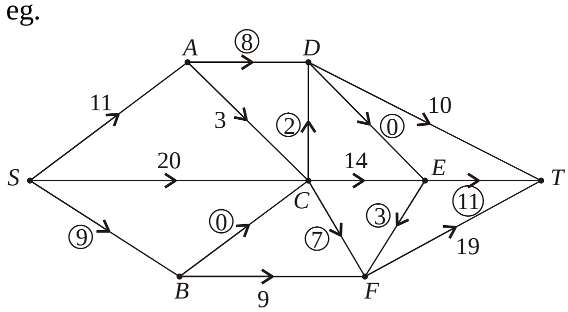
e.g.

S A C D T – 2   S C F T – 6   A1

S A C E F T – 3   S A C F T – 1   A1 3

max flow 40

(ii) eg.



M1 A1 2

(iii) Max flow – min cut theorem M1

cut AD, CD, DE, ET, CF, BC, SB ie {S A C E } {B D F T} A2, 0 3

(d) Idea of a <u>directed</u> flow through a <u>system</u> of arcs from <u>S to T</u> B1 1

<u>practical</u>

[14]

Solutions

1. To maximise, subtract all entries from $n \geq 278$ — M1

e.g.

$$\begin{bmatrix} 11 & 6 & 2 & 17 \\ 14 & 7 & 0 & 15 \\ 11 & 5 & 3 & 15 \\ 17 & 9 & 4 & 21 \end{bmatrix}$$

A1 — 2

Reduce rows

$$\begin{bmatrix} 9 & 4 & 0 & 15 \\ 14 & 7 & 0 & 15 \\ 8 & 2 & 0 & 12 \\ 13 & 5 & 0 & 17 \end{bmatrix}$$

then columns

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 6 & 5 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 5 \end{bmatrix}$$

M1 A1ft A1ft — 3

Min element = 1

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 5 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}$$

M1 A1ft A1 — 3

Min element = 1

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 5 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 4 & 1 & 0 & 3 \end{bmatrix}$$

or

Min element = 2

$$\begin{bmatrix} 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

M1 A1ft A1ft — 3

then min element 1

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 4 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$$

optimal

So A – H or H — M1
B – P or S — A1 — 2
C – S or I
D – I or P
(both £1077)

[13]

2. e.g.

Stage	State	Action	Dest	Value
1 (Sept)	2	2	0	200 + 200 = 400 *
	1	3	0	200 + 100 = 300 *
	0	4	0	200 = 200 *
2 (Aug)	2	5	2	200 + 200 + 500 + 400 = 1300
		4	1	200 + 200 + 300 = 700
		3	0	200 + 200 + 200 = 600 *
	1	5	1	200 + 100 + 500 + 300 = 1100
		4	0	200 + 100 + 200 = 500 *
	0	5	0	200 + 500 + 200 = 900 *
3 (Jul)	2	5	0	200 + 200 + 500 + 900 = 1800 *
4 (Jun)	2	3	2	200 + 200 + 1800 = 2200 *
	1	4	2	200 + 100 + 1800 = 2100 *
	0	5	2	200 + 500 + 1800 = 2500 *
5 (May)	0	5	2	200 + 500 + 2200 = 2900
	1	4	2	200 + 2100 = 2300 *
	0	5	2	200 + 2500 = 2700 *

Month	May	June	July	August	September	
production schedule	4	4	5	5	4	M1 A1ft

Cost £2300 A1ft 3

[12]

3. Let x_{ij} be the number of units transported from i to j , in 1000 litres
where $i \in \{F, G, H\}$ and $j \in \{S, T, U\}$ B2, 1, 0 2

Minimise $C = 23\,x_{fs} + 31x_{fft} + 46x_{fu} +$ B1
$35\,x_{gs} + 38x_{gt} + 51\,x_{gu} +$ B1 2
$41\,x_{hs} + 50x_{ht} + 63x_{hu}$

Unbalanced

Subject to $x_{fs} + x_{ft} + x_{fu} \le 540$
$x_{gs} + x_{gt} + x_{gu} \le 789$ M1
$x_{hs} + x_{ht} + x_{hu} \le 673$ A1
$x_{fs} + x_{gs} + x_{hs} \le 257$ }
$x_{ft} + x_{gt} + x_{ht} \le 348$ } accept = here A1 3
$x_{fu} + x_{gu} + x_{hu} \le 412$ }

$x_{ij} \ge 0$ B1 1

Accepted introduction of a dummy demand methods.

[8]

4. (a) Adds zero for costs in third column — B1
Adds 14 as the demand value — B1 — 2

(b) The total supply is greater than the total demand — B2, 1, 0 — 2

(c) The solution would otherwise be degenerate — B2 — 1

(d)

		10	15	0
		J	K	L
0	A		8	1
0	B			13
–6	C	9	3	

$I_{AJ} = 12 - 0 - 10 = 2$ — M1 A1
$I_{BJ} = 8 - 0 - 10 = -2$ — A1
$I_{BK} = 17 - 0 - 15 = 2$ — A1
$I_{CL} = 0 + 6 - 0 = 6$ — 4

	J	K	L
A		$8 - \theta$	$1 + \theta$
B	θ		$13 - \theta$
C	$9 - \theta$	$3 + \theta$	

$\theta = 8$
Entering square BJ — M1
Exiting square AK — A1ft — 2

		8	13	0
		J	K	L
0	A			9
0	B	8		5
–4	C	1	11	

$I_{AJ} = 12 - 0 - 8 = 4$ — M1 A1ft
$I_{AK} = 15 - 0 - 13 = 2$ — A1ft
$I_{BK} = 17 - 0 - 13 = 4$ — A1ft
$I_{CL} = 0 + 4 - 0 = 4$ — A1 — 5

No negatives, so optimal

[16]

5. (a) Row minimums {–2, –1, –4, –2} row maximum = –1 — M1
Column maximums {1, 3, 3, 3} column minimum = 1 — A1

Since $1 \neq -1$ not stable — A1 — 3

(b) Row 2 dominates Row 3 — B1
Column 1 dominates column 4 — B1 — 2

(c) Let A play row R, with probability P_1, R_2 with probability P_2 and "R_3" with probability P_3. — B1

$$\begin{pmatrix} -2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{pmatrix} \underset{+3}{\overset{\text{eg}}{\rightarrow}} \begin{pmatrix} 1 & 4 & 6 \\ 2 & 6 & 5 \\ 4 & 1 & 2 \end{pmatrix}$$ — M1 — 2

e.g. maximise P = V — M1 A1

subject to $V - p_1 - 2p_2 - 4p_3 \leq 0$ — A4ft, 3ft, 2ft, 1ft, 0 — 6

$V - 4p_1 - 6p_2 - p_3 \leq 0$

$V - 6p_1 - 5p_2 - 2p_3 \leq 0$

$p_1 + p_2 - p_3 \leq 1$

$V, p_1, p_2, p_3 \geq 0$

OR

e.g. Let $x_i = \dfrac{p_i}{v}$ $\therefore$ $\dfrac{1}{v} = x_1 + x_2 + x_3$ — M1

minimise $P = x_1 + x_2 + x_3$ — A1

subject to $x_1 + 2x_2 + 4x_3 \geq 1$

$4x_1 + 6x_2 + x_3 \geq 1$ — A4ft 3ft 2ft 1ft 0 — 6

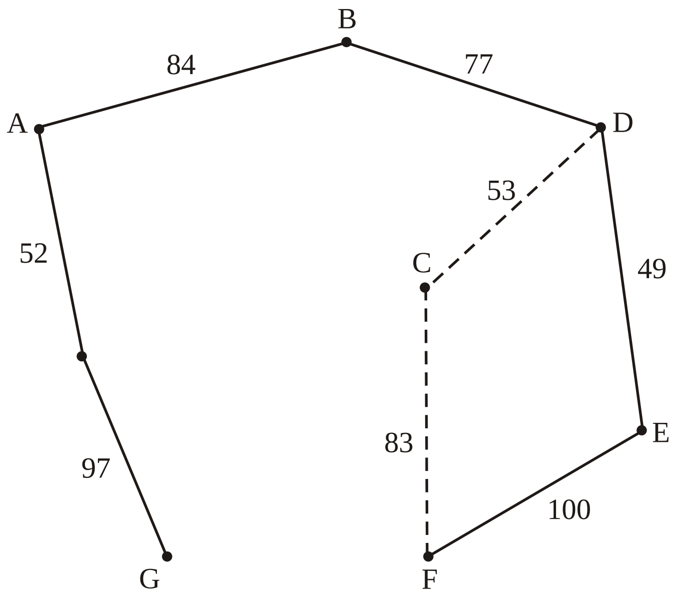
$6x_1 + 5x_1 + 2x_3 \geq 1$

$x_1 + x_2 + x_3 \geq 0$

+ other equivalent methods.

[13]

6. (a)



R.M.S.T

e.g. AH, AB, BD, DE — M1

HG, EF using prim — A1

length of R M S T = 459

$\therefore$ lower bound = 459 + 53 + 83 = 595 km (deleting c) — A1

$\therefore$ Best lower bound is 595 km, by deleting c — M1 A1ft — 5

(b) Adds 167 to AF and FA

137 to CH and HC — B1, 3, 2, 1, 0 — 4

136 to DF and FD

145 to DG and GD

(c) C D E F H A B G C — M1 A1

C		D		E		F		H		A		B		G		C
	53		49		120		115		52		84		222		92	

$\therefore$ Best upper bound is 707 starting at F — B1ft — 4

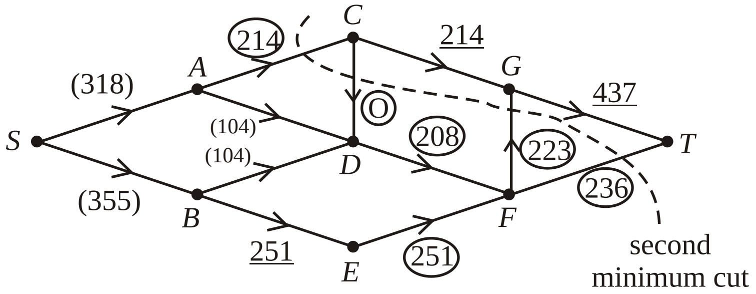
[13]

7. (a) (i) A cut is a division of the vertices of a flow network into 2 sets, one containing the source(s) and the other containing the sink(s). B1

(ii) A cut whose capacity is least B1 2

(b) $C_1 = 1038$, $C_2 = 673$ B1, B2, 0 3

(c) e.g.



M1A1A1 3

O = saturated

– = compulsory

(d) AC, CD, GF, FT B1 1

(e) DE would not allow any further flow into EF B1, 1, 0 2

DG would cross both minimum cuts – D can take extra flow, G can accept it. Flow increased by 8.6 to 759 (accept either number)

[11]

Solutions

1. (a) Any part of an optimal path is itself optimal B1

(b) The route chosen such that the maximum arc length is as small as possible B1

(c) e.g. Maximising freight by minimising fuel needed when planning multiple stage light aircraft journey B2, 1, 0

B1 cao ("port", "section", OK; "arc", "stage", activity", "event", not)
B1 cao (not min of max rate, not minimize largest arc)
B2 cao
B1 cloze "Bod" gets B1

[4]

2. Let x_{ij} = 1 if worker does task, 0 otherwise B1

where x_{ij} indicates the arc from node i to node j i.e P, Q, R j E 1, 2, 3 B1

$x_{p1} + x_{p2} + x_{p3} = 1$		$x_{p1} + x_{q1} + x_{r1} = 1$	M1	
$x_{q1} + x_{q2} + x_{q3} = 1$	and	$x_{p2} + x_{q2} + x_{r2} = 1$	A1	
$x_{r1} + x_{r2} + x_{r3} = 1$		$x_{p3} + x_{q3} + x_{r3} = 1$	A1	3

Minimise, $C = 8x_{p1} + 7x_{p2} + 3x_{p3} + 9x_{q1} + 5x_{q2} + 6x_{q3} + 10x_{r1} + 4x_{r2} + 4x_{r3}$
where C is in hundreds of pounds B1, B1 2

B1 cao
B1 defining variable – attempt
M1 at least 3 ***equations*** *– coefficients of one*
A1 cao 3 correct
A1 cao 6 correct
B1 Minimise
B1 cao (condone a slip) (- accept cost in pounds)

[7]

3. (a) Each activity must be visited once and then we return to the starting activity, this must be done in a minimum time B2, 1, 0 2

B2 cao – all 3 bits in the context
B1 cloze 'Bod' is B1 (e.g. not in contect; just 'each activity once' – but not all 3; …)

(b) 108 + 54 + 150 + 68 + 100 = 480 minutes (= 8 hours) M1 A1 2

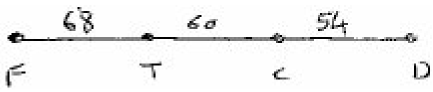
M1 (maybe implicit) attempting to add 5 values
A1 cao

(c) Use nearest neighbour B F T C D B M1 A1

64 + 68 + 60 + 54 + 150 = 396 minutes (67 hours) A1 3

M1 each vertex visited once – either NN or 2 x mst-shortcut (BD)
A1 cao incl return to B (BFTCDB)
A1 cao (396)

(d)



CT, TF, CD (Prim or Kruskal) M1 A1

182 +64 + 100 = 346 minutes M1 A1ft 4

M1 Finding correct minimum spanning tree (maybe implicit) 182 sufficient
A1 cao tree or 182
M1 adding 2 least arcs to B i.e. 100 and 64 only
A1ft cao ft from their m.s.t. value i.e. 164 and their ***<u>tree</u>*** *length*

[11]

4. (a) Adding n $\geq$ 20 to table to give B1

	H	P	R	W
A	3	5	11	9
B	3	7	8	N
C	2	5	10	7
D	8	3	7	6

Reducing rows first $\begin{bmatrix} 0 & 2 & 8 & 6 \\ 0 & 4 & 5 & n-3 \\ 0 & 3 & 8 & 5 \\ 5 & 0 & 4 & 3 \end{bmatrix}$ then columns $\begin{bmatrix} 0 & 2 & 4 & 3 \\ 0 & 4 & 1 & n-6 \\ 0 & 3 & 4 & 2 \\ 5 & 0 & 0 & 0 \end{bmatrix}$ M1 A13

Either $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 3 & 0 & n-7 \\ 0 & 2 & 3 & 1 \\ 6 & 0 & 0 & 0 \end{bmatrix}$ M1 A1ft

$\downarrow$ $\downarrow$

$\begin{bmatrix} 0 & 0 & 2 & 1 \\ 1 & 3 & 0 & n-7 \\ 0 & 1 & 2 & 0 \\ 7 & 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 3 & 1 \\ 0 & 2 & 0 & n-8 \\ 0 & 1 & 3 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}$ M1 A1ft 4

A – H P
B – R or R cost £21 000
C – W H A1

D	–	P	W	A1	2

(b) Not unique – gives the other solution M1 A1ft 2

[11]

5.

Stage	State	Action	Value		
1	H	HT	4*		
	I	IT	3*		
	J	JT	12*		
	K	KT	20*	M1 A1	2
2	D	DH	2 +4 = 6		
		DI	4 + 3 = 7*	M1 A1	
	E	EH	3 + 4 = 7*		
		EI	4 + 3 = 7*		
	F	FJ	10 + 12 = 22*		
		FK	–8 + 20 = 12		
	G	GJ	10 + 12 = 22		
		GK	17 + 20 = 37*	A1	3
3	A	AD	3 + 7 = 10		
		AE	2 + 7 = 9	M1 A1ft	
		AF	–5 + 22 = 17*		
	B	BD	3 + 7 = 10		
		BE	2 + 7 = 9		
		BF	–6 + 22 = 16*		
	C	CF	8 + 22 = 30*		
		CG	–15 + 37 = 22	A1 ft	3
4	S	SA	2 + 17 = 19		
		SB	3 + 16 = 19	M1 A1ft	2
		SC	–10 + 30 = 20*		

Route S C F J T £20 000 M1 A1 2

[12]

6. (a) Either e.g.

In an n × m problem, a degenerate solution occurs when the number of cells used is less than (n + m – 1) B2,1,0 2

or e.g. when all the demand for one destination is satisfied by all the supply from a source, before the final demand and supplies are allocated

B2 cao
B1 cloze "bod" is B1

(b) If the total supply > total demand a dummy is used to absorb the excess — B1 — 1

B1 cao must (cannot decipher copy properly)

(c) $\begin{bmatrix} 15 & & \\ 1 & 11 & 0 \\ & & 17 \end{bmatrix}$ — B1 — 1

B1 cao total of five numbers

(d) Shadow costs $S_A = 0 \quad S_B = -1 \quad S_C = -1$

$D_1 = 62 \quad D_2 = 49 \quad D_3 = 1$

Improvement indices $I_{A2} = 47 - 0 - 49 = -2^*$

$I_{A3} = 0 - 0 - 1 = -1$

$I_{C1} = 68 + 1 - 62 = 7$

$I_{C2} = 58 + 1 - 49 = 10$

	1 (62)	2 (49)	3 (1)
(0) A	15–θ	θ	
(–1) B	1+θ	11–θ	0
(–1) C			17

M1A1A1ft — 3

Entering A2, exiting B2, $\theta = 0$

Shadow costs $S_A = 0 \quad S_B = -1 \quad S_C = -1$

$D_1 = 62 \quad D_2 = 47 \quad D_3 = 1$

Improvement indices $I_{A3} = 0 - 0 - 1 = -1^*$

$I_{B2} = 48 + 1 - 47 = 2$

$I_{C1} = 68 + 1 - 62 = 7$

$I_{C2} = 58 + 1 - 47 = 12$

	1 (62)	2 (47)	3 (1)
(0) A	4–θ	11	θ
(–1) B	12+θ		0–θ
(–1) C			17

M1A1A1ft — 3

Entering A3, exiting B3, $\theta = 0$

	1 (62)	2 (47)	3 (1)
(0) A	4	11	θ
(–1) B	12		
(–1) C			17

Shadow costs $S_A = 0 \quad S_B = -1 \quad S_C = 0$

$D_1 = 62 \quad D_2 = 47 \quad D_3 = 0$ — M1 A1

Improvement indices $I_{B2} = 48 + 1 – 47 = 2$

$I_{B3} = 0 + 1 – 0 = 1$

$I_{C1} = 68 – 0 – 62 = 6$ B1

$I_{C2} = 58 – 0 – 47 = 11$

∴ Optimal

Cost 1497 units B1 4

[14]

7. (a) e.g. Maximise P = V B1

Subject to: $V – 5p_1 – 3p_2 – 6p_3 + r = 0$ M1

$V – 7p_1 – 8p_2 – 4p_3 + s = 0$ A2,1,0

$V – 2p_1 – 4p_2 – 9p_3 + t = 0$

$p_1 + p_2 + p_3 (+ u) = 1$

where V = value of game to A, P_i = probability of A playing row i

$P_i \geq 0$ and r, s, t, u are stack variables all ≥ 0 B1 5

B1 Maximise/minimise and consistent function
M1 constraints (condone non-negativity)
– at least one correct must be equations
A2 all correct
A1 at least two correct
B1 defining variables

(b) Not reducible and a three variable problem B1 1

B1 cao – both

(c) e.g.

b v	V	P_1	P_2	P_3	r	s	t	u	value		
r	**1**	–5	–3	–6	1	0	0	0	0	M1	
s	1	–7	–8	–4	0	1	0	0	0	A1	
t	1	–2	–4	–9	0	0	1	0	0		2
u	0	1	1	1	0	0	0	1	1		
P	–1	0	0	0	0	0	0	0	0		

b v	V	P_1	P_2	P_3	r	s	t	u	value	Row ops		
V	1	–5	–3	–6	1	0	0	0	0	$R_1 / 1$	M1 A1	
s	0	–2	–5	**2**	–1	1	0	0	0	$R_2 – R_1$	A1	
t	0	–3	–1	–3	–1	0	1	0	0	$R_3 – R_1$	B1ft	
u	0	1	1	1	0	0	0	1	1	R_4 stet		4
P	0	–5	–3	–6	1	0	0	0	0	$R_5 + R_1$		

b v	V	P_1	P_2	P_3	r	s	t	u	value	Row ops	
V	1	–11	–18	0	–2	3	0	0	0	$R_1 + 6R_2$	M1 A1ft

P_3	0	–1	$-\frac{5}{2}$	1	½	½	0	0	0	$R_2 / 2$	A1
t	0	0	$-\frac{17}{2}$	0	$-\frac{5}{2}$	$\frac{5}{2}$	1	0	0	$R_3 + 3R_2$	B1ft
u	0	2	$\frac{7}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	1	$R_4 – R_2$	4
P	0	–11	–18	0	–2	3	0	0	0	$R_5 + 6R_2$	

[16]

8. (a) $7x+10y+10z+r=3600$

$6x+9y+12z+s=3600$ B2,1,0

$2x+3y+4z+t=2400$

$P-35x-55y-60z=0$ B2,0 4

(b) (i)

b.v.	x	y	z	r	s	t	value	Row ops		
r	2	$\mathbf{\frac{5}{2}}$	0	1	$-\frac{5}{6}$	0	600	$R_1 – 10R_2$	A1	
z	$\frac{1}{2}$	$\frac{3}{4}$	1	0	$\frac{1}{12}$	0	300	$R_2 \div 12$	M1	
t	0	0	0	0	$-\frac{1}{3}$	1	1200	$R_3 – 4R_2$	A1ft	
P	–5	–10	0	0	5	0	1800	$R_4 + 60R_2$	B1	5

(ii)

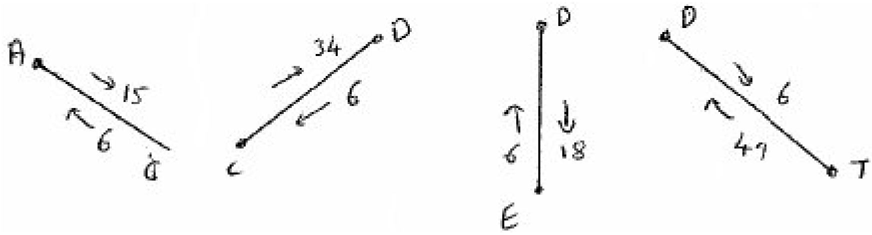
b.v.	x	y	z	r	s	t	value	Row ops	M1	
y	$\frac{4}{5}$	1	0	$\frac{2}{5}$	$-\frac{1}{3}$	0	240	$R_1 \div \frac{5}{2}$	M1	
z	$-\frac{1}{10}$	0	1	$-\frac{3}{10}$	$\frac{1}{3}$	0	120	$R_2 - \frac{3}{4}R_1$	A1ft	
t	0	0	0	0	$-\frac{1}{3}$	1	1200	R_3 stet	M1	
P	3	0	0	4	$\frac{5}{3}$	0	20400	$R_4 + 10R_1$	A1	4

(c) P = 20400 x = 0 y = 240 z = 120 M1

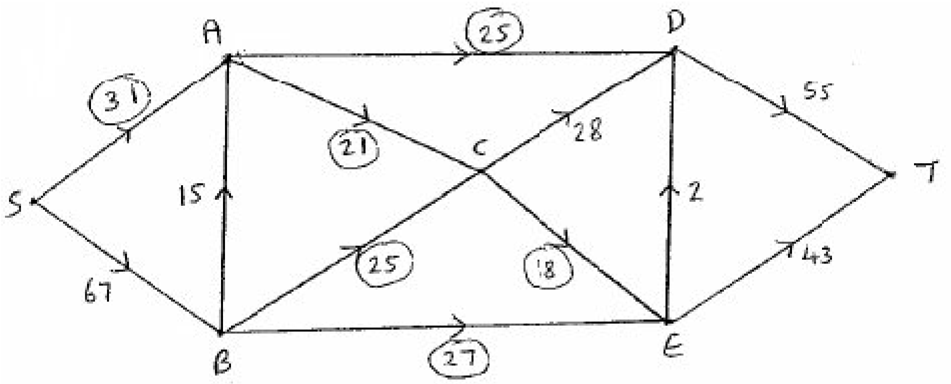
r = 0 s = 0 t = 1200 A2ft, A1ft, 0 2

[16]

9.	(a)	$C_1 = 103$, $C_2 = 177$, flow = 76	B1, B1, B1	3
	(b)		M1A1	2



(c)	e.g.	S B C D T – 6	M1	
		S B C D E T – 1	A3,2,1,0	
		S B A C D E T – 15		
		Max flow is 98	B1	5
(d)			M1A1	2



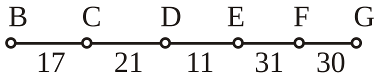
(e)	Maximum flow = minimum cut	M1	
	Cut through AD, AC, BC and BE	A1	2

[14]

Solutions

1. (a) Adds 32 to $AB + BA$ (ACB) B1
47 to $AE + EA$ ($ACDE$) B1
32 to $CE + EC$ (CDE) B1
53 to $DG + GD$ (DCG) B1 4

(b) $A \quad C \quad B \quad D \quad E \quad F \quad G \quad A$ M1A1
15 + 17 + 38 + 11 + 31 + 30 + 23 = 165 miles A1 3

(c) e.g. BC, CD, DE, EF, FG 

M1

weight of RSMT = 110 miles A1
Lower bound = 110 + 15 + 23 M1
= 148 miles A1ft 4

[11]

2. (a)

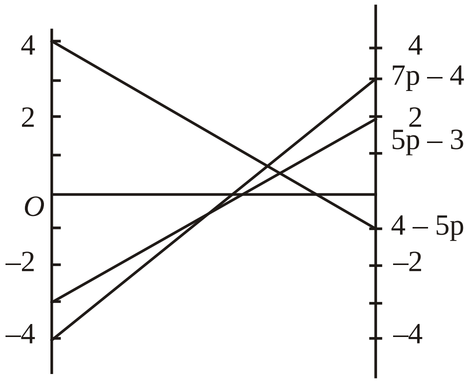
$$\begin{bmatrix} 2 & -1 & 3 \\ -3 & 4 & -4 \end{bmatrix} \quad \begin{matrix} \text{Row min} \\ -1 \leftarrow \\ -4 \end{matrix}$$

M1A1

col max 2 ↑ 4 3 $2 \neq -1 \therefore$ not stable A1 3

(b) Let Denis play 1 with probability p
So he'll play 2 with probability $1 - p$

If Hilary plays 1 Denis wins: $2p - 3(1 - p) = 5p - 3$ M1
If Hilary plays 2 Denis wins: $-p + 4(1 - p) = 4 - 5p$ A2,1,0
If Hilary plays 3 Denis wins: $3p - 4(1 - p) = 7p - 4$



M1A2,1,0

$5p - 3 = 4 - 5p$
$10p = 7$
$p = \frac{7}{10}$ M1A1ft

Denis should play 1 with probability $\frac{7}{10}$
2 with probability $\frac{3}{10}$

the value of the game is $\frac{1}{2}$ B1ftB1 10

[13]

3. (a)

$$\begin{bmatrix} 66 & 101 & 85 & 36 \\ 66 & 98 & 74 & 38 \\ 63 & 97 & 71 & 34 \\ 67 & 102 & 78 & 35 \end{bmatrix}$$

reducing rows first — then columns

$$\begin{bmatrix} 30 & 65 & 49 & 0 \\ 28 & 60 & 36 & 0 \\ 29 & 63 & 37 & 0 \\ 32 & 67 & 43 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 & 13 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 7 & 7 & 0 \end{bmatrix}$$

M1A1

$$\begin{bmatrix} 1 & 4 & 12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 3 & 6 & 6 & 0 \end{bmatrix}$$

M1A1ftA1ft

$$\begin{bmatrix} 0 & 3 & 11 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 2 & 5 & 5 & 0 \end{bmatrix}$$

M1A1ftA1ft

A – cutting
B – stitching
C – filling
D – dressing — A1 — 9

(b) 66 + 98 + 71 + 35 =270 seconds — B1 — 1

(c) 20 × 98 + 66 +71 + 35 = 2132 seconds — M1A1ft
= 35 minutes 32 seconds — A1 — 3

[13]

4. (a) — B2,1,0 — 2

	A	S	D	Seats
1			0	94
2			0	65
3			0	80
	18	200	21	

(b) total supply > total demand — B1 — 1

(c)(d)

	A	S	D
1	18	76	
2		65	
3		59	21

B1
M1A1ft

S(1) = 0 D(A) = 5

S(2) = –0.7 D(S) = 4.5
S(3) = –0.5 D(D) = 0.5

I_{1D} = 0 – 0 – 0.5 = –0.5 *
I_{2A} = 4.2 + 0.7 – 5 = –0.1
I_{2D} = 0 + 0.7 – 0.5 = 0.2
I_{3A} = 4.6 + 0.5 – 5 = 0.1 A1

	A	S	D
1	18	$76 - \theta$	θ
2		65	
3		$59 + \theta$	$21 - \theta$

Entering 1D
Exiting 3D
$\theta = 21$

	A	S	D
1	18	55	21
2		65	
3		80	

M1A1ft
A1 7

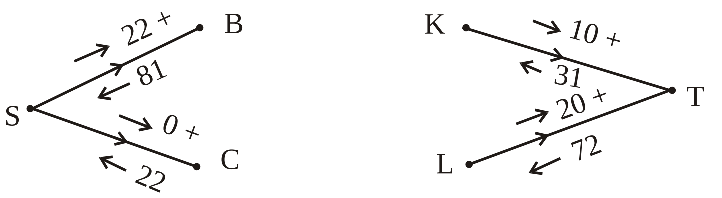
(e) S(1) = 0 D(A) = 4.9 M1
S(2) = –0.7 D(B) = 4.5 A1
S(3) = –0.5 D(B) = 0

I_{1A}= 5 – 0 – 4.9 = 0.1
I_{2D} = 0 + 0.7 – 0 = 0.7
I_{3A} = 4.6 + 0.5 – 4.9 = 0.2
I_{3D} = 0 + 0.5 – 0 = 0.5 A1

Optimal since all II's ≥ 0 A1
cost £902.70 M1A1 6

[16]

5. (a)

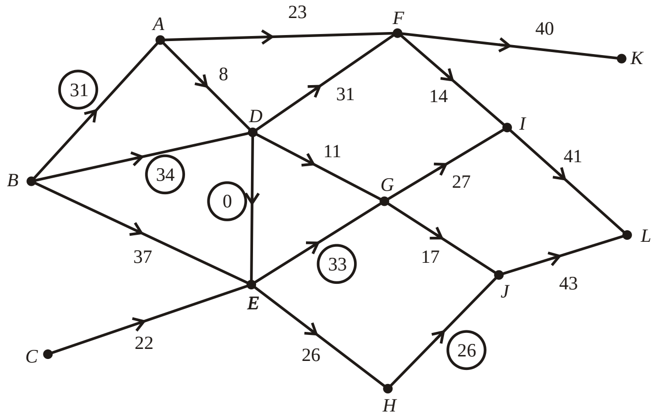


M1A1
A1 3

(b) <u>103</u> B1 1

(c) e.g. S B E G I L T – 3 M1
S B E D F K T – 5 A4,3,2,1,0 5
S B E H J G D F K T – 4
S B E G D F I L T – 9

(d) e.g.



M1A1
A1 3

Flow value <u>124</u> (given)

(e) Max flow = min cut
cut through <u>AB, BD, DE, EG, HJ</u> M1A1 2

[14]

6. <u>Alt 1</u>

Game from R's point of view.

	A1	A2	A3
R_1	–6	3	–5
R_2	2	–1	–4
R_3	3	–2	1

Add 7

	A1	A2	A3
R_1	1	10	2
R_2	9	6	3
R_3	10	5	8

B1, B1

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 B1
V = value of the game

Maximise P = V B1

Subject to $V - P_1 - 9P_2 - 10P_3 \leq 0$ M1A1ft
$V - 10P_1 - 6P_2 - 5P_3 \leq 0$ A1ft
$V - 2P_1 - 3P_2 - 8P_3 \leq 0$ A1ft
$P_1 + P_2 + P_3 \leq 1$ accept= A1 8
$V, P_1, P_2, P_3 \geq 0$

<u>Alt 2</u>

Add 4 to all entries B1

	R_1	R_2	R_3
A1	10	2	1
A2	1	5	6
A3	9	8	3

Let R play 1 with probability P_1
2 with probability P_2
3 with probability P_3 let V = value of game. B1

Let $x_1 = \frac{P_1}{V}, x_2 = \frac{P_2}{V}, x_3 = \frac{P_3}{V}$ B1

Maximise $P = x_1 + x_2 + x_3$ B1

Subject to $10x_1 + 2x_2 + x_3 \leq 1$ M1A1ft

$x_1 + 5x_2 + 6x_3 \le 1$ A1ft

$9x_1 + 8x_2 + 3x_3 \le 1$ A1

$x_1, x_2, x_3 \ge 0$ accept $P_i \ge 0$

[8]

7. (a)

Stage	State	Action	Destination	Value	Marks
	J	JY	Y	98*	
1	K	KY	Y	94*	B1
	L	LY	Y	86*	
	G	GJ	J	max(79, 98) = 98*	M1
		GK	K	max(98, 94) = 98*	
2	H	HK	K	max(95, 94) = 95	A1A1
		HL	L	max(72, 86) = 86*	
	I	IL	L	max(56, 86) = 86*	
	C	CG	G	max(50, 98) = 98*	
	D	DG	G	max(92, 98) = 98	M1
3		DH	H	max((81, 86) = 86*	A1A1ft
	E	EH	H	max(89, 86) = 89*	
	F	FH	H	max(84, 86) = 86*	
		FI	I	max(72, 86) = 86*	
	A	AC	C	max(95, 98) = 98	M1
		AD	D	max(86, 86) = 86*	A1ft
4		AE	E	max(63, 89) = 89	
	B	BE	E	max(88, 89) = 89	
		BF	F	max(87, 86) = 87*	
5	X	XA	A	max(55, 86) = 86*	A1ft
		XB	B	max(85, 87) = 87	

X A D H L Y (minimax = 86) M1A1ft 12

(b) X B F (H or I) L Y (minimax = 87) one M1A1 2

[14]

8. (a) $P - 2x - 4y - 3z = 0$ (o.e.) B2,0 2

(b) $12x + 4y + 5z \le 246$ B1

$9x + 6y + 3z \le 153$ B1

$5x + 2y - 2z \le 171$ B1 3

(c)

basic variable	x	y	z	r	s	t	Value
r	12	4	5	1	0	0	246
s	9	(6)	3	0	1	0	153
t	5	2	–2	0	0	1	171
P	–2	–4	–3	0	0	0	0

b.v.	x	y	z	r	s	t	Value	Row operations	
r	6	0	3	1	$-\frac{2}{3}$	0	144	$R_1 - 4R_2$	M1A1
y	$\frac{3}{2}$	1	½	0	$\frac{1}{6}$	0	25.5	$R_2 \div 6$	M1A1ft
t	2	0	–3	0	$-\frac{1}{3}$	1	120	$R_3 - 2R_2$	B1ft
P	4	0	–1	0	$\frac{2}{3}$	0	102	$R_4 + 4R_2$	

b.v.	x	y	z	r	s	t	Value	Row operations	
z	2	0	1	$\frac{1}{3}$	$-\frac{2}{9}$	0	48	$R_1 \div 3$	M1A1
y	½	1	0	$-\frac{1}{6}$	$\frac{5}{18}$	0	1.5	$R_2 - \frac{1}{2}R_1$	M1A1
t	8	0	0	1	–1	1	264	$R_3 + 3R_1$	
P	6	0	0	$\frac{1}{3}$	$\frac{4}{9}$	0	150	$R_4 + R_1$	

9

(d) $P = 150$ $x = 0$ $y = 1.5$ $z = 48$ M1A1ft
$r = 0$ $s = 0$ $t = 264$ A1ft 3

(e) (The third constraint) $t \neq 0$ B1ft 1

[18]

9. (a) 85 B1 1

(b) $c_1 = 140$, $c_2 = 104$ B1, B1 2

(c) e.g.

S	B	D	F	H	J	T		– 4	M1A1
S	B	D	F	G	T			– 1	
S	B	D	F	C	H	I	T	– 2	A1
S	B	D	F	C	H	J	T	– 2	A1
S	B	D	E	G	T			– 10	A1 5

(d) Max flow – min cut theorem, flow is 104, min cut is c_2 M1A1 2

[10]

Solutions

1.	(a)	$x = 9$, $y = 11$ 1B1: cao (permit B1 if 2 correct answers, but transposed) 2B1: cao	B1, B1	2
	(b)	AC DC DT ET 1B1: correct (condone one error – omission or extra) 2B1: all correct (no omissions or extras)	B2,1,0	2
	(c)	36 1B1: cao	B1	1
	(d)	$C_1 = 49$, $C_2 = 48$, $C_3 = 39$ 1B1: cao 2B1: cao 3B1: cao	B1, B1, B1	3
	(e)	e.g. SAECT 1B1: A correct route (flow value of 1 given)	B1	1
	(f)	maximum flow = minimum cut cut through DT, DC, AC and AE 1M1: Must have attempted (e) and made an attempt at a cut. 1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of fours.	M1A1	2

[11]

2.	(a)	A walk is a **finite sequence of arcs** such that the **end vertex of one arc is the start vertex of the next**. 1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous. 2B1: A good clear complete answer: End vertex = start vertex + finite.	B2,1,0	2
	(b)	A tour is a walk that visits **every vertex, returning to its stating vertex**. 1B1: Probably one of the two below but accept correct relevant statement– bod gets B1, generous. 2B1: A good clear complete answer: Every vertex + return to start.	B2,1,0	2

<u>From the D1 and D2 glossaries</u>

<u>D1</u>

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, <u>and in which no vertex appears more than once</u>.

A **cycle (circuit)** is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

<u>D2</u>

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

[4]

3.	(a)	Total supply > total demand	B2,1,0	2

(b) Adds 0, 0 and 5 to the dummy column — B2,1,0 — 2

(c) — B1 — 1

	L	E	D
A	35	20	
B		40	5

(d) — M1A1

		80	70	20
		L	E	D
0	A	35	20	
–20	B		40	5

$I_{AD} = 0 – 0 – 20 = –20$

$I_{BL} = 60 + 20 – 80 = 0$ — A1 — 3

M1

	L	E	D
A	35	$20 - \theta$	θ
B		40+θ	5–θ

θ = 5; entering square is AD; exiting square is BD — A1ft — 2

B1ft

		80	70	0
		L	E	D
0	A	35	15	5
–20	B		45	

$I_{BL} = 60 + 20 – 80 = 0$

$I_{BD} = 0 + 20 – 0 = 20$ — B1ft — 2

(e) Cost is (£) 6100 — B1 — 1

[13]

4. (a) Maximin : we seek a route where the shortest arc used is a great as possible.
Minimax : we seek a route where the longest arc used is a small as possible. — B2,1,0 — 2

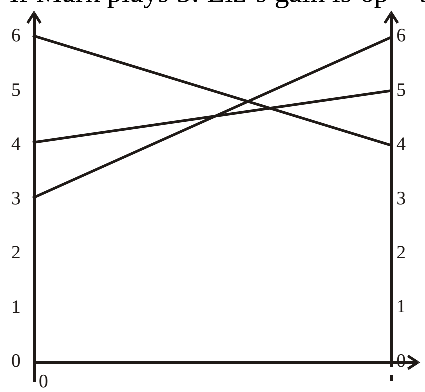
(b)

Stage	State	Action	Dest.	Value	
	G	GR	R	132*	
1	H	HR	R	175*	M1A1
	I	IR	R	139*	
	D	DG	G	min (175,132) = 132	M1A1
		DH	H	min (160,175) = 160*	
2	E	EG	G	min (162,132) = 132	
		EH	H	min (144,175) = 144*	A1
		EI	I	min (102,139) = 102	
	F	FH	H	min (145,175) = 145*	
		FI	I	min (210,139) = 139	
	A	AD	D	min (185,160) = 160*	
		AE	E	min (279,144) = 144	M1A1ft
3	B	BD	D	min (119,160) = 119	
		BE	E	min (250,144) = 144*	A1ft
		BF	F	min (123,145) = 123	
	C	CE	E	min (240,144) = 144	
		CF	F	min (170,145) = 145*	
	L	LA	A	min (155,160) = 155*	A1ft
4		LB	B	min (190,144) = 144	
		LC	C	min (148,145) = 145	

		Maximin route: LADHR	A1ft	5
				[12]
5.	(a)	For each row the element in column x must be less than the element in column y.	B2,1,01	2
	(b)	Row minimum {2,4,3} row maximin = 4	M1	
		Column maximum {6,5,6} column minimax = 5	A1	
		$4 \neq 5$ so not stable	A1	3
	(c)	Row 3 dominates row 1, so matrix reduces to	B1	

	M1	M2	M3
L2	4	5	6
L3	6	4	3

Let Liz play 2 with probability p and 3 with probability (1– p)		
If Mark plays 1: Liz’s gain is 4p + 6 (1– p) = 6 – 2p		
If Mark plays 2: Liz’s gain is 5p + 4 (1– p) = 4 + p	M1	
If Mark plays 3: Liz’s gain is 6p + 3 (1– p) = 3 + 3p	A1	3



	B2,1,0	2
$4 + p = 6 - 2p$	M1A1	
$p = \frac{2}{3}$	A1ftA1	4

(d)	Liz should play row 1 – never, row 2 – $\frac{2}{3}$ of the time, row 3 – $\frac{1}{3}$ of the time and the value of the game is $4\frac{2}{3}$ to her.	B1	
	Row 3 no longer dominates row 1 and so row 1 can not be deleted. Use Simplex (linear programming).	B1	2
			[16]

6. (a) Since maximising, subtract all elements from some n ≥ 53

$$\begin{bmatrix} 5 & 4 & 11 & 11 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 6 & 3 & 7 & 10 \end{bmatrix}$$

M1A1 2

Reduce rows $\begin{bmatrix} 1 & 0 & 7 & 7 \\ 0 & 4 & 2 & 3 \\ 2 & 0 & 5 & 5 \\ 3 & 0 & 4 & 7 \end{bmatrix}$ then columns $\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 4 & 0 & 0 \\ 2 & 0 & 3 & 2 \\ 3 & 0 & 2 & 4 \end{bmatrix}$

M1A1ft 2

Minimum element 1

M1

A1ft

$$\begin{bmatrix} 0 & 0 & 4 & 3 \\ 0 & 5 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 3 \end{bmatrix}$$

A1ft 3

M1

A1ftA1ft 3

(b)

$$\begin{bmatrix} 0 & 1 & 4 & 3 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 3 & 2 \\ 1 & 6 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

M1A1ft 2

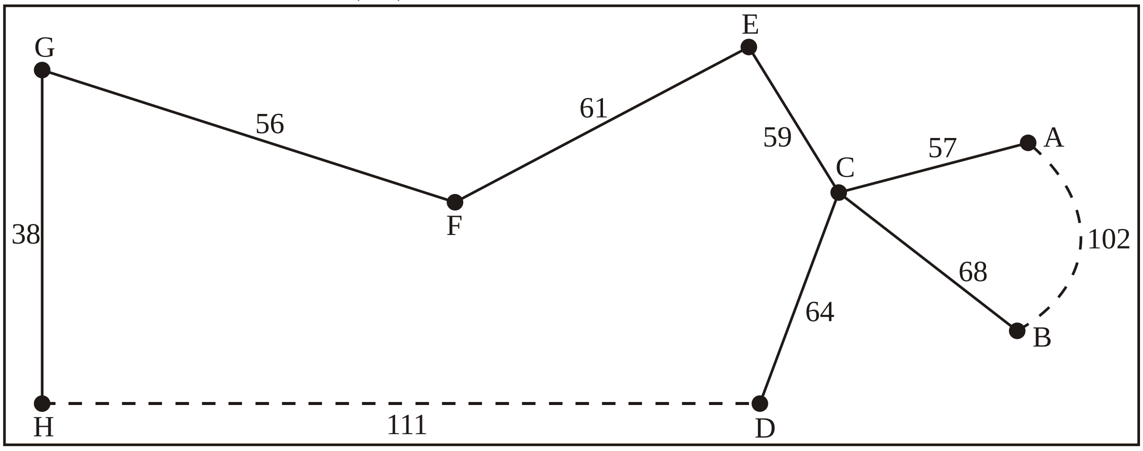
M1A1 2

Joe	A	A
Min-Seong	C	D
Olivia	D	B
Robert	B	C

Value £197 000

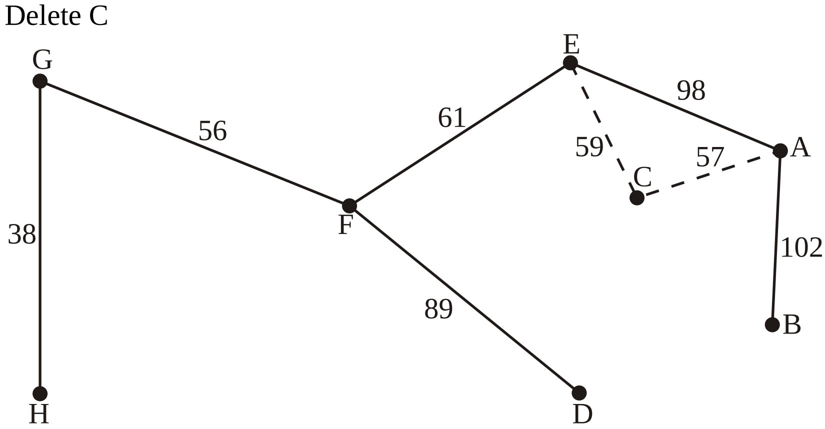
[14]

7.	(a)	GH(38) GF(56) CA(57) EC(59) FE(61) CD(64) CB(68)	M1A1ft	2
	(b)	2 × 403 = 806 (km)	B1	1
	(c)	e.g. DH saves 167 AB saves 23	M1A1	
		806 – 190 = 616 (km)	A1	



A1 4

	(d)	eg A B C E F G H D C A	M1A1	
		B C A E F G H D B 68 + 57 + 98 + 61 + 56 + 38 + 111 + 108 = 597 (km)	A1	3
	(e)	Delete C		



M1A1M1A1ft 4

	(f)	RMST weight = 444 Lower bound = 444 + 59 + 57 = 560 (km) 560 < length ≤ 597	B2,1,0	2

[16]

8. (a)

b.v.	x	y	z	R	s	t	Value
r	4	$\frac{7}{3}$	$\frac{5}{2}$	1	0	0	64
s	1	3	0	0	1	0	16
t	4	2	2	0	0	1	60
P	–5	$-\frac{7}{2}$	–4	0	0	0	0

b.v.	x	y	z	R	s	t	Value	Row ops	
r	0	$\frac{1}{3}$	$\frac{1}{2}$	1	0	–1	4	$R_1 - 4R_3$	M1A1
s	0	$\frac{5}{2}$	$-\frac{1}{2}$	0	1	$-\frac{1}{4}$	1	$R_2 - R_3$	M1A1ftA1
x	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{4}$	15	$R_3 \div 4$	
P	0	–1	$-\frac{3}{2}$	0	0	$\frac{5}{4}$	75	$R_4 + 5R_3$	

b.v.	x	y	z	R	s	t	Value	Row ops		
z	0	$\frac{2}{3}$	1	2	0	–2	8	$R_1 \div \frac{1}{2}$	M1A1ft	
s	0	$\frac{17}{6}$	0	1	1	$-\frac{5}{4}$	5	$R_2 + \frac{1}{2}R_1$	M1A1	9
x	1	$\frac{1}{6}$	0	–1	0	$\frac{5}{4}$	11	$R_3 - \frac{1}{2}R_1$		
P	0	0	0	3	0	$-\frac{7}{4}$	87	$R_4 + \frac{3}{2}R_1$		

(b) There is still negative numbers in the profit row. B1 1

[10]

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Mark Scheme (Results)

Summer 2009

GCE

GCE Mathematics (6690/01)

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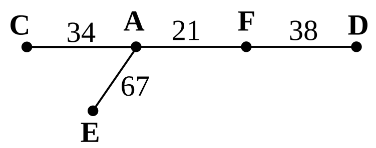
June 2009
6690 Decision Mathematics D2
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	There are more tasks than people.	B1 (1)
(b)	Adds a row of zeros	B1 (1)
(c)	$\begin{bmatrix} 15 & 11 & 14 & 12 \\ 13 & 8 & 17 & 13 \\ 14 & 9 & 13 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 3 & 1 \\ 5 & 0 & 9 & 5 \\ 5 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \rightarrow \begin{bmatrix} 3 & 0 & 2 & 0 \\ 4 & 0 & 8 & 4 \\ 4 & 0 & 3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	B1;M1A1
	Either $\begin{bmatrix} 3 & 3 & 2 & 0 \\ 1 & 0 & 5 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}$	M1 A1
	Or $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 \\ 2 & 0 & 1 & 5 \\ 0 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 4 & 0 & 2 \end{bmatrix}$	
	J – 4, M – 2, R – 3, (D – 1)	A1 (6)
(d)	Minimum cost is (£)33.	B1 (1)
		[9]

Question Number	Scheme	Marks
Q2		
(a)	In the classical problem each vertex must be visited **only** once. In the practical problem each vertex must be visited **at least** once.	B2, 1, 0 (2)
(b)	A F D B E C A {1 4 6 3 5 2 } 21 + 38 + 58 + 36 + 70 + 34 = 257	M1 A1 A1 (3)
(c)	257 is the better upper bound, it is lower.	B1ft (1)
(d)	R.M.S.T. 	M1 A1
	Lower bound is 160 + 36 + 58 = 254	M1A1 (4)
(e)	Better lower bound is 254, it is higher	B1ft
(f)	$254 < \text{optimal} \leq 257$	B1 (2)
		[12]
	Notes: (a) 1B1: Generous, on the right lines bod gets B1 2B1: cao, clear answer. (b) 1M1:Nearest Neighbour each vertex visited once (condone lack of return to start) 1A1: Correct route cao – must return to start. 2A1: 257 cao (c) 1B1ft: ft their lowest. (d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient 1A1: cao tree or 160. 2M1:Adding 2 least arcs to B, 36 and 58 only 2A1: 254 (e) 1B1ft: ft their highest (f) 1B1: cao	

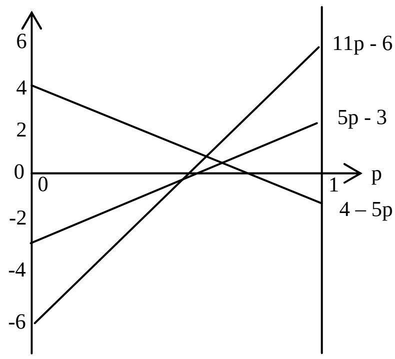
Question Number	Scheme	Marks
Q3 (a)	Row minima {-5, -4, -2} row maximin = -2 Column maxima {1, 6, 13} col minimax = 1 $-2 \neq 1$ therefore not stable.	M1 A1 A1 (3)
(b)	Column 1 dominates column 3, so column 3 can be deleted.	B1 (1)
(c)	see table below	B1 B1 (2)
(d)	Let B play row 1 with probability p and row 2 with probability (1-p) If A plays 1, B's expected winnings are 11p – 6 If A plays 2, B's expected winnings are 4 – 5p If A plays 3, B's expected winnings are 5p – 3	M1 A1
		M1 A1
	$5p-3=4-5p$ $10p=7$ $p=\frac{7}{10}$	M1
	B should play 1 with a probability of 0.7 2 with a probability of 0.3 and never play 3	A1
	The value of the game is 0.5 to B	A1 (7)
		[13]

Table for (c):

	A plays 1	A plays 2	A plays 3
B plays 1	5	-1	2
B plays 2	-6	4	-3

Question Number	Scheme	Marks
Q4 (a)	Value of cut $C_1 = 34$; Value of cut $C_2 = 45$	B1; B1 (2)
(b)	S B F G T or S B F E T – value 2 Maximum flow = 28 **Notes:** (a) 1B1: cao 2B1: cao (b) 1M1: feasible flow-augmenting route and a value stated 1A1: a correct flow-augmenting route and value 1A1= B1: cao	M1 A1 A1=B1 (3) **[5]**
Q5 (a)	$x = 0,\ y = 0,\ z = 2$	B2,1,0 (2)
(b)	$P - 2x - 4y + \frac{5}{4}r = 10$ **Notes:** (a) 1B1: Any 2 out of 3 values correct 2B1: All 3 values correct. (b) 1M1: One equal sign, modulus of coefficients correct. All the right ingredients. 1A1: cao – condone terms of zero coefficient	M1 A1 (2) **[4]**

Question Number	Scheme	Marks
Q6 (a)	The supply is equal to the demand	B1 (1)

(b) B1 (1)

	A	B	C
X	16	6	
Y		9	8
Z			15

(c) M1 A1

	A	B	C
X	16- θ	6+ θ	
Y		9- θ	8+ θ
Z	θ		15- θ

Value of θ = 9, exiting cell is YB — A1 (3)

(d) M1 A1

		17	8	20
		A	B	C
0	X	7	15	
-5	Y			17
-11	Z	9		6

XC = 7 – 0 – 20 = -13
YA = 16 + 5 – 17 = 4
YB = 12 + 5 – 8 = 9
ZB = 10 + 11 – 8 = 13 — A1 (3)

	A	B	C
X	7- θ	15	θ
Y			17
Z	9+ θ		6- θ

Value of θ = 6, entering cell XC, exiting cell ZC — M1 A1

	A	B	C
X	1	15	6
Y			17
Z	15		

A1 (3)

Cost (£) 524 — B1 (1)

[12]

Question Number	Scheme	Marks
Q7 (a)		

Stage	State (in £1000s)	Action (in £1000s)	Dest. (in £1000s)	Value (in £1000s)	Marks
	250	*250*	*0*	*300**	
1	*200*	*200*	*0*	*240**	
	150	*150*	*0*	*180**	
	100	*100*	*0*	*120**	
	50	*50*	*0*	*60**	
	0	*0*	*0*	*0**	
	250	*280*	*0*	*200 + 0 = 280*	
		200	*50*	*235 + 60 = 295*	
		150	*100*	190 + 120 = 310*	
		100	*150*	125 + 180 = 305	1M1 A1
		50	*200*	65 + 240 = 305	
		0	*250*	0 + 300 = 300	
2	*200*	*200*	*0*	235 + 0 = 235	
		150	*50*	190 + 60 = 250*	
		100	100	125 + 120 = 245	A1
		50	150	65 + 180 = 245	
		0	200	0 + 240 = 240	
	150	*150*	0	190 + 0 = 190*	2M1
		100	50	125 + 60 = 185	
		50	100	65 + 120 = 185	A1
		0	150	0 + 180 = 180	
	100	100	0	125 + 0 = 125*	A1
		50	50	65 + 60 = 125*	
		0	100	0 + 120 = 120	
	50	50	0	65 + 0 = 65*	
		0	50	0 + 60 = 60	
	0	0	0	0 + 0 = 0*	3M1 A1ft
3	250	250	0	300 + 0 = 300	
		200	50	230 + 65 = 295	
		150	100	170 + 125 = 295	
		100	150	110 + 190 = 300	
		50	200	55 + 250 = 305	
		0	250	0 + 310 = 310*	

Maximum income £310 000 — B1

Scheme	1	2	3
Invest (in £1000s)	100	150	0

B1 (10)

(b)

Stage: Scheme being considered — B1
State: Money available to invest — B1
Action: Amount chosen to invest — B1 (3)

[13]

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Question Number	Scheme	Marks
Q8		
	E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$	B1
	Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively Let V = value of game + 6	B1
	e.g. Maximise P = V Subject to:	B1
	$V - 4p_1 - 13p_2 - 7p_3 \le 0$ $V - 14p_1 - 10p_2 - p_3 \le 0$ $V - 5p_1 - 3p_2 - 10p_3 \le 0$ $p_1 + p_2 + p_3 \le 1$ $p_1, p_2, p_3 \ge 0$	M1 A3,2ft,1ft,0 (7)
	Notes: 1B1: Making all elements positive 2B1: Defining variables 3B1: Objective, cao word and function 1M1: At least one constraint in terms of their variables, must be going down columns. Accept = here. 1A1ft: ft their table. One constraint in V correct. 2A1ft: ft their table. Two constraints in V correct. 3A1: CAO all correct .	**[7]**
	Alt using x_i method Now additionally need: let $x_i = \frac{p_i}{v}$ for 2B1 minimise $(P) = x_1 + x_2 + x_3 = \frac{1}{v}$ subject to: $4x_1 + 13x_2 + 7x_3 \ge 1$ $14x_1 + 10x_2 + x_3 \ge 1$ $5x_1 + 3x_2 + 10x_3 \ge 1$ $x_i \ge 0$	

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Summer 2010

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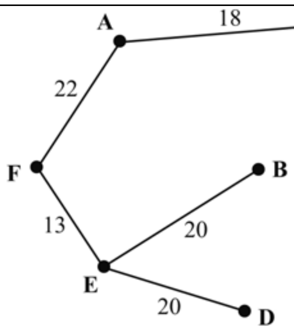
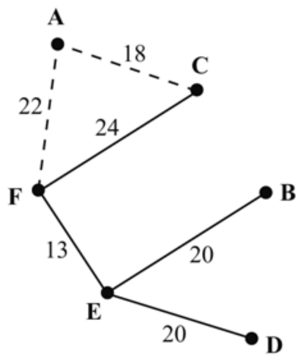
Summer 2010
Decision Mathematics D2 6690
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)		M1 A1 (2)
(b)	Minimum Spanning tree length 93, so upper bound is £186	B1ft (1)
(c)	A C F E B D A 18 24 13 20 22 28 Length 125 A C F E D B A 18 24 13 20 22 36 Length 133	M1 A1 A1 (3)
(d)	Best upper bound is £125	B1ft (1)
(e)	Delete A 	M1 A1
	RMST weight = 77 Lower bound = 77 + 18 + 22 = £117	M1 A1 (4) **[11]**

Question Number	Scheme	Marks
Q2 (a)	Since maximising, subtract all elements from some n ≥ 27	
	$\begin{bmatrix} 12 & 6 & 8 & 13 \\ 10 & 5 & 11 & 60 \\ 5 & 6 & 3 & 8 \\ 11 & 4 & 7 & 16 \end{bmatrix}$	1M1 2M1
	Reduce rows $\begin{bmatrix} 6 & 0 & 2 & 7 \\ 5 & 0 & 6 & 55 \\ 2 & 3 & 0 & 5 \\ 7 & 0 & 3 & 12 \end{bmatrix}$ then columns $\begin{bmatrix} 4 & 0 & 2 & 2 \\ 3 & 0 & 6 & 50 \\ 0 & 3 & 0 & 0 \\ 5 & 0 & 3 & 7 \end{bmatrix}$	3M1 A1
	$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 0 & 4 & 48 \\ 0 & 5 & 0 & 0 \\ 3 & 0 & 1 & 5 \end{bmatrix}$	4M1 A1ft
	$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 47 \\ 0 & 6 & 0 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix}$	5M1A1 (8)
(b)	Three optimal allocations:	
	$\begin{array}{\|l\|c\|c\|c\|} \hline \text{Harry} & 3 & 4 & 4 \\ \hline \text{Jess} & 1 & 1 & 2 \\ \hline \text{Louis} & 4 & 3 & 1 \\ \hline \text{Saul} & 2 & 2 & 3 \\ \hline \end{array}$	M1
	Total amount earned by team: £90	A1 (2) **[10]**

Question Number	Scheme	Marks

Q3 (a)

	A	B	C	D
X	18	31	4	
Y			18	29

B1 (1)

(b) e.g.

		28	20	19	22
		A	B	C	D
0	X	x	x	x	-6
-5	Y	-8	-3	x	x

M1 A1

	A	B	C	D
X	18 – θ	31	4+θ	
Y	θ		18 – θ	29

2M1

Entering cell: AY θ = 18

Either

Exiting cell: XA

	A	B	C	D
X		31	22	
Y	18		0	29

		20	20	19	22
		A	B	C	D
0	X	8	x	x	-6
-5	Y	x	-3	x	x

Entering cell: XD

	A	B	C	D
X		31	22 – θ	θ
Y	18		0 + θ	29 – θ

Exiting cell: XC

θ = 22

	A	B	C	D
X		31		22
Y	18		22	7

		14	20	13	16
		A	B	C	D
0	X	14	X	6	x
1	Y	x	-9	x	x

Or

Exiting cell: YC

	A	B	C	D
X	0	31	22	
Y	18			29

		28	20	19	30
		A	B	C	D
0	X	x	x	x	-14
-13	Y	x	5	8	x

Entering cell: XD

	A	B	C	D
X	0 – θ	31	22	θ
Y	18 + θ			29 – θ

Exiting cell: XA

θ = 0

	A	B	C	D
X		31	22	0
Y	18			29

		14	20	19	16
		A	B	C	D
0	X	14	x	x	x
1	Y	x	-9	-6	x

Marks: 2A1ft; 3M1 3A1; 4M1; 4A1ft; 5A1 (9)

(c) Negative improvement index so not optimal B1ft (1)

[11]

Q4 (a)

Minimax route

Question Number	Scheme	Marks

Stage	State	Action	Dest.	Value
	G	GT	T	17*
1	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	max(22, 17) = 22*
		DH	H	max(31, 21) = 31
	E	EH	H	max(34, 21) = 34*
		EI	I	max(39, 29) = 39
	F	FI	I	max(52, 29) = 52*
3	A	AD	D	max(41, 22) = 41
		AE	E	max(38, 34) = 38*
	B	BE	E	max(44, 34) = 44*
	C	CE	E	max(36, 34) = 36*
		CF	F	max(35, 52) = 52
4	S	SA	A	max(37, 38) = 38*
		SB	B	max(39, 44) = 44
		SC	C	max(41, 36) = 41

1M1
A1

2M1 A1

A1

3M1 A1ft

A1ft

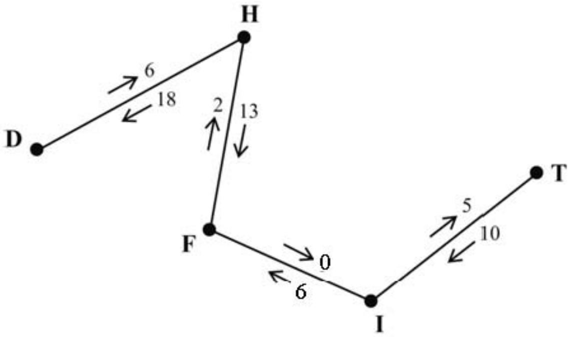
A1ft

(9)

(b) Route: SAEHT Greatest annual cost: £38 000 — M1 A1ft (2)

(c) Average expenditure $\frac{37+38+34+21}{4} = \frac{130}{4} =$ £32 500 — M1A1 (2)

[13]

Question Number	Scheme	Marks
Q5 (a)	Initial flow = 41	B1 (1)
(b)	Capacity of C_1 = 69 Capacity of C_2 = 64	B1 B1 (2)
(c)		M1 A1 (2)
(d)	e.g. SBADHT – 2 SCGEDHT – 2	M1 A1 A1 (3)
(e)	maximum flow = minimum cut e.g. cut through SA, SB, CE, GE, GI or HT, FI, GI	DM1 A1 (2) **[10]**
	Notes: (a) 1B1: cao (b) 1B1: cao (permit B1 if 2 correct answers, but transposed) 2B1: cao (c) 1M1: Two numbers on each arc 1A1: cao (d) 1M1: One valid flow augmenting route, S to T, found and value (≤ 4) stated. 1A1: Flow increased by at least 2 2A1: Flow increased by 4 (e) 1DM1: Must have attempted (d) and made an attempt at a cut. 1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of four.	

Question Number	Scheme	Marks
Q6 (a)	$P - x - 2y - 6z = 0$	B1 (1)
(b)	(tables below)	M1 A1 M1 A1ft A1 (5) M1 A1ft M1 A1 (4)
(c)	P = 52 x = 0 y = 20 z = 2 r = 0 s = 0 t = 6	M1 A1ft A1 (3) **[13]**

(b)

b.v	x	y	z	r	s	t	Value
r	0	1	2	1	0	0	24
s	2	1	4	0	1	0	28
t	-1	$\frac{1}{2}$	3	0	0	1	22
P	-1	-2	-6	0	0	0	0

b.v.	x	y	z	r	s	t	Value	Row Ops.
r	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 4$
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	1	1	$R_3 - 3R_2$
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 6R_2$

b.v.	x	y	z	r	s	t	Value	Row Ops.
y	-2	1	0	2	-1	0	20	$R_1 \div \frac{1}{2}$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + \frac{1}{4}R_1$
P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$

Notes:

(a) 1B1: cao

(b) 1M1: correct pivot located, attempt to divide row
 1A1: pivot row correct including change of b.v.
 2M1: (ft) Correct row operations used at least once or stated correctly.
 1A1ft: Looking at non zero-and-one columns, one column ft correct
 2A1: cao.
 3M1: (ft)Correct pivot identified – negative pivot gets M0 M0
 1A1: ft pivot row correct including change of bv – but don't penalise b.v. twice.
 4M1: (ft) Correct row operations used at least once or stated correctly.
 1A1: cao

(c) 1M1: At least 4 values stated. No negative. Reading off bottom row gets M0.
 1A1ft: At least 4 values correct.
 2A1: cao

Question Number	Scheme	Marks
Q7	$\begin{bmatrix} -4 & 5 & 1 \\ 3 & -1 & -2 \\ -3 & 0 & 2 \end{bmatrix} \rightarrow \text{add 5 to all entries} \begin{bmatrix} 1 & 10 & 6 \\ 8 & 4 & 3 \\ 2 & 5 & 7 \end{bmatrix}$	M1
	Either: Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively. **Or:** Define variables e.g. let p_1, p_2 and p_3 be the probability that A plays rows 1, 2 and 3 respectively. Let $x_i = \frac{p_i}{V}$	B1
	Either: Maximise P = V **Or:** Minimise P $= x_1 + x_2 + x_3$	B1
	Either: Subject to: $V - p_1 - 8p_2 - 2p_3 \leq 0$ $V - 10p_1 - 4p_2 - 5p_3 \leq 0$ $V - 6p_1 - 3p_2 - 7p_3 \leq 0$ $p_1 + p_2 + p_3 \leq 1$ $p_1, p_2, p_3 \geq 0$	M1 A1 A1 A1
	Or: Subject to $x_1 + 8x_2 + 2x_3 \geq 1$ $10x_1 + 4x_2 + 5x_3 \geq 1$ $6x_1 + 3x_2 + 7x_3 \geq 1$ $x_1, x_2, x_3 \geq 0$	
		[7]
	Notes: 1M1: Adding n (≥ 4) to all entries 1B1: Defining variables 1B1: Objective correct 2M1: At least 3 constraints, using columns, one of correct form 1A1ft: one correct constraint – excluding non-negativity constraint 2A1ft: two correct constraints – excluding non-negativity constraint 3A1: cao including non-negativity constraint	

Notes for Question 1

(a) 1M1: Spanning tree found. Allow 1x2x43 across top of table or 93
1A1: CAO must see tree or list of arcs

(b) 1B1ft: 186 their ft93 x 2

(c) 1M1: One Nearest Neighbour each vertex visited at least once (condone lack of return to start)
1A1: One correct route and length CAO – must return to start.
2A1: Second correct route and length CAO – must return to start.

(d) 1B1ft: ft but only on three different values.

(e) 1M1: Finding correct RMST (maybe implicit) 77 sufficient, or correct numbers. 4 arcs.
1A1: CAO tree or 77.
2M1: Adding 2 least arcs to A, 18 and 22 or 40 only
2A1: CAO 117

Notes for Question 2

(a) 1M1: Subtracting from some n ≥ 27, condone up to two errors
2M1: Dealing with (Jess, 4) entry.
3M1: Reducing rows then columns
1A1: cao (pick up (J,4) value here)
4M1: Double covered +e; one uncovered – e; and one single covered unchanged.
2 lines needed to 3 lines needed.
2A1ft: ft correct - no errors
5M1: Double covered +e; one uncovered – e; and one single covered unchanged.
3 line to 4 line solution.
3A1: correct - no errors

(b) 1M1: A complete, correct solution.
1A1: cao

Q2 Special case (Minimises)

M0
M1

$$\begin{bmatrix} 18 & 24 & 22 & 17 \\ 20 & 25 & 19 & 60 \\ 25 & 24 & 27 & 22 \\ 19 & 26 & 23 & 14 \end{bmatrix} \xrightarrow{\textit{row reduction}} \begin{bmatrix} 1 & 7 & 5 & 0 \\ 1 & 6 & 0 & 41 \\ 3 & 2 & 5 & 0 \\ 5 & 12 & 9 & 0 \end{bmatrix}$$

M1

$$\xrightarrow{\textit{column reductions}} \begin{bmatrix} 0* & 5 & 5 & 0 \\ 0 & 4 & 0* & 41 \\ 2 & 0* & 5 & 0 \\ 4 & 10 & 9 & 0* \end{bmatrix}$$

A1

M0
M0

Solution:
Harry - 1 M1
Jess - 3
Louis - 2
Saul - 4

Total £75 A1

Maximum 5 marks

Notes for Question 3

(a) 1B1: Cao

(b) 1M1: 6 shadow costs and precisely 3 improvement indices stated. (no extra zeros)
1A1: cao.
2M1: A valid route, negative II chosen, only one empty square used, θ's balance.
2A1ft: improved solution (no extra zeros)
3M1ft: 6 shadow costs and precisely 3 improvement indices stated (no extra zeros)
3A1: cao.
4M1ft:A valid route, negative II chosen, only one empty square used, θ's balance.
4A1ft: improved solution (no extra zeros)
5A1=5M1: 6 shadow costs and precisely 3 improvement indices, (or 1 negative improvement index), stated (no extra zeros).

(c) 1B1ft=1A1ft: cao for conclusion, but must follow from at least one negative in a third 'set' of IIs.

Misreads for Q3b Not choosing most negative.

	A	B	C	D
X	18	31	4	
Y			18	29

		28	20	19	22
		A	B	C	D
0	X	x	x	x	-6
-5	Y	-8	-3	x	x

Either

Entering cell: XD

	A	B	C	D
X	18	31	4– θ	θ
Y			18+ θ	29– θ

Exiting cell: XC

θ =4

	A	B	C	D
X	18	31		4
Y			22	25

		28	20	13	16
		A	B	C	D
0	X	x	x	6	x
1	Y	-14	-9	x	x

Or

Entering cell: YB

	A	B	C	D
X	18	31- θ	4+ θ	
Y		θ	18- θ	29

Exiting cell: YC

θ = 18

	A	B	C	D
X	18	13	22	
Y		18		29

		28	20	19	25
		A	B	C	D
0	X	x	x	x	-9
-8	Y	-5	x	3	x

Candidates can get

2M1 2A1 for first route and the improved solution

3M1 3A0 – 6 shadow costs and 3 IIs

4M1 for finding a valid route and 4A1 if their route leads to an improved solution

[A0 – 6 shadow costs and 3 IIs but it is CAO]

Notes for Question 4

Throughout section (a):

- **Condone lack of destination column and/or reversed stage numbers throughout.**
- **Only penalise incorrect result in Value – ie ignore working values.**
- **Penalise absence of state or action column with first two A marks earned only**
- **Penalise empty/errors in stage column with first A mark earned only.**

(a) 1M1: First, T, stage complete and working backwards.
1A1: CAO (condone lack of *)
2M1: Second stage completed. Penalise reversed states here and in (b). Bod if something in each column.
2A1: Any 2 states correct. Penalise * errors, with an A mark, only once in the question).
3A1: All 3 states correct. (Penalise * errors only once in the question).
3M1: 3rd and 4th stages completed. Bod if something in each column.
4A1ft: Any 2 states correct. (Penalise * errors only once in the question). A, B or C
5A1ft: All 3 states correct. (Penalise * errors only once in the question). A, B and C.
6A1ft: Final, S, state correct. (Penalise * errors only once in the question).

(b) 1M1: Route (S to T or vv.) and cost stated
1A1ft: CAO (Penalise reversed states here)

(c) 1M1: Sum of four arcs /4 (do not isw here if they 'add' to this method)
1A1: CAO (32 500 gets both marks)

Special cases (and misreads)

SC1 Maximin: treat as misread. **MAX 11/13**

SC2 Maximum: 1M1,1A1; 2M0; 3M1,4A1ft,5A0,6A1ft, M1A1ft M1A1ft **MAX 9/13**

SC3 Minimum: Marks awarded as above SC2

SC4 Maximax: 1M1,1A1; 2M0; 3M1,4A0,5A0,6A0,M1A1ft M1A1ft **MAX 7/13**

SC5 Minimin: Marks awarded as above SC4

SC6 Working forwards:
1M1,1A0; 2M0; 3M1,4A0,5A0,6A0,M1A1ft M1A1ft **MAX6/13**

Anything else annotate and send to review.

Q4 Misreads

SC 1 Maximin

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	min(22, 17) = 17
		DH	H	min(31, 21) = 21*
	E	EH	H	min(34, 21) = 21
		EI	I	min(39, 29) = 29*
	F	FI	I	min(52, 29) = 29*
3	A	AD	D	min(41, 21) = 21
		AE	E	min(38, 29) = 29*
	B	BE	E	min(44, 29) = 29*
	C	CE	E	min(36, 29) = 29*
		CF	F	min(35, 29) = 29*
4	S	SA	A	min(37, 29) = 29*
		SB	B	min(39, 29) = 29*
		SC	C	min(41, 29) = 29*

SC 2 Maximum route

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	22 + 17 = 39
		DH	H	31 + 21 = 52*
	E	EH	H	34 + 21 = 55
		EI	I	39 + 29 = 68*
	F	FI	I	52 + 29 = 81*
3	A	AD	D	41 + 52 = 93
		AE	E	38 + 68 = 106*
	B	BE	E	44 + 68 = 112*
	C	CE	E	36 + 68 = 104
		CF	F	35 + 81 = 116*
4	S	SA	A	37 + 106 = 143
		SB	B	39 + 112 = 151
		SC	C	41 + 116 = 157*

Route: SCFIT

SC3 Minimum route

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	22 + 17 = 39*
		DH	H	31 + 21 = 52
	E	EH	H	34 + 21 = 55*
		EI	I	39 + 29 = 68
	F	FI	I	52 + 29 = 81*
3	A	AD	D	41 + 39 = 80*
		AE	E	38 + 55 = 93
	B	BE	E	44 + 55 = 99*
	C	CE	E	36 + 55 = 91*
		CF	F	35 + 81 = 116
4	S	SA	A	37 + 80 = 117*
		SB	B	39 + 99 = 138
		SC	C	41 + 91 = 132

Route: SADGT

SC 4 Maximax route

Stage	State	Action	Dest.	Value
	G	GT	T	17*
1	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	max(22, 17) = 22
		DH	H	max(31, 21) = 31*
	E	EH	H	max(34, 21) = 34
		EI	I	max(39, 29) = 39*
	F	FI	I	max(52, 29) = 52*
3	A	AD	D	max(41, 31) = 41
		AE	E	max(38, 39) = 39*
	B	BE	E	max(44, 39) = 44*
	C	CE	E	max(36, 39) = 39
		CF	F	max(35, 52) = 52*
4	S	SA	A	max(37, 39) = 39
		SB	B	max(39, 44) = 44
		SC	C	max(41, 52) = 52*

Route SCFIT

SC 5 Minimin

Stage	State	Action	Dest	Value
1	G	GT	T	17*
	H	HT	T	21*
	I	IT	T	29*
2	D	DG	G	min(22, 17) = 17*
		DH	H	min(31, 21) = 21
	E	EH	H	min(34, 21) = 21*
		EI	I	min(39, 29) = 29
	F	FI	I	min(52, 29) = 29*
3	A	AD	D	min(41, 17) = 17*
		AE	E	min(38, 21) = 21
	B	BE	E	min(44, 21) = 21*
	C	CE	E	min(36, 21) = 21*
		CF	F	min(35, 29) = 29
4	S	SA	A	min(37, 17) = 17*
		SB	B	min(39, 21) = 21
		SC	C	min(41, 21) = 21

Route SADGT

SC 6 Working forwards S to T

Stage	State	Action	Dest	Value
1	A	AS	S	37*
	B	BS	S	39*
	C	CS	S	41*
	D	DA	A	max(41, 37) = 41*
	E	EA	A	max(38, 37) = 38*
		EB	B	max(44, 39) = 44
		EC	C	max(36, 41) = 41
	F	FC	C	max(35, 41) = 41*
3	G	GD	D	max(22, 41) = 41*
	H	HD	D	max(31, 41) = 41
		HE	E	max(34, 38) = 38*
	I	IE	E	max(39, 38) = 39*
		IF	F	max(52, 41) = 52
4	T	TG	G	max(17, 41) = 41
		TH	H	max(21, 38) = 38*
		TI	I	max(29, 39) = 39

Route SAEHT

Q6b Misreads Alternative 1

Increasing *x* first,

b.v.	*x*	*y*	*z*	*r*	*s*	*t*	value	row ops
r	0	1	2	1	0	0	24	R_1 no change
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	14	$R_2 \div 2$
t	0	1	5	0	$\frac{1}{2}$	1	36	$R_3 + R_2$
P	0	$-\frac{3}{2}$	–4	0	$\frac{1}{2}$	0	14	$R_4 + R_2$

then *y* next

b.v.	*x*	*y*	*z*	*r*	*s*	*t*	value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
x	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{2}R_1$
t	0	0	3	–1	$\frac{1}{2}$	1	12	$R_3 - R_1$
P	0	0	–1	$\frac{3}{2}$	$\frac{1}{2}$	1	50	$R_4 + \frac{3}{2}R_1$

then *z*.

b.v.	*x*	*y*	*z*	*r*	*s*	*t*	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 - 3R_2$
P	0	0	0	1	1	1	52	$R_4 + R_2$

Q6b Misreads Alternative 2

Increasing *x* first

b.v.	x	y	z	r	s	t	value	row ops
r	0	1	2	1	0	0	24	R_1 no change
x	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	0	14	$R_2 \div 2$
t	0	1	5	0	$\frac{1}{2}$	1	36	$R_3 + R_2$
P	0	$-\frac{3}{2}$	–4	0	$\frac{1}{2}$	0	14	$R_4 + R_2$

Increasing *z* next

b.v.	x	y	z	r	s	t	value	row ops
r	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	10	$R_1 - 2R_2$
z	$\frac{1}{2}$	$\frac{1}{4}$	1	0	$\frac{1}{4}$	0	7	$R_2 \div 2$
t	$-\frac{5}{2}$	$-\frac{1}{4}$	0	0	$\frac{3}{4}$	1	1	$R_3 - 5R_2$
P	2	$-\frac{1}{2}$	0	0	$\frac{3}{2}$	0	42	$R_4 + 4R_2$

then increasing *y*

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 \div \frac{1}{2}$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 - \frac{1}{4}R_1$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + \frac{1}{4}R_1$
P	1	0	0	1	1	0	52	$R_4 + \frac{1}{2}R_1$

Q6b Misreads Alternative 3

Increasing *y* first

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
s	2	0	2	-1	1	0	4	$R_2 - R_1$
t	-1	0	2	$-\frac{1}{2}$	0	1	10	$R_3 - \frac{1}{2}R_1$
P	-1	0	-2	2	0	0	48	$R_4 + 2R_1$

Increasing *x* next

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	R_1 no changw
x	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	0	0	3	-1	$\frac{1}{2}$	1	12	$R_3 - 3R_2$
P	0	0	-1	$\frac{3}{2}$	$\frac{1}{2}$	0	50	$R_4 + R_2$

then increasing *z*

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 1$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 + R_2$
P	1	0	0	1	1	0	52	$R_4 + R_2$

Q6b Misreads   Alternative 4

Increasing *y* first

b.v.	x	y	z	r	s	t	value	row ops
y	0	1	2	1	0	0	24	$R_1 \div 1$
s	2	0	2	-1	1	0	4	$R_2 - R_1$
t	-1	0	2	$-\frac{1}{2}$	0	1	10	$R_3 - \frac{1}{2}R_1$
P	-1	0	-2	2	0	0	48	$R_4 + 2R_1$

Increasing *z* next

b.v.	x	y	z	r	s	t	value	row ops
y	-2	1	0	2	-1	0	20	$R_1 - 2R_2$
z	1	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	2	$R_2 \div 2$
t	-3	0	0	$\frac{1}{2}$	-1	1	6	$R_3 - 2R_2$
P	1	0	0	1	1	0	52	$R_4 + 2R_2$

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GCE Decision D2 (6690) Paper 1

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3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
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- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

June 2011
Decision Mathematics D2 6690
Mark Scheme

Question Number	Scheme	Marks

1. (a)

	A	B	C	D	E	F
A	-	19	11	23	20	**37**
B	19	-	8	**42**	17	32
C	11	8	-	**34**	9	26
D	23	**42**	**34**	-	27	31
E	20	17	9	27	-	17
F	**37**	32	26	31	17	-

B3, 2, 1, 0 (3)

(b)

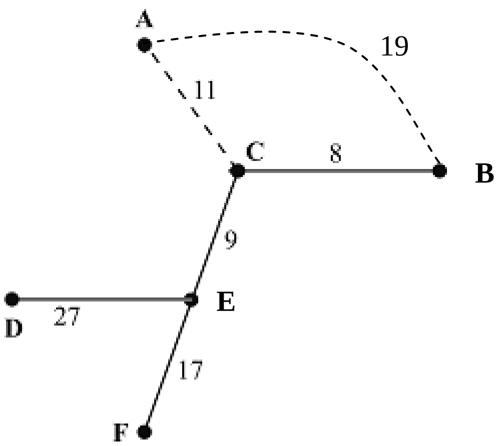
A C B E F D A
11 8 17 17 31 23 = 107

M1 A1
A1 (3)

(c)

Delete A

M1



RMST weight = 61 — A1

Lower bound = 61 + 11 + 19 = 91 km — M1 A1

(4)

10

Question Number	Scheme	Marks
	Notes:	
(a)1B1	One double entry correct	
2B1	Two double entries correct	
3B1	Three double entries correct	
(b)M1	NN route, each letter appearing once, condone lack of return vertex	
1A1	CAO	
2A1	CAO	
(c)1M1	Finding my RMST – accept 61 for both marks	
1A1	Either 8 + 9 + 17 + 27 or 61 seen	
2M1	Adding on two least arcs, accept 11 and 19 or AC and AB	
2A1	91 CAO	
2. (a)	Adds a column of four zeros and 10.	B1 (1)

Shadow costs		**31**	**42**	**47**	**9**
		A	B	C	D
0	1	x	-13	**-15**	-9
-9	2	x	x	-11	**0**
-15	3	**9**	x	x	**6**
-9	4	1	-7	x	x

Marks: M1, A1, M1, A1 (4)

(b)

	A	B	C	D
1	20 – θ		θ	
2	15 + θ	7 – θ		
3		18 + θ	2 – θ	
4			28	10

M1

Entering cell: 1C Exiting cell: 3C — A1ft

(c)

	A	B	C	D
1	18		2	
2	17	5		
3		20		
4			28	10

DM1 A1 (4)

9

Question Number	Scheme	Marks
	Notes:	
(a)	1B1: cao	
(b)	1M1: Finding all 8 shadow costs 1A1: cao 2M1: Finding missing four improvement indices – no extra zeros 2A1: cao	
(c)	1M1: A valid route, their most negative II chosen, only one empty square used, θ's balance. 1A1ft: consistent; their entering and exiting cells **stated clearly** 2DM1: An **improved** solution 7 entries only, (so must now be using one of my negative indices as the entering cell). Must ft from their valid route. 2A1: cao	
3. (a)	$P - 7x + z + 4s = 320$	M1 A1 (2)
(b)	see tables below	2M1 2A1ft 1M1 2A1 3A1 (5)
(c)	$P = 376 \quad x = 8 \quad y = 1 \quad z = 0 \quad r = 14 \quad s = 0 \quad t = 0$	M1 A1ft A1 (3) **10**

3(b):

b.v	x	y	z	r	s	t	Value
r	$-\frac{1}{2}$	0	2	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	5
t	$\frac{1}{2}$	0	1	0	$-\frac{1}{4}$	1	4
P	-7	0	1	0	4	0	320

b.v.	x	y	z	r	s	t	Value	Row ops.
r	0	0	3	1	$-\frac{3}{4}$	1	14	$R_1 + \frac{1}{2}R_3$
y	0	1	$-\frac{1}{4}$	0	$\frac{1}{2}$	- 1	1	$R_2 - \frac{1}{2}R_3$
x	1	0	2	0	$-\frac{1}{2}$	2	8	$R_3 \div \frac{1}{2}$
P	0	0	15	0	$\frac{1}{2}$	14	376	$R_4 + 7R_3$

Question Number	Scheme	Marks
	Notes:	
(a)	1M1: One equal sign, P and 320 present 1A1: cao	
(b)	1M1: correct pivot located, attempt to divide row. If choosing negative pivot M0M0 in (b) 1A1: pivot row correct including change of b.v. 2M1: (ft) Correct row operations used at least once or stated correctly. 2A1ft: Looking at non zero-and-one columns, one column ft correct 3A1: cao.	
(c)	1M1: At least 4 values stated. Reading off bottom row, or negative values get M0. 1A1ft: Their four basic variables correct ft from their table. 2A1: cao	

Question Number	Scheme	Marks

4.

(a)

	S plays 1	S plays 2	S plays 3
L plays 1	-4	-1	1
L plays 2	3	-1	-2
L plays 3	-3	0	2

Row 3 dominates row 1 so row 1 may be deleted.

	S plays 1	S plays 2	S plays 3
L plays 2	3	-1	-2
L plays 3	-3	0	2

M1

Let Laura play 2 with probability p and 3 with probability (1- p) — M1

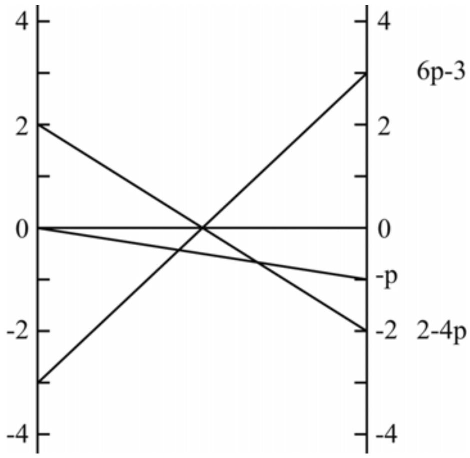
If Sam plays 1: Laura's gain is $3p - 3(1-p) = -3 + 6p$

If Sam plays 2: Laura's gain is $-p + 0\,(1-p) = -p$ — A1

If Sam plays 3: Laura's gain is $-2p + 2\,(1-p) = 2 - 4p$

(3)

(b)



B2,1ft,0 (2)

(c)

$-3+6p=-p$ — M1

$7p=3$ — A1

$p=\dfrac{3}{7}$

Laura should play row 1: never,

row 2: $\frac{3}{7}$ of the time and

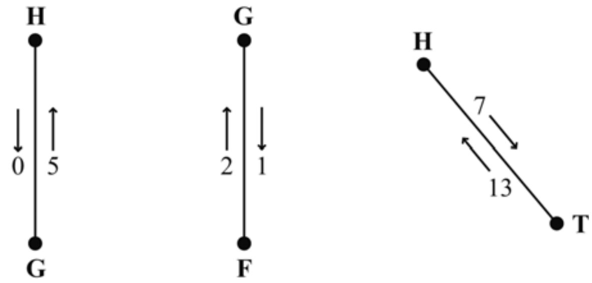
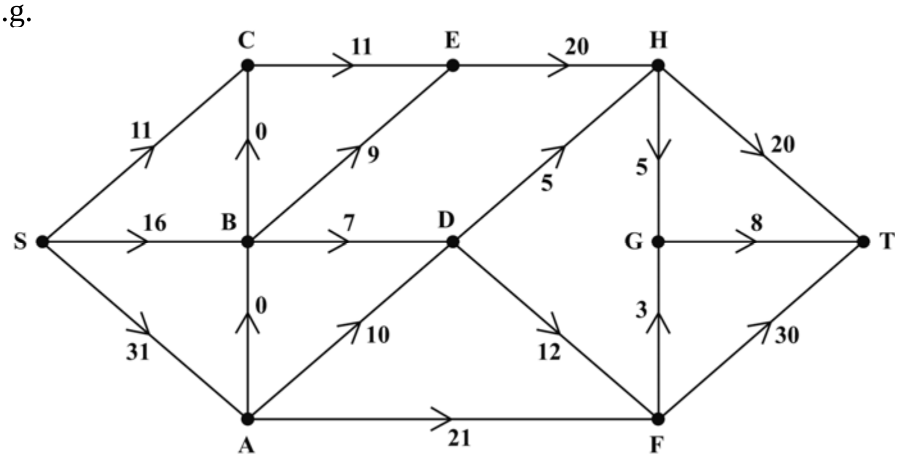
row 3: $\frac{4}{7}$ of the time — A1ft

and the value of the game is $-\frac{3}{7}$ to her. — A1

(4)

9

Question Number	Scheme	Marks
	<u>Notes</u>:	
(a)	1M1: Matrix reduced correctly. Could be implicit from equations. 2M1: Setting up three probability equations, implicit definition of p. 1A1: CAO	
(b)	1B1ft: At least two lines correct, accept p>1 or p<0 here. Must both be function of p. 2B1: 3 lines cao, $0 \leq p \leq 1$, scale clear (or 1 line = 1), condone lack of labels. Rulers used.	
(c)	3M1: Finding their correct optimal point, must have three lines, and setting up an equation to find $0 \leq p \leq 1$. 1A1: CAO 2A1ft: All three options listed must ft from their p, check page 1, no negatives. 3A1: CAO	

Question Number	Scheme	Marks
5. **(a)**	$a = 1$ $b = 5$ $c = 13$ Flow = 49	B1, B1 B1, B1 (4)
(b)		M1 A1 (2)
(c)	e.g. SBEHT - 7 together with either SBEHDAFGT – 2 or SBCEHDAFGT - 2	M1 A1 A2,1,0 (4)
(d)	58	B1 (1)
(e)	e.g. 	M1 A1 (2)
(f)	Max flow = min cut Cut through HT, HG, GF, FT Value 58	M1 A1 (2) **15**

Question Number	Scheme	Marks
	Notes:	
(a)	1B1: a = 1 cao 2B1: b = 5 cao 3B1: c = 13 cao 4B1: 49 cao	
(b)	1M1: Two numbers on each arc 1A1: cao	
(c)	1M1: One valid flow augmenting route found and value stated. 1A1: Flow increased by at least 2 2A1: A second correct flow 3A1: Flow increased by 9 and no more	
(d)	1B1: cao	
(e)	1M1: Consistent flow pattern > 51 1A1: cao	
(f)	1M1: Must have attempted (e), S to T, and made an attempt at a cut. 1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of four.	

Question Number	Scheme	Marks
6. **(a)**		

	Task A	Task B	Task C
Worker P	27	31	25
Worker Q	26	30	34
Worker R	35	29	32

Let $x_{ij} = \begin{cases} 1 \text{ if worker does the task} \\ 0 \text{ otherwise} \end{cases}$ — B1

Where x_{ij} indicates worker i being assigned to task j, $i \in \{P, Q, R\}$, $j \in \{A, B, C\}$ — B1

Minimise
$27x_{PA} + 31x_{PB} + 25x_{PC} + 26x_{QA} + 30x_{QB} + 34x_{QC} + 35x_{RA} + 29x_{RB} + 32x_{RC}$ — B1 B1

Subject to:

$x_{PA} + x_{PB} + x_{PC} = 1$

$x_{QA} + x_{QB} + x_{QC} = 1$

$x_{RA} + x_{RB} + x_{RC} = 1$

$x_{PA} + x_{QA} + x_{RA} = 1$

$x_{PB} + x_{QB} + x_{RB} = 1$

$x_{PC} + x_{QC} + x_{RC} = 1$

M1 A1 A1

(7)

(b) Since we need to maximise first subtract all entries from some n $\geq$ 41 — M1

	Task A	Task B	Task C
Worker P	8	4	10
Worker Q	9	5	1
Worker R	0	6	3

A1

(2)

9

Question Number	Scheme	Marks
	Notes:	
(a)	1B1: defining variables 2B1: defining variables 3B1: minimise 4B1: cao 1M1: At least 3 equations, coefficients of 1. Accept inequalities here should be precisely 9 variables. 1A1: cao 3 equations correct accept slack variables if defined 2A1: cao 6 equations correct accept slack variables if defined	
(b)	1M1: subtracting from some n ≥ 41 condone up to two errors 1A1: correct	
7.		
(a)		

Stage	State	Action	Dest.	Value	Marks
0	H	H- London	London	36 – 5 = 31*	
	I	I – London	London	38 – 4 = 34*	
1	F	FH	H	29 – 6 + 31 = 54	
		FI	I	29 – 7 + 34 = 56*	
	G	GH	H	27 – 5 + 31 = 53	1M1 1A1
		GI	I	27 – 6 + 34 = 55*	(2)
2	C	CF	F	42 – 6 + 56 = 92*	2M1 2A1
		CG	G	42 – 5 + 55 = 92*	
	D	DF	F	41 – 6 + 56 = 91	
		DG	G	41 – 3 + 55 = 93*	
	E	EF	F	39 – 4 + 56 = 91*	3A1
		EG	G	39 – 4 + 55 = 90	(3)
3	A	AC	C	22 – 5 + 92 = 109	3M1 4A1ft
		AD	D	22 – 4 + 93 = 111*	
		AE	E	22 – 2 + 91 = 111*	
	B	BC	C	17 – 4 + 92 = 105	
		BD	D	17 – 4 + 93 = 106*	5A1ft
		BE	E	17 – 3 + 91 = 105	(3)
4	London	London – A	A	-5 + 111 = 106*	4M1 6A1ft
		London – B	B	-3 + 106 = 103	

Optimal expected income is £10 600 — 7A1ft (3)

Question Number	Scheme	Marks
(b)	Optimal schedules are: London – A – D – G – I – London (or v.v.) London – A – E – F – I – London (or v.v)	B1ft B1 (2) **13**

Question Number	Scheme	Marks
	<u>Notes:</u> **Throughout section (a):** • **Condone lack of destination column and/or reversed stage numbers throughout.** • **Only penalise incorrect result in Value – ie ignore working values.** • **Penalise absence of state or action column with first two A marks earned only** • **Penalise empty/errors in stage column with first A mark earned only.**	
(a)	1M1: First stage completed. 1A1: CAO Penalise * errors only once in the question on the first occurrence 2M1: Second stage completed. Penalise reversed states here and at end. Bod if something in each cell. 2A1: Any 2 states correct. (Penalise * errors only once in the question). 3A1: All 3 states correct. (Penalise * errors only once in the question). 3M1: 3rd stage completed. Bod if something in each cell. 4A1ft: A or B state correct. (Penalise * errors only once in the question). 5A1ft: A and B states correct. (Penalise * errors only once in the question). 4M1: 4th stage completed. Bod if something in each cell. 6A1ft: Final, state correct. (Penalise * errors only once in the question). 7A1ft: CAO	
(b)	1B1ft: 1 route correct, consistent with their working penalise reversed states again here. Condone absence of London 2B1: both routes cao. London to London.	

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Rewarding Learning

Mark Scheme (Results)

Summer 2012

GCE Decision D2
(6690) Paper 1

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Summer 2012
Publications Code UA031968

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

1st Validity (Wed/Thur 13/14th June)
2nd Validity (Wednesday 20th June)
3rd Validity (Tuesday 26th June)
4th Validity (Sunday 1st July)

12 each time

(Not classified) – 'I think this is good' add your initials.
RFFU – Use it I agree
Poor example – don't use it.
Duplicate – I've changed the marks one this one - note marks changed.
Susie will mark them up as good example once they are commissioned.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
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- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Special case for Q1

If they reduce columns then rows they get

$$\begin{bmatrix} 2 & 2 & 1 & 3 & 3 \\ 0 & 0 & 2 & 0 & 0 \\ 15 & 6 & 0 & 9 & 7 \\ 0 & 2 & 1 & 0 & 4 \\ 14 & 9 & 8 & 13 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 & 2 \\ 0 & 0 & 2 & 0 & 0 \\ 15 & 6 & 0 & 9 & 7 \\ 0 & 2 & 1 & 0 & 4 \\ 6 & 1 & 0 & 5 & 3 \end{bmatrix}$$

Which is a three line situation. They have not followed the instructions on the QP and so missed out the first iteration.

Please give 1M1 1A1 (if earned) for their column then row reductions, but then 2M0 (and therefore 2A0) since they have not done the first iteration of the Hungarian Algorithm.

So 1M1 1A1 2M0 2A0 then other marks as usual.

June 2012
6690 Decision Mathematics D2
Mark Scheme

Question Number	Scheme	Marks
Q1(a)	$\begin{bmatrix} 129 & 127 & 122 & 134 & 135 \\ 127 & 125 & 123 & 131 & 132 \\ 142 & 131 & 121 & 140 & 139 \\ 127 & 127 & 122 & 131 & 136 \\ 141 & 134 & 129 & 144 & 143 \end{bmatrix}$	
	Reducing rows then columns	
	$\begin{bmatrix} 7 & 5 & 0 & 12 & 13 \\ 4 & 2 & 0 & 8 & 9 \\ 21 & 10 & 0 & 19 & 18 \\ 5 & 5 & 0 & 9 & 14 \\ 12 & 5 & 0 & 15 & 14 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 3 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 17 & 8 & 0 & 11 & 9 \\ 1 & 3 & 0 & 1 & 5 \\ 8 & 3 & 0 & 7 & 5 \end{bmatrix}$	1M1 1A1
	$\begin{bmatrix} 2 & 2 & 0 & 3 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 16 & 7 & 0 & 10 & 8 \\ 0 & 2 & 0 & 0 & 4 \\ 7 & 2 & 0 & 6 & 4 \end{bmatrix}$	2M1 2A1ft
	$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 14 & 5 & 0 & 8 & 6 \\ 0 & 2 & 2 & 0 & 4 \\ 5 & 0 & 0 & 4 & 2 \end{bmatrix}$	3M1 3A1ft 4A1 cso
	Allocation: A – 1, B – 5, C – 3, D – 4, E – 2.	5A1= B1 **8**
(b)	Cost is £ 647	B1 **1**
		Total 9

Notes for question 1

a1M1 Reducing rows **and then** columns – **See special case**

a1A1 CAO

a2M1 Double covered +e; one uncovered – e; and one single covered unchanged. 2 lines needed to 3 lines needed.

a2A1ft ft on their previous table.

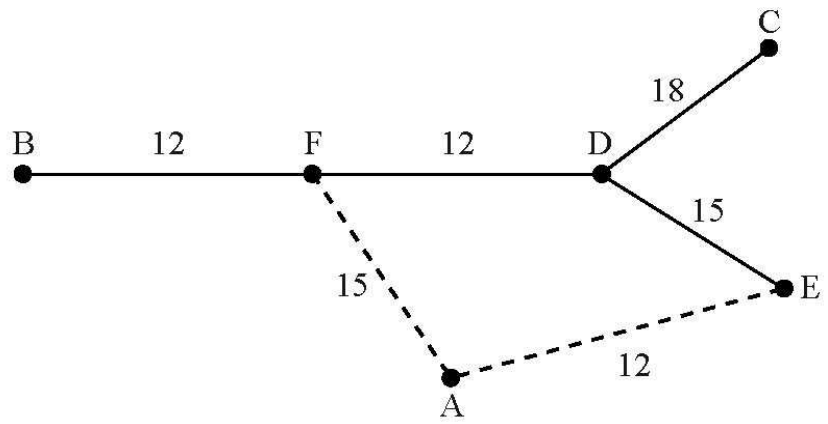
a3M1 Double covered +e; one uncovered – e; and one single covered unchanged. 3 lines needed to 5 lines needed. Watch out for 'slow Hungarian' (e.g. 2 'iterations' each subtracting 1), give M0 if seen.

a3A1ft ft on their previous table. Condone one 'new' error in table here.

a4A1 CSO on final table

a5A1= B1 CAO

b1B1 CAO

Question Number	Scheme	Marks
Q2		
(a)	A E D F B C A (arc lengths 12, 15, 12, 12, 24, 25) = 100 km	1M1 1A1 2A1
		3
(b)	Delete A 	1M1 1A1
	RMST weight = 12 + 12 + 15 + 18 = 57 (km)	
	Lower bound = 57 + 12 + 15 = 84 (km)	2M1 2A1
		4
		Total 7

Notes for question 2

a1M1 NN Each vertex visited at least once, accept 156324 across top of table (condone lack of return to start).
a1A1 Route CAO must be stated, must return to A, accept link back to A.
a2A1 Length CAO 100. Do not ISW if candidates then go on to double the route length.
b1M1 Finding correct RMST (maybe implicit) 57 sufficient; or 12, 12, 15 and 18. Must have 4 arcs.
b1A1 CAO; tree **or** list of arcs **or** 57 **or** 12 + 12 + 15 + 18 seen.
b2M1 Adding 2 least arcs from A to ‘tree’; 12 and 15 **or** AF and AE **or** 27 only. Must add these arcs distinctly.
b2A1 CAO 84

Notes for question 3

Some candidates are starting by confirming that they should use AG as their first entering square. So if the candidate starts by finding initial shadow costs and II's to confirm that AG has the most negative II, ignore this work and start marking from their first route. Do not credit shadow costs and IIs found here.

a1M1 A valid route, AG used as the empty square, θ's balance. **If AG not used mark as a misread.**
a1A1 A correct route, correctly stating exiting cell, up to my improved solution with no extra zeros.
a2M1 Finding 7 shadow costs and 6 IIs.
a2A1 Shadow costs CAO [Alt: A(17), B(18), C(18), D(0), E(2), F(-2), G(3)]
a3A1 Improvement indices CAO
a3M1 A valid route, their most negative II chosen, only one empty square used, θ's balance.
a4A1ft a correct route, correctly stating entering cell, exiting cell.
a5A1 CSO, my solution no extra zeros.
b1M1 Finding 7 shadow costs **and** all 6 IIs **or** at least1 negative II found.
b1A1 Shadow costs CAO [Alt SC: A(17), B(21), C(18), D(0), E(-1), F(-2), G(3)]
b2A1 BG = -2 found as an II.
b3A1ft CAO + conclusion. If candidates go on to perform a third iteration and determine that it is optimal, please allow this final mark. Must make link between negative II and not optimal.

Question Number	Scheme	Marks

Q3

(a)

	D	E	F	G	Supply
A	15	3 □ θ		θ	18
B		21 + θ	2 – θ		□3
C			16 + θ	13 – θ	29
Demand	15	24	18	13	70

1M1 1A1

Exiting square is BF, (θ = 2).

Shadow costs		17	19	15	20	
		D	E	F	G	Supply
0	A	15	1		2	18
1	B		23			23
1	C			18	11	29
	Demand	15	24	18	13	70

2M1 2A1

Improvement indices:

AF = 21 – 0 – 15 = 6 BG = 22 – 1 – 20 = 1

BD = 21 – 1 – 17 = 3 CD = 18 – 1 – 17 = 0

BF = 19 – 1 – 15 = 3 CE = 17 – 1 – 19 = -3

3A1

Entering square CE

	D	E	F	G	Supply
A	15	1 – θ		2 + θ	18
□		23			23
C		θ	18	11 – θ	29
Dem□nd	15	24	18	13	70

3M1 4A1ft

Exiting square is AE, (θ = 1).

	D	E	F	G	Supply
A	15			3	18
B		23			23
C		1	18	10	29
Demand	15	24	18	13	70

5A1 cso

8

(b)

Shadow costs		17	16	15	20	
		D	E	F	G	Supply
0	A	15			3	18
4	B		23			23
1	C		1	18	10	□□
	Demand	15	24	18	13	

1M1
1A1

Improvement indices:

AE = 19 – 0 – 16 = 3 BF = 19 – 4 – 15 = 0

AF = 21 – 0 – 15 = 6 BG = 22 – 4 – 20 = -2

BD = 21 – 4 – 17 = 0 CD = 18 – 1 – 17 = 0

2A1

Not optimal since a negative improvement index

3A1ft **4**

Total 12

Question Number	Scheme	Marks
Q4		

(a)

b.v	x	y	z	r	s	t	value	θ values
r	5	$\frac{1}{2}$	0	1	0	0	5	10
s	1	-2	4	0	1	0	3	$-\frac{3}{2}$
t	8	4	6	0	0	1	6	$\frac{3}{2}\leftarrow$
P	-5	-7	-4	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row ops
r	4	0	$-\frac{3}{4}$	1	0	$-\frac{1}{8}$	$\frac{17}{4}$	R1- $\frac{1}{2}$R3
s	5	0	7	0	1	$\frac{1}{2}$	6	R2 + 2R3
y	2	1	$\frac{3}{2}$	0	0	$\frac{1}{4}$	$\frac{3}{2}$	R3 ÷ 4
P	9	0	$\frac{13}{2}$	0	0	$\frac{7}{4}$	$\frac{21}{2}$	R4 + 7R3

1M1 1A1
B1
2M1 2A1
5

(b)

$$P + 9x + \frac{13}{2}z + \frac{7}{4}t = \frac{21}{2}$$

M1 A1 **2**

(c) $P = \frac{21}{2} - 9x - \frac{13}{2}z - \frac{7}{4}t$, so increasing x or z or t would decrease P

B1 **1**

Total 8

Notes for question 4

a1M1 Correct pivot located, attempt to divide row. If choosing negative number as pivot M0B0M0
a1A1 pivot row correct including change of b.v.
a1B1 Row operations CAO – allow if given in terms of old row 3.
a2M1 (ft) Correct row operations used at least once, column x, z, t or value correct.
a2A1 CAO on the three non-pivot rows.
b1M1 One equal sign, P, terms in x, z, t plus a non-zero number term.
b1A1 CAO
c1B1 **Explanation**, must refer to increasing x, z and t, condone no ref to $x = z = t = 0$, must have correct signs in equation in (b). Do not accept 'no negatives in profit row' o.e. alone.

Notes on question 5

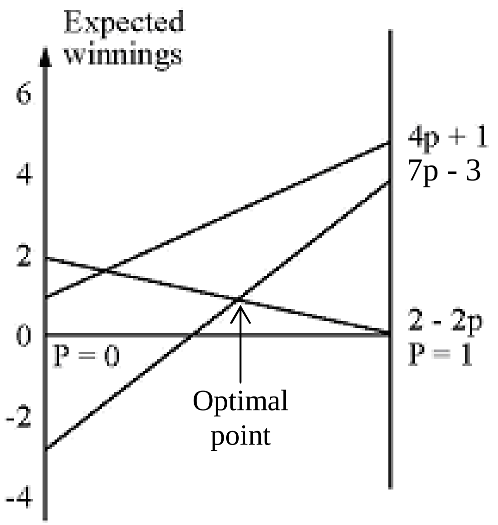
a1B1 CAO. Accept 'air dominates land' etc. Must have a named row dominating a named row
b1M1 Setting up three probability equations, implicit definition of p.
b1A1 CAO
b2M1 Three lines drawn, accept $p > 1$ or $p < 0$ here. Must be functions of p.
b2A1 CAO $0 \le p \le 1$, scale clear (or 1 line = 1), condone lack of labels. Rulers used.
b3DM1 Must have drawn 3 lines. Finding their correct optimal point, must have three lines and set up an equation to find $0 \le p \le 1$. If solving each pair of SE's must clearly select the correct one or M0, but allow recovery if their choice is clear from (c).
b3A1 CAO 5/9
b4A1ft All three options listed must ft from their p, check page 1, no negatives.
c1B1 CAO

Question Number	Scheme	Marks
Q5 (a)	Row 1 (air) dominates row 3(land), (so Row 3 can be deleted)	B1 **1**
(b)		

	Plan 1	Plan 2	Plan 3
Air	0	4	5
Sea	2	-3	1

Let Goodie play row 1 with probability p,
and row 2 with probability $1 - p$.

If F plays 1 G's expected winnings are $0 + 2(1-p) = 2 - 2p$
If F plays 2 G's expected winnings are $4p - 3(1-p) = 7p - 3$ — 1M1 1A1
If F plays 3 G's expected winnings are $5p + (1-p) = 4p + 1$



2M1 2A1

$$7p - 3 = 2 - 2p$$ — 3DM1

$$9p = 5$$

$$p = \frac{5}{9}$$ — 3A1

Goodie should play

Row 1 (air) with probability $\frac{5}{9}$, row 2 (sea) with probability $\frac{4}{9}$ and never row 3 (land). — 4A1ft **7**

(c) The value of the game to Goodie is $\frac{8}{9}$. — B1 **1**

Total 9

Notes for question 6

a1B1 CAO

b1M1 Two numbers on each arc

b1A1 CAO do give bod since they might well cross these number out.

c1M1 One valid flow augmenting route found and a valid value stated.

c1A1 Flow increased by at least 2

c2M1 A second correct flow route and value correct.

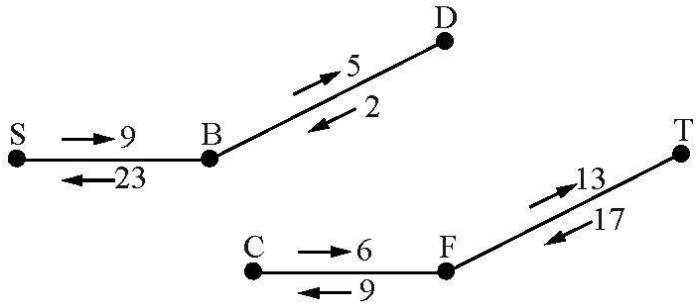
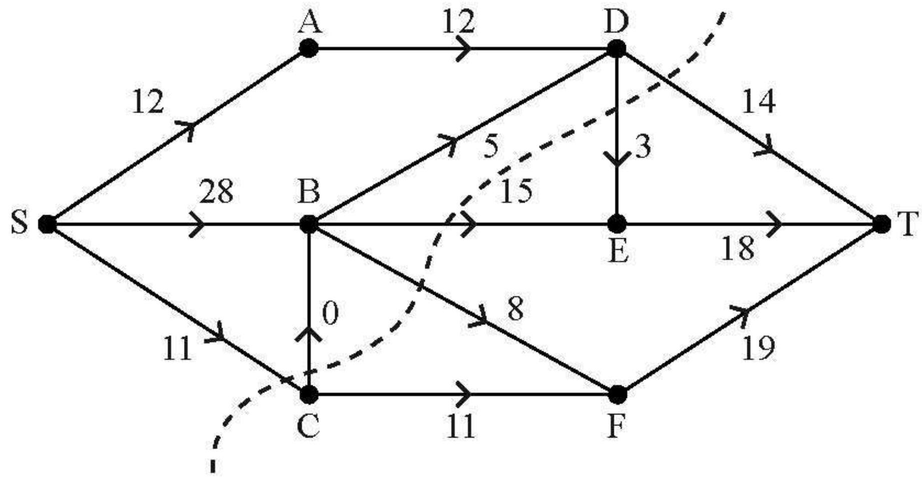
c2A1 CSO Flow increased by 5 and no more.

d1M1 Consistent flow pattern ≥ 48. One number only per arc. No unnumbered arcs.

d1A1 CAO must follow from their routes.

e1M1 Must have attempted (d) - at least one number on all but one arc, and made an attempt at a cut, condone one missing arc if listed. (Accept sum of arcs as evidence of cut here only.)

e1A1CSO For (d) and (e) Cut and (d) correct, Cut may be drawn. Must refer to max flow-min cut theorem three words out of four.

Question Number	Scheme	Marks
Q6		
(a)	Initial flow = 46	B1 **1**
(b)		M1 A1 **2**
(c)	E.g. SBDET – flow 3 SBCFT – flow 2	1M1 1A1 2M1 2A1 **4**
(d)		M1 A1 **2**
(e)	(The value of the flow is 51). The cut through DT, DE, BE, BF, CB and SC has value 51 By max flow-min cut theorem flow is maximal	M1 A1cso **2**
		Total 11

Question Number	Scheme	Marks
Q7	Let x_{ij} be 0 or 1 $\begin{cases} 1 \text{ if worker } (i) \text{ does task } (j) \\ 0 \text{ otherwise} \end{cases}$ where $i \in \{\text{A, B, C, D}\}$ and $j \in \{\text{P, Q, R, S}\}$	B1
	minimise $P = 23x_{AP} + 41x_{AQ} + 34x_{AR} + 44x_{AS}$ $+21x_{BP} + 45x_{BQ} + 33x_{BR} + 42x_{BS}$ $+26x_{CP} + 43x_{CQ} + 31x_{CR} + 40x_{CS}$ $+20x_{DP} + 47x_{DQ} + 35x_{DR} + 46x_{DS}$	1M1 1A1
	Subject to	
	$x_{AP} + x_{AQ} + x_{AR} + x_{AS} = 1$ or $\sum x_{Aj} = 1$	
	$x_{BP} + x_{BQ} + x_{BR} + x_{BS} = 1$ or $\sum x_{Bj} = 1$	2M1
	$x_{CP} + x_{CQ} + x_{CR} + x_{CS} = 1$ or $\sum x_{Cj} = 1$	
	$x_{DP} + x_{DQ} + x_{DR} + x_{DS} = 1$ or $\sum x_{Dj} = 1$	2A1 3M1
	$x_{AP} + x_{BP} + x_{CP} + x_{DP} = 1$ or $\sum x_{iP} = 1$	
	$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} = 1$ or $\sum x_{iQ} = 1$	
	$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$ or $\sum x_{iR} = 1$	3A1 **7**
	$x_{AS} + x_{BS} + x_{CS} + x_{DS} = 1$ or $\sum x_{iS} = 1$	**Total 7**

Notes for question 7

1B1 Defining variables fully both 'bits' values and subscripts. Penalise poor variable choice, (AP etc.) here.
1M1 Attempt at a 16 term expression, coefficients 'correct', but condone 2 slips.
1A1 CAO + minimise. Penalise reversed subscripts once only per question.
2M1 Four eqns, each in four vars, coeffs of 1, all 16 vars included, = 1, accept $\leq 1, \geq 1$ here for this M only
2A1 Any 4 CAO. Penalise reversed subscripts once only per question.
3M1 All 8 equations, each in four variables, unitary coefficients, all 16 variables included = 1.
3A1 CAO. Penalise reversed subscripts once only per question.

Notes for question 8 – see alts too

<u>ALL M marks - Must bring earlier optimal results into calculations. Ignore extra rows. Must have necessary right 'ingredients' (– storage costs, overheads, extra worker costs) at least once per stage.</u>

1M1 First stage completed. 3 rows.
1A1 CAO condone missing * here. No extra rows.
2M1 Second stage completed. Expect 3 states.
2A1ft Any 2 states correct. Ft for * values only No missing/extra rows. (Penalise * errors only once in the qn).
3A1 CAO All 3 states correct. No missing rows. (Penalise * errors only once in the question).
3M1 3rd stage completed. Expect 3 states.
4A1ft Any state correct. Ft on * values only. No missing rows. (Penalise * errors only once in the qn).
5A1ft Any 2 states correct. Ft on * values only. No missing rows. (Penalise * errors only once in the qn).
6A1 CAO All 3 states correct. No missing/extra rows. (Penalise * errors only once in the question).
4M1 4th stage completed.
7A1 CAO Final, state correct. No missing/extra rows. (Penalise * errors only once in the question).
1B1 CAO. Must have attempted algorithm, getting at least one M mark.

Question Number	Scheme	Marks
Q8	E.g.	

Stage	State	Action	Dest.	Value	Marks
April	0	4	0	400+ 300 = 700*	
(4)	1	3	0	150+300 = 450*	1M1 1A1 **2**
	2	2	0	300+300 = 600*	
March	0	3	0	300+700 = 1000*	2M1
(3)		4	1	400+ 300+450 = 1150	
	1	2	0	150+300+700 = 1150	2A1ft
		3	1	150+300+450 = 900*	
		4	2	400+150+300+600 =1450	
	2	1	0	300+300+700 = 1300	
		2	1	300+300+450 = 1050*	3A1
		3	2	300+300+600 = 1200	**3**
Feb.	0	2	0	300+1000 = 1300	
(2)		3	1	300+ 900 = 1200*	3M1
		4	2	400 +300+1050 =1750	4A1ft
	1	1	0	150+300+1000 = 1450	
		2	1	150+300+ 900 = 1350*	5A1ft
		3	2	150+300+1050 =1500	
	2	0	0	300+ 1000 = 1300*	6A1
		1	1	300+300+ 900 = 1500	**4**
		2	2	300+300+1050 =1650	
Jan.	0	2	0	300+1200 = 1500*	4M1 7A1
(2)		3	1	300+1350 = 1650	**2**
		4	2	400 +300+1300 = 2000	

Month	Jan	Feb	March	April
Number made	2	3	3	3

B1 **1**

Total 12

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Rewarding Learning

Alt correct solution – adding the storage costs at start of month.

Stage	**State**	**Action**	**Dest**	**Value**	
April	0	4	0	400 + 300 = 700*	1M1
(4)	1	3	0	300 = 300*	1A1
	2	2	0	300 = 300*	
March	0	3	0	300 + 700 = 1000*	2M1
(3)		4	1	400 + 150 + 300 + 300 = 1150	
	1	2	0	300 + 700 = 1000	
		3	1	150 + 300 + 300 = 750*	2A1ft
		4	2	400 + 300 + 300 + 300 = 1300	
	2	1	0	300 + 700 = 1000	
		2	1	150 + 300 + 300 = 750*	3A1
		3	2	300 + 300 + 300 = 900	
Feb	0	2	0	300 + 1000 = 1300	
(2)		3	1	150 + 300 + 750 = 1200*	3M1
		4	2	400 + 300 + 300 + 750 = 1750	4A1ft
	1	1	0	300 + 1000 = 1300	
		2	1	150 + 300 + 750 = 1200*	5A1ft
		3	2	300 + 300 + 750 = 1350	
	2	0	0	1000 = 1000*	
		1	1	150 + 300 + 750 = 1200	6A1
		2	2	300 + 300 + 750 = 1350	
Jan	0	2	0	300 + 1200 = 1500*	4M1
(2)		3	1	150 + 300 + 1200 = 1650	
		4	2	400 + 300 + 300 + 1000 = 2000	7A1

Month	Jan	Feb	March	April	
Number made	2	3	3	3	B1

Special Case 1: Working forward Max 7/12 version 1

Stage	**State**	**Action**	**Dest**	**Value**	
Jan	0	2	0	300 = 300*	1M1
(2)		3	1	300 = 300*	1A1
		4	2	400 + 300 = 700*	
Feb	0	2	0	300 + 300 = 600*	2M1
(2)		3	1	300 + 300 = 600*	
		4	2	400 + 300 + 300 = 1000	
	1	1	0	150 + 300 + 300 = 750	
		2	1	150 + 300 + 300 = 750	
		3	2	150 + 300 + 300 = 750*	2A0
	2	0	0	300 + 700 = 1000	
		1	1	300 + 300 + 700 = 1300	
		2	2	300 + 300 + 700 = 1300	3A0
March	0	3	0	300 + 600 = 900*	3M1
(3)		4	1	400 + 300 + 600 = 1300	4A0
	1	2	0	150 + 300 + 600 = 1050*	
		3	1	150 + 300 + 600 = 1050	
		4	2	400 + 150 + 300 + 600 = 1450	5A0
	2	1	0	300 + 300 + 750 = 1350	
		2	1	300 + 300 + 750 = 1350	
		3	2	300 + 300 + 750 = 1350*	6A0
April (4)	0	4	0	400 + 300 + 900 = 1600	4M1
	1	3	0	150 + 300 + 1050 = 1500*	
	2	2	0	300 + 300 + 1350 = 1950	7A1

Month	Jan	Feb	March	April	
Number made	2	3	3	3	B1

Special Case 2: Working forward Max 7/12 version 2

Stage	State	Action	Dest	Value	
Jan	0	2	0	300 = 300*	1M1
(2)		3	1	150 + 300 = 450*	1A1
		4	2	400 + 300 + 300 = 1000*	
Feb	0	2	0	300 + 300 = 600*	2M1
(2)		3	1	150+ 300 + 300 = 750*	
		4	2	400 + 300 + 300 + 300 = 1300	
	1	1	0	300 + 450 = 750	
		2	1	150 + 300 + 450 = 900	
		3	2	300 + 300 + 450 = 1050*	2A0
	2	0	0	1000 = 1000	
		1	1	150 + 300 + 1000 = 1450	
		2	2	300 + 300 + 1000 = 1600	3A0
March	0	3	0	300 + 600 = 900*	3M1
(3)		4	1	400 + 150 + 300 + 600 = 1450	4A0
	1	2	0	300 + 750 = 1050	
		3	1	150 + 300 + 750 = 1200*	
		4	2	400 + 300 + 300 + 750 = 1750	5A0
	2	1	0	300 + 1050 = 1350	
		2	1	150 + 300 + 1050 = 1500	
		3	2	300 + 300 + 1050 = 1650*	6A0
April (4)	0	4	0	400 + 300 + 900 = 1600	4M1
	1	3	0	300 + 1200 = 1500*	
	2	2	0	300 + 1650 = 1950	7A1

Month	Jan	Feb	March	April	
Number made	2	3	3	3	B1

Special Case 3: Reversed states Max 7/12 version 1

Stage	**State**	**Action**	**Dest.**	**Value**	
April	0	4	0	400+ 300 = 700*	1M1
(4)	1	3	0	150+300 = 450*	1A1 CAO
	2	2	0	300+300 = 600*	
March	0	3	0	300+700 = 1000*	2M1
(3)	1	2	0	150+300+700 = 1150	
	2	1	0	300+300+700 = 1300	
	0	4	1	400+ 300+450 = 1150	
	1	3	1	150+300+450 = 900*	2A0
	2	2	1	300+300+450 = 1050*	
	1	4	2	400+150+300+600 =1450	3A0
	2	3	2	300+300+600 = 1200	
Feb.	0	2	0	300+1000 = 1300	3M1
(2)	1	1	0	150+300+1000 = 1450	4A0
	2	0	0	300+ 1000 = 1300*	
	0	3	1	300+900 = 1200*	
	1	2	1	150+300+900 = 1350*	5A0
	2	1	1	300+300+900 = 1500	
	0	4	2	400 +300+1050 =1750	
	1	3	2	150+300+1050 =1500	6A0
	2	2	2	300+300+1050 =1650	
Jan.	0	2	0	300+1200 = 1500*	4M1
(2)	0	3	1	300+1350 = 1650	
	0	4	2	400 +300+1300 = 2000	7A1 CAO

Month	Jan	Feb	March	April	
Number made	2	3	3	3	B1

Special Case 4: Reversed states Max 7/12 version 2

Stage	State	Action	Dest	Value	
April	0	4	0	400 + 300 = 700*	1M1
(4)	1	3	0	300 = 300*	1A1 CAO
	2	2	0	300 = 300*	
March	0	3	0	300 + 700 = 1000*	2M1
(3)	1	2	0	300 + 700 = 1000	
	2	1	0	300 + 700 = 1000	2A0
	0	4	1	400 + 150 + 300 + 300 = 1150	
	1	3	1	150 + 300 + 300 = 750*	
	2	2	1	150 + 300 + 300 = 750*	
	1	4	2	400 + 300 + 300 + 300 = 1300	
	2	3	2	300 + 300 + 300 = 900	3A0
Feb	0	2	0	300 + 1000 = 1300	3M1
(2)	1	1	0	300 + 1000 = 1300	4A0
	2	0	0	1000 = 1000*	
	0	3	1	150 + 300 + 750 = 1200*	
	1	2	1	150 + 300 + 750 = 1200*	5A0
	2	1	1	150 + 300 + 750 = 1200	
	0	4	2	400 + 300 + 300 + 750 = 1750	
	1	3	2	300 + 300 + 750 = 1350	6A0
	2	2	2	300 + 300 + 750 = 1350	
Jan	0	2	0	300 + 1200 = 1500*	4M1
(2)		3	1	150 + 300 + 1200 = 1650	
		4	2	400 + 300 + 300 + 1000 = 2000	7A1 CAO

Month	Jan	Feb	March	April	
Number made	2	3	3	3	B1

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Summer 2013
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks
1.(a)	Subtracting all elements from some n ≥228	1M1
	Reducing rows and then columns to get	2M1
	0 11 16 14 0 0 0 0 0 17 20 17 then 0 6 4 3 0 17 18 16 0 6 2 2 0 16 25 18 0 5 9 4	1A1
	Using two lines and 2 to get	3M1
	2 0 0 0 0 4 2 1 0 4 0 0 0 3 7 2	2A1
	Using three lines and 1 to get	4M1
	3 0* 0 0 0 3 1 0* 1 4 0* 0 0* 2 6 1	3A1ft 4A1 **(8)**
(b)	So C = 2, J = 4, K = 3 and N = 1 maximum profit of £664	M1 A1 **(2)** **10 marks**
	Note ' minimise' gives this special case 0 4 0 0 0* 4 1 0 C = 1 4 2 0 1 then 3 1 0 0* then J = 4 2 0 0 0 2 0* 1 0 K = 2 9 8 0 4 8 7 0* 3 N = 3 Profit £651 Gives 5 max: (a) 1M0 2M1 1A1 3M0 2A0 4M1 3A1ft 4A0 (b) M1A0	

Notes for Question 1

a1M1: Subtracting all elements from some $n \geq 228$, condone up to 2 errors
a2M1: Reducing rows **and then** columns
a1A1: CAO
a3M1: Double covered +e; one uncovered – e; and one single covered unchanged. 2 lines needed to 3 lines needed.
a2A1: CAO
a4M1: One double covered +e; one uncovered – e; and one single covered unchanged. 3 lines needed to 4 lines needed.
a3A1ft: on their previous table.
a4A1: CSO on final table
b1M1: Their optimal allocation (of workers to tasks) and an attempt to calculate the profit – this mark is dependent on all M marks in (a) have been earned.
b1A1: CAO

Question Number	Scheme	Marks
2.(a)	E.g. If use CD as shortcut get 807 or if use CF + AD get 793	M1 A1 **(2)**
(b)	A F E D B C A 82 113 98 130 110 217 = 750	B1 B1 **(2)**
(c)	length of RMST = 439 439 + 82 + 113 = 634	B1 M1 A1 **(3)**
(d)	634 < optimal ≤750	B1ft **(1)** **8 marks**

Notes for Question 2

a1M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated.
a1A1: CAO – shortcut and length must be consistent.

(Examples shortcuts: CD = 807, CF + AD = 793, CF + BD = 664, AD + EF + FC = 715, DF + FC = 785 etc.)

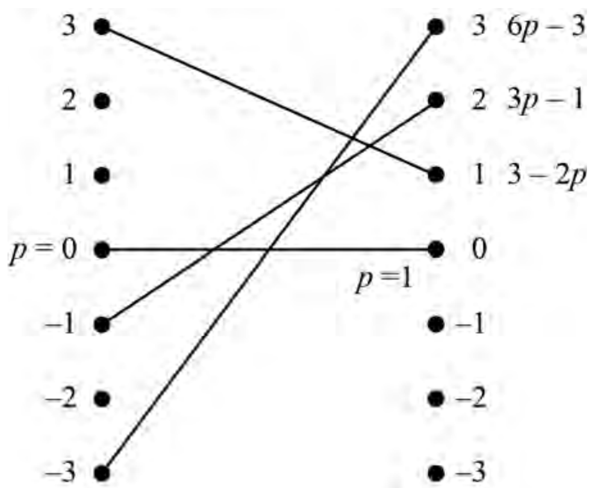
b1B1: CAO
b2B1: CAO
c1B1: CAO
c1M1: Adding two least weighted arcs to their RMST length
c1A1: CAO
d1B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)

Question Number	Scheme	Marks
3.(a)	The solution would otherwise be **degenerate**	B1 (1)
(b)	22 36 31 46 **1** **2** **3** **4** 0 **A** x x -12 -9 -1 **B** 8 x x -9 -6 **C** (8) (2) x (1) -8 **D** (9) (2) x x	M1 A1 M1 A1 (4)
(c)	Route is e.g. A3 – B3 – B2 – A2 entering cell A3, Exiting cell B3	M1 A1 A1 (3)
		8 marks

Notes for Question 3

a1B1: CAO
b1M1: 8 shadow costs stated.
b1A1: CAO
b2M1: Remaining 4 IIs stated.
b2A1: CAO
c1M1: A valid route (possibly drawn), their most negative II chosen, only one empty square used, θ's balance.
c1A1: CAO – stepping stone route **stated** or clearly shown on **separate** diagrams
c2A1: CAO for entering and exiting cells.

(b) Alternative shadow costs:
1(0) 2(14) 3(9) 4(24)
A(22) B(21) C(16) D(14)

Question Number	Scheme	Marks
4.	R1 dominates R2, so deleted R2 to give	
	2 1 3 -1 3 -3	B1
	If S plays 1; R's gain is 2p – (1 – p) = 3p – 1	M1
	If S plays 2; R's gain is p + 3(1 – p) = 3 – 2p	
	If S plays 3; R's gain is 3p – 3(1 – p) = 6p – 3	A1
		B1ft B1
	3 – 2p = 3p – 1 giving p = 4/5	M1 A1
	Robin should play R1 with probability 4/5 R2 never R3 with probability 1/5	A1ft
	The value of the game is 7/5 to Robin	A1
		9 marks

Notes for Question 4

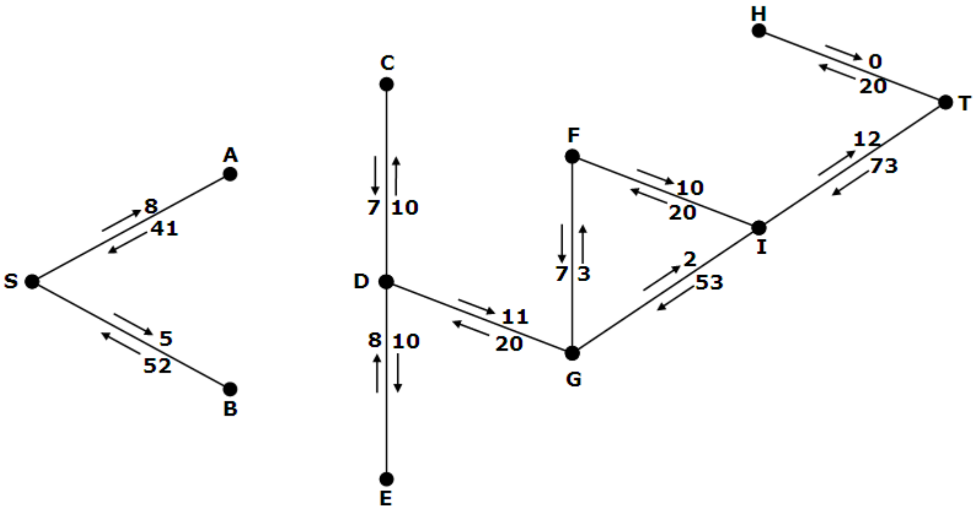
1B1: CAO
1M1: Setting up three probability expressions, implicit definition of 'p'.
1A1: CAO (condone incorrect simplification)
2B1ft: Attempt at three lines (correct gradients and correct order of intersection with 'axes'), accept p > 1 or p < 0 here. Must be functions of p.
3B1: CAO 0 ≤ p ≤ 1, scale clear (or 1 line = 1), condone lack of labels. Rulers used.
2M1: Finding their correct optimal point, must have three lines and three intersection points and set up an equation to find 0 ≤ p ≤ 1. Dependent on the second B mark being earned. Solving all three simultaneous equations only is M0.
2A1: CSO (all previous marks must have been awarded)
3A1ft: All three options listed must ft from their p, check page 1 for R should never play 2. $0 \le \text{probabilities} \le 1$ Dependent on **both** previous M marks being awarded.
4A1: CAO for the value of the game (7/5)

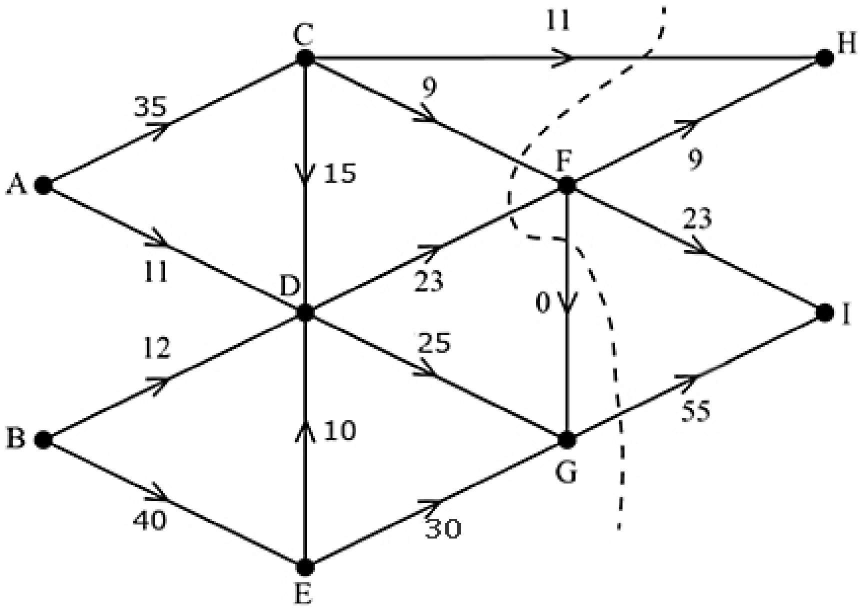
Question Number	Scheme	Marks
5. (a)	(see tableau below)	M1 A1 M1 A1 A1 **(5)**
(b)	$P + x + 4s + 18t = 240$	B1 **(1)**
(c)	$P = 240 - x - 4s - 18t$ and at present x, s and t are zero. If we increase any of these the profit will decrease.	B2, 1, 0 **(2)**
		8 marks

b.v.	x	y	z	r	s	t	Value	Row ops
r	$\frac{4}{5}$	0	0	1	$\frac{1}{5}$	$-\frac{3}{5}$	11	$R_1 + \frac{1}{2}R_2$
y	$\frac{3}{5}$	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	2	$R_2 \div 2.5$
z	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	$\frac{3}{5}$	4	$R_3 - \frac{1}{2}R_2$
P	1	0	0	0	4	18	240	$R_4 + 10R_2$

Notes for Question 5

a1M1: correct pivot located, attempt to divide row. If choosing negative pivot M0M0.
a1A1: pivot row correct including change of b.v.
a2M1: (ft) One row (excluding the pivot row) correct or one column either the value, *x, s* or *t* column correct.
a2A1ft: Correct row operations used at least once. One column either the value, *x, s* or *t* column correct on the ft.
a3A1: CAO.
b1B1: CAO
c1B1: Using their profit equation to make a pertinent statement. Maybe muddled, if bod give this mark only. No ‘negatives’ in their profit equation.
c2B1: Good explanation – dependent on the correct equation being stated in (b).

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>6.(a)</td><td>Initial flow = 93</td><td>B1
(1)</td></tr>
<tr><td>(b)</td><td>Adds supersource S plus arcs SA(49) and SB(57)
Adds supersink T plus arcs HT(20) and IT(85)</td><td>M1
A1
(2)</td></tr>
<tr><td></td><td>


</td><td>M1 A1 A1
(3)</td></tr>
<tr><td>(d)</td><td>E.g. SACDGIT – 2 and SACDGFIT - 3
Maximum flow = 98</td><td>M1 A1
A1
(3)</td></tr>
<tr><td>(e)</td><td>E.g.


</td><td>M1 A1
(2)</td></tr>
<tr><td>(f)</td><td>Max flow= min cut, cut through CH, CF, DF, FG, GI</td><td>M1 A1
(2)</td></tr>
</table>

Question Number	Scheme	Marks
		13 marks

Notes for Question 5
a1B1: CAO b1M1: All relevant arcs added OR all arcs and numbers from supersource OR from supersink correct. b1A1: CAO all arcs and numbers correct. c1M1: 2 numbers and arrows on each arc. c1A1: CAO Condone 4 errors. c2A1: CAO. d1M1: One valid flow augmenting route (from S to T) found and a value stated. d1A1: Flow increased by 5 and no more. d2A1: CAO 98 (allow if seen in (f) but must be clearly labelled as the maximum flow) e1M1: Consistent flow pattern > 95 – condone S and T's presence. Must have exactly one number on each arc. e1A1: CAO must follow from their routes (allow if routes in (d) do not include S and/or T). f1M1: Must have attempted (e) and made an attempt at a cut. f1A1: cut correct – may be drawn. Must have shown a correct flow of 98 in (e). Refer to max flow-min cut theorem all four words. Examples of flow augmenting routes: • SACDGFIT (3), SACDGIT (2) • SBEDGFIT (3), SBEDGIT (2) • SBEDGFIT (3), SACDGIT (2) • SACDGFIT (3), SBEDGIT (2)

Question Number	Scheme	Marks
7.	E.g. Add 4 to each element	B1
	Let p_1, p_2, p_3 be the probability of (A) playing 1, 2 and 3 respectively (where $p_1, p_2, p_3 \geq 0$)	B1
	let V = value of the game (to player A)	B1
	maximise P = V subject to:	B1
	$5p_1 + 2p_2 + 9p_3 \geq V$ $p_1 + 7p_2 + 3p_3 \geq V$ $6p_1 + 3p_2 + 4p_3 \geq V$	M1 A1
	$p_1 + p_2 + p_3 \leq 1$	A1 **(7)** **7 marks**

Notes for Question 7

1B1: Making all terms non-negative.
2B1: Defining probability variables
3B1: Defining V
4B1: 'maximise' + function/expression
1M1:At least three equations/inequations in (V), p_1, p_2 and p_3
1A1: The three inequalities in V, p_1, p_2 and p_3 CAO
1A1: probability sum inequality (or equation) correct.

Question Number	Scheme				Marks
8.	Stage	State	Action	Dest.	Value	

Stage	State	Action	Dest.	Value	Marks
Fresh	0	0	0	0	
	1	1	0	45	
	2	2	0	85	1M1 1A1
	3	3	0	120	(stage 1)
	4	4	0	150	
	5	5	0	175	
Frozen	0	0	0	0	
	1	1	0	45 + 0 = 45*	
		0	1	0 + 45 = 45*	
	2	2	0	70 + 0 = 70	
		1	1	45 + 45 = 90*	2M1 2A1
		0	2	0 + 85 = 85	(1st 4 states of stage 2)
	3	3	0	100 + 0 = 100	
		2	1	70 + 45 = 115	
		1	2	45 + 85 = 130*	3A1
		0	3	0 + 120 = 120	(state 3)
	4	4	0	120 + 0 = 120	
		3	1	100 + 45 =145	
		2	2	70 + 85 = 155	3M1 4A1
		1	3	45 + 120 = 165*	(Last 2 states
		0	4	0 + 150 = 150	of stage 2)
	5	5	0	130 + 0 = 130	
		4	1	120 + 45 = 165	
		3	2	100 + 85 = 185	
		2	3	70 + 120 = 190	5A1
		1	4	45 + 150 = 195*	(state 5)
		0	5	0 + 175 = 175	
Canned	5	5	0	195 + 0 = 195	
		4	1	155 + 45 = 200	
		3	2	125 + 90 = 215*	4M1 6A1ft
		2	3	75 + 130 = 205	(3rd stage)
		1	4	35 + 165 = 200	
		0	5	0 + 195 = 195	

Fresh = 1, Frozen = 1, Canned = 3 — 5M1

Monthly income = £ 21 500 — 7A1ft

12 marks

Notes for Question 8
• **ALL M marks - Must bring earlier optimal results into calculations. Must have necessary right 'ingredients' (profit values from the table) at least once per stage.** • **Penalise inconsistency / errors with the state / destination columns with the first two A marks earned only.** • **Penalise empty / errors in stage column with first A mark earned only.** 1M1: First stage completed (bod something in each cell). Condone missing state 0 for this mark. Must have columns for stage, state, value and one of either action or destination. 1A1: CAO for first stage – condone missing * in this stage. 2M1: Second stage – states 0, 1 and 2 complete (bod something in each cell). Condone missing state 0 for this mark. 2A1: States 0, 1 and 2 correct for second stage. Penalise * errors only twice in the question on the first occurrences. 3A1: State 3 correct for second stage. Penalise * errors only twice in the question. 3M1: Second stage – states 3 and 4 complete (bod something in each cell). 4A1: State 4 correct for second stage. Penalise * errors only twice in the question. 5A1: State 5 correct for second stage. Penalise * errors only twice in the question. 4M1: Third stage complete (bod something in each cell). 6A1ft: Third stage correct (ft from previous stage). Penalise * errors only twice in the question. 5M1: For Fresh = 1, Frozen = 1, Canned = 3 – dependent on all previous M marks having been awarded. 7A1ft: Income correct for their table. Must have earned the first 4 M marks only (**not** dependent on 5M1).

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Mark Scheme (Results)

Summer 2013

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Summer 2013
Publications Code UA035688

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

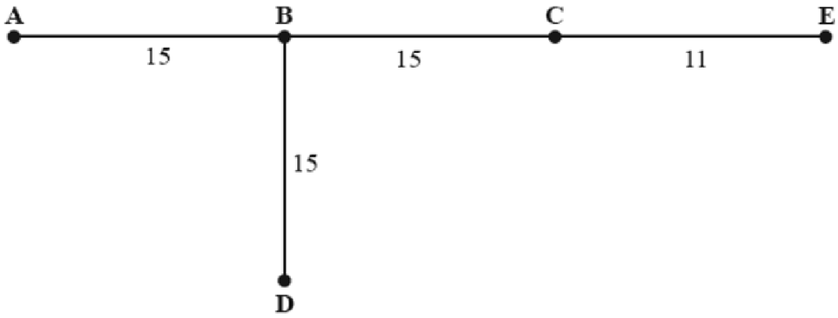
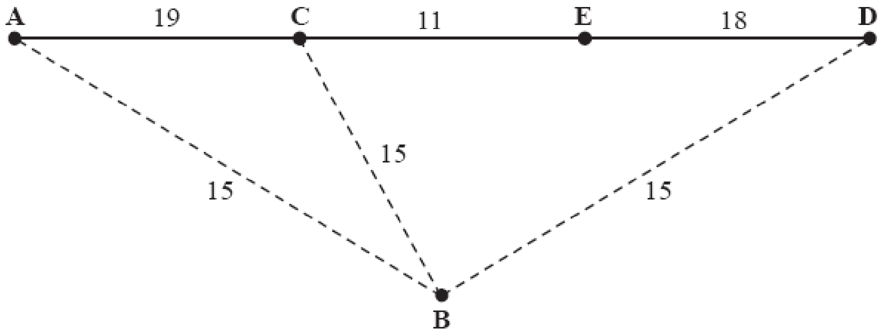
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks
1(a)	 e.g. starting from A: AB, BD, BC, CE or AB, BC, CE, BD	M1A1 (2)
(b)	2 x 56 = 112	B1 (1)
(c)	A B C E D A and A B D E C A 15 15 11 18 25 = 84 15 15 18 11 19 = 78	M1 A1 A1 (3)
(d)	78 is the better upper bound	B1ft (1)
(e)	 Lower bound = 48 + 15 + 15 = 78	1M1A1 2M1A1 (4)

Question Number	Scheme	Marks
(f)	The route is ABDECA (The optimal route length is 78, since upper bound = lower bound) a1M1 First three arcs (or all 5 nodes / or numbers across the top of the matrix) selected correctly (may start from any node). Award M1 only for a correct tree with no working. a1A1 CAO (order of arc selection clear) b1B1 112 CAO c1M1 Nearest Neighbour either A-B-C-E-D- or A-B-D-E-C- (condone lack of return to start). Accept 12354 or 12534 across the top of the matrix. c1A1 1 route and length CAO (Do not ISW if route length is doubled) c2A1 both routes and lengths CAO (Do not ISW if route lengths are doubled) d1B1ft their stated shortest (must be a number) e1M1 Finding correct RMST (maybe implicit) 48 sufficient, or correct numbers. 3 arcs. e1A1 CAO; tree **or** 48 **or** 11 + 18 + 19 seen. e2M1 Adding 2 least arcs to B; 15 and 15 **or** two out of BA, BC or BD **or** 30 only e2A1 CAO 78 f1B1 CAO, accept any start point for the correct tour, but must return to start. Dependent on their answer to part (d) = their answer to part (e).	B1 (1) **Total 12**

Question Number	Scheme	Marks

2(a)

	1	2	3	Supply
A	18			18
B	9	5		14
C		13	8	21
D			12	12
Demand	27	18	20	65

B1
(1)

(b)

	1	2	3
A	18		
B	9-θ	5+θ	
C		13-θ	8+θ
D	θ		12-θ

giving

	1	2	3
A	18		
B		14	
C		4	17
D	9		3

M1A1
(2)

(c)

Shadow costs		10	22	19
		1	2	3
0	A	X	-11	1
-15	B	20	X	9
-7	C	21	X	X
-1	D	X	0	X

1M1A1

	1	2	3
A	18-θ	θ	
B		14	
C		4-θ	17+θ
D	9+θ		3-θ

giving

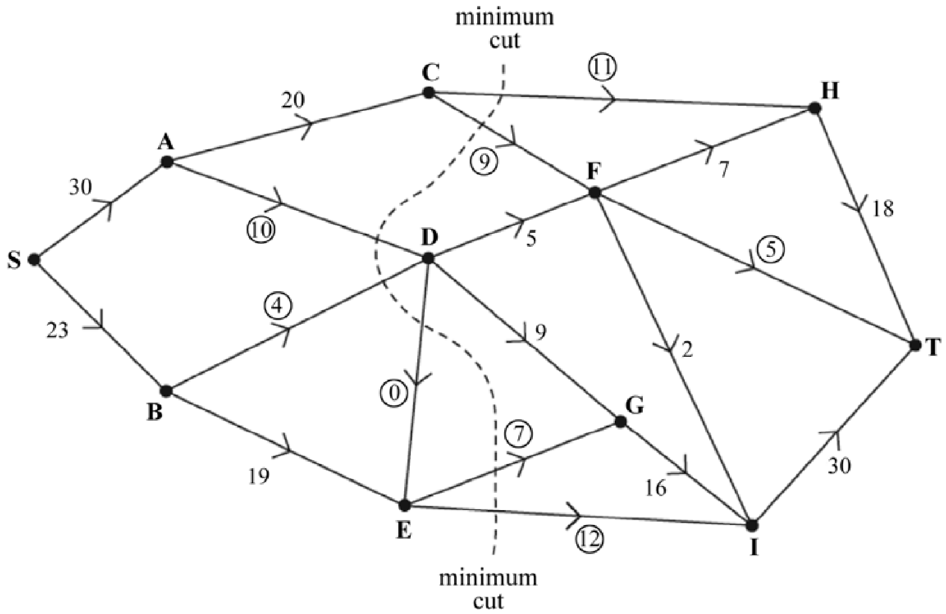
	1	2	3
A	15	3	
B		14	
C		1	20
D	12		

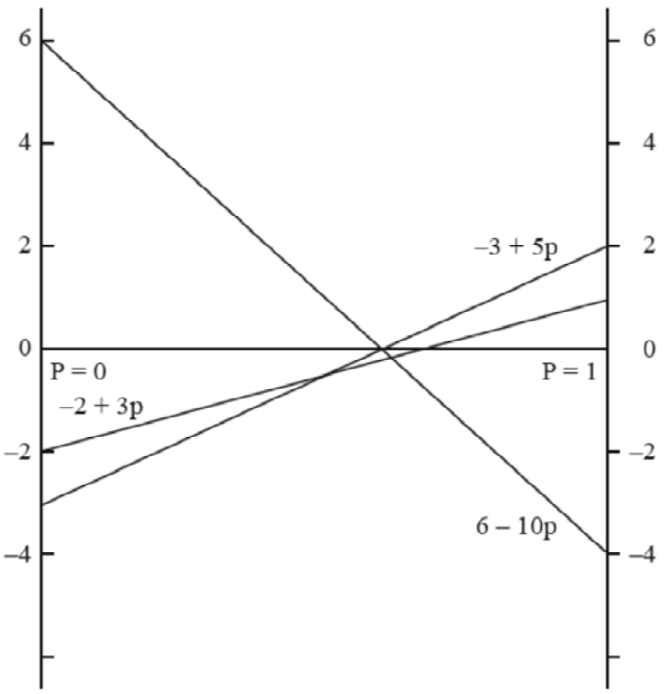
2M1A1

(θ = 3) entering cell A2, exiting cell D3

(4)

Question Number	Scheme	Marks
(d)	<table><tr><td>Shadow costs</td><td></td><td>10</td><td>11</td><td>8</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td>3</td></tr><tr><td>0</td><td>A</td><td>X</td><td>X</td><td>12</td></tr><tr><td>-4</td><td>B</td><td>9</td><td>X</td><td>9</td></tr><tr><td>4</td><td>C</td><td>10</td><td>X</td><td>X</td></tr><tr><td>-1</td><td>D</td><td>X</td><td>11</td><td>11</td></tr></table>	M1 A1
	Optimal since no negative improvement indices.	A1 (3)
	a1B1 CAO b1M1 A valid route, only one empty square, D1, used, θ's balance. b1A1 Correct route, up to an improved solution (six numbers no zeros) c1M1 Finding 7 shadow costs and 6 Improvement indices c1A1 Shadow costs [Alt: A(10), B(-5), C(3), D(9), 1(0), 2(12), 3(9)] and improvement indices CAO c2M1 A valid route, their most negative II chosen, only one empty square used, θ's balance. c2A1 CSO (entering A2, and exiting D3 stated) d1M1 Finding 7 shadow costs **and** all 6 IIs **or** at least1 negative II found. d1A1 CAO for the shadow costs [Alt: A(10), B(6), C(14), D(9), 1(0), 2(1), 3(-2)] and 6 positive IIs d2A1 CSO (for part (d)) + optimal.	
		Total 10

Question Number	Scheme	Marks
3(a)	Initial flow = 44	B1
(b)	Value of cut = 12+7+4+10+2+5+31 = 71	B1 (2)
(c)	e.g.SACFHT – 3; SADGIT – 4; SBEDFHT – 2 e.g. SACFHT – 3; SADFHT – 2; SADGIT – 2; SBEDGIT - 2	M1A1;A1; A1 (4)
(d)	e.g.  	M1A1 (2)
(e)	Maximum flow=minimum cut e.g. cut through CH, CF, AD, BD, DE, EG and EI a1B1 CAO b1B1 CAO c1M1 One valid flow augmenting route found and a value stated. c1A1 Flow increased by at least 2 c2A1 A second correct flow route (and value at least 2) correct c3A1 CSO Flow increased by 9 and no more. d1M1 Consistent flow pattern > 50 (check each node, must have exactly 1 number per arc) d1A1 CAO, showing flow of 53, must follow from their routes. e1DM1 Must have attempted (d) and made an attempt at a cut. e1A1 cut correct – may be drawn. Refer to max flow-min cut theorem all four words (alternative cut: CH, CF, AD, BD, BE). Guidance for 3(c) SA +7 SB +2 AC +3 AD +4 BD none BE + 2 ED + 2 CH none CF +3 EG none EI none (DF+2 DG+2 FH +5 FT none FI none GI +4 HT +5 IT +4)	DM1 A1 (2) **Total 10**

Question Number	Scheme	Marks
4(a)	$\begin{bmatrix} 4 & -6 \\ -2 & 3 \\ -1 & 2 \end{bmatrix}$ column 2 dominates column 1	B1 (1)
(b)	$\begin{bmatrix} -4 & 2 & 1 \\ 6 & -3 & -2 \end{bmatrix}$	B1 B1 (2)
(c)	Let p = probability that B plays new row 1 If A plays 1: B's expected winnings = -4p + 6(1- p) = 6 – 10p If A plays 2: B's expected winnings = 2p – 3(1 - p) = – 3 + 5p If A plays 3: B's expected winnings = p – 2(1 - p) = – 2 + 3p	1M1A1 (2)
		B2, 1ft, 0 (2)
	$6-10p=-2+3p$ $8=13p$ $p=\frac{8}{13}$	2M1 A1 (2)
	B should play 1: never, play 2 with probability $\frac{8}{13}$ and play 3 with probability $\frac{5}{13}$ The value of the game is $-\frac{2}{13}$ to B	B1 B1 (2) **Total 11**

Question Number	Scheme	Marks
	a1B1 CAO (accept reduced matrix or 'column 2 dominates column 1' or column crossed out). Allow recover in part (b) b1B1 either 3 × 2 matrix with correct values (including signs) or 2 × 3 matrix with correct values (condone incorrect signs) b2B1 CAO c1M1 Setting up three probability expressions, implicit definition of 'p'. c1A1 CAO (condone incorrect simplification) c1B1ft Attempt at three lines (correct gradients and intersection with 'axes'), accept $p > 1$ or $p < 0$ here. Must be functions of p. c2B1 CAO $0 \leq p \leq 1$, scale clear (or 1 line = 1), condone lack of labels. Rulers used. c2M1 Finding their correct optimal point, must have three lines and set up an equation to find $0 \leq p \leq 1$. Dependent on first B mark in part (c). Must have three intersection points. Solving all three simultaneous equations only is M0. c2A1 CSO c3B1 All three options listed must ft from their p, check page 1 for B should never play 1. $0 \leq$ probabilities ≤ 1. c4B1 -2/13 CAO (accept awrt 0.154) **SC1**: If column 2 deleted in (a) candidates can earn a **maximum** of (a) B0 (b) B1 B0 (c) M1 A0 B1 B0 M1 A0 B1 B1 (max. of 6) – the final B mark is for the value of the game being -4/3 **SC2**: If column 3 is deleted in (a) candidates can earn a **maximum** of (a) B0 (b) B1 B0 (c) M1 A0 B1 B0 M0 A0 B0 B0	

Question Number	Scheme	Marks
5(a)	Variable z was increased first, since it has become a basic variable.	B1

(b)

b.v	x	y	z	r	s	t	value
r	-1	[2]	0	1	0	1	8
s	-1	3	0	0	1	1	22
z	-2	1	1	0	0	1	11
P	2	-5	0	0	0	$\frac{1}{2}$	15

b.v	X	y	z	r	s	t	value	row ops
y	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	4	$R_1 \div 2$
s	$\frac{1}{2}$	0	0	$-\frac{3}{2}$	1	$-\frac{1}{2}$	10	$R_2 - 3R_1$
z	$-\frac{3}{2}$	0	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	7	$R_3 - R_1$
P	$-\frac{1}{2}$	0	0	$\frac{5}{2}$	0	3	35	$R_4 + 5R_1$

1M1A1
2M1A1
(4)

b.v	X	y	z	r	s	t	value	row ops
y	0	1	0	-1	1	0	14	$R_1 + \frac{1}{2}R_2$
x	1	0	0	-3	2	-1	20	$R_2 \div \frac{1}{2}$
z	0	0	1	-5	3	-1	37	$R_3 + \frac{3}{2}R_2$
P	0	0	0	1	1	$\frac{5}{2}$	45	$R_4 + \frac{1}{2}R_2$

3M1A1ft
4M1A1
(4)

(c) P =45; x = 20; y = 14; z = 37; r = s = t = 0.

M1 A1 (2)
Total 11

a1B1 Identifies *z*, refers to basic variable.
b1M1 Correct pivot located, attempt to divide row. If choosing negative pivot M0M0.
b1A1 CAO pivot row correct including change of b.v.
b2M1 (ft) Correct row operations used at least once, column *x*, *r*, *t* or value correct.
b2A1 CAO including row operations
b3M1 Their correct pivot located, attempt to divide row. If choosing negative pivot M0M0.
b3A1ft pivot row correct including change of b.v.
b4M1 (ft) Correct row operations used at least once, column *r*, *s*, *t* or value correct.
b4A1 CAO
c1M1 Their correct values stated for at least *P*, *x*, *y*, *z* from their ‘optimal’ iteration. No negatives. Two M marks in part (b) must have been awarded
c1A1 CAO for all 7 values.

Question Number	Scheme	Marks
6 (a)	Since maximising subtract all elements from some n ≥ 257, say 260. $\begin{bmatrix} 9 & 17 & 3 \\ 16 & 13 & 5 \\ 11 & 8 & 14 \end{bmatrix} \quad \left(n = 257 \begin{bmatrix} 6 & 14 & 0 \\ 13 & 10 & 2 \\ 8 & 5 & 11 \end{bmatrix}, \quad n = 258 \begin{bmatrix} 7 & 15 & 1 \\ 14 & 11 & 3 \\ 9 & 6 & 12 \end{bmatrix} \right)$	1B1 (1)
(b)	$x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ does task } j \\ 0 & \text{otherwise} \end{cases}$	1B1
	Where x_{ij} indicates worker i being assigned to task j $i \in \{H, K, J\}$ and $j \in \{1, 2, 3\}$	2B1 (2)
	E.g. Minimise P = $9x_{H1} + 17x_{H2} + 3x_{H3} + 16x_{J1} + 13x_{J2} + 5x_{J3} + 11x_{K1} + 8x_{K2} + 14x_{K3}$ $(P = 6x_{H1} + 14x_{H2} + 13x_{J1} + 10x_{J2} + 2x_{J3} + 8x_{K1} + 5x_{K2} + 11x_{K3})$ $(P = 7x_{H1} + 15x_{H2} + x_{H3} + 14x_{J1} + 11x_{J2} + 3x_{J3} + 9x_{K1} + 6x_{K2} + 12x_{K3})$	3B1 4B1
	OR maximise P = $251x_{H1} + 243x_{H2} + 257x_{H3} + 244x_{J1} + 247x_{J2} + 255x_{J3} + 249x_{K1} + 252x_{K2} + 246x_{K3}$	(2)
	Subject to: $x_{H1} + x_{H2} + x_{H3} = 1$ or $\sum x_{Hj} = 1$ $x_{J1} + x_{J2} + x_{J3} = 1$ or $\sum x_{Jj} = 1$ $x_{K1} + x_{K2} + x_{K3} = 1$ or $\sum x_{Kj} = 1$ $x_{H1} + x_{J1} + x_{K1} = 1$ or $\sum x_{i1} = 1$ $x_{H2} + x_{J2} + x_{K2} = 1$ or $\sum x_{i2} = 1$ $x_{H3} + x_{J3} + x_{K3} = 1$ or $\sum x_{i3} = 1$	M1 1A1 2A1 (3)
	a1B1 CAO (o.e.) b1B1 possible values of x_{ij} defined b2B1 Defining x_{ij} including the set of values for i and j b3B1 Objective function b4B1 Minimise/Maximise but consistent with objective function b1M1 Three equations, unit coefficients, =1 b1A1 Any three equations CAO (condone inconsistent notation)	**Total 8**

Question Number	Scheme	Marks
	b2A1 All six equations CAO (consistent notation required)	

Question Number	Scheme	Marks
7		

Stage	State	Action	Destination	Value
end	4	Sell	-	1*
	3	Sell	-	2*
	2	Sell	-	4*
	1	Sell	-	6*
4	3	K	4	1 + 2 – 3 = 0
		R	1	6 + 11 – 9 = 8*
	2	K	3	2 + 5 – 2 = 5
		R	1	6 + 11- 8 = 9*
	1	K	2	4 + 8 – 1 = 11*
		R	1	6 + 11 – 7 = 10
3	2	K	3	8 + 5 – 2 = 11
		R	1	11 + 11 – 8 = 14*
	1	K	2	9 + 8 – 1 = 16*
		R	1	11 + 11 – 7 = 15
2	1	K	2	14 + 8 – 1 = 21*
		R	1	16 + 11 – 7 = 20
1	new	K	1	21 + 11 = 32*

Marks: 1M1A1A1; 2M1 A1; 3M1A1ft; A1; 4M1 A1ft; A1

The actions Nigel should take are:
Keep, Keep, Replace, Keep in years 1, 2, 3 and 4 respectively — B1
His income will be £32 000. — B1ft

Total 13

1M1 At least 3 columns in Stage 4 completed, something in each cell.
1A1 For stage 4 at least two columns of state, action, destination entries correct
2A1 Two rows in Stage 4 CAO. Penalise * errors only twice in the question on the first occurrences
All future M marks must bring all optimal results from previous stage into current stage at least once (or three out of four previous results correct).
2M1 All four rows in stage 4 completed. Bod if something in each cell.
3A1 CAO. Stage 4 correct. (Penalise * errors only twice in the question).
3M1 Stage 3 completed. Bod if something in each cell.
4A1ft Any state correct (Penalise * errors only twice in the question).
5A1 CAO Both states correct. (Penalise * errors only twice in the question).
4M1 Stage 2 and 1 completed. Bod if something in each cell.
6A1ft CAO Stage 2 correct. (Penalise * errors only twice in the question).
7A1 CAO Stage 1 correct.
1B1 Actions correct. Must have earned all previous M marks
2B1ft Income correct for their table. Must have earned all previous M marks.
Penalise extra rows for stage 4 with the 3rd A mark, stage 3 with the 5th A mark and stage 2 with the 6th A mark.

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Rewarding Learning

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Decision Mathematics 2R (6690/01R)

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Summer 2014
Publications Code UA038495

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (a)	(table below)	B1 **(1)**

	P	Q	R	S	Supply
A	11	2			13
B		4			4
C		4	8		12
D			3	8	11
Demand	11	10	11	8	

(b) M1 A1, M1 A1 **(4)**

Shadow costs		**28**	**32**	**35**	**41**	
		P	Q	R	S	Supply
0	A	x	x	**-2**	**-14**	13
- 3	B	6	x	**-6**	**-7**	4
-6	C	8	x	x	**-3**	12
-7	D	4	5	x	x	11
	Demand	11	10	11	8	

(c) M1 A1

	P	Q	R	S	Supply
A	11	2 – θ		θ	13
B		4			4
C		4 + θ	8 – θ		12
D			3 + θ	8 – θ	11
Demand	11	10	11	8	

Entering square AS

Exiting square is AQ, (θ = 2). A1ft

	P	Q	R	S	Supply
A	11			2	13
B		4			4
C		6	6		12
D			5	6	11
Demand	11	10	11	8	70

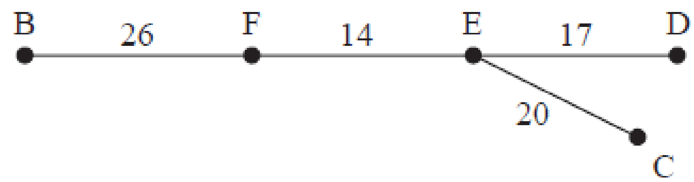
A1 **(4)**

(d) Current cost = £1152 B1 **(1)**

10 marks

Notes for Question 1

a1B1: CAO
b1M1: Finding 8 shadow costs.
b1A1: Shadow costs CAO.
b2M1: Finding the 5 missing improvement indices.
b2A1: Improvement indices CAO [Shadow costs: A(28), B(25), C(22), D(21), P(0), Q(4), R(7), S(13)].
c1M1: A valid route, their most negative II chosen, only one empty square used, θ's balance.
c1A1: CAO correct route.
c2A1ft: Correctly stating their entering and exiting cells.
c3A1: CSO
d1B1: CAO

Question Number	Scheme	Marks
2. (a)	In the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once.	B2, 1, 0 **(2)**
(b)	A D E F B C A 15 + 17 + 14 + 26 + 50 + 48 = 170	M1 A1 A1 **(3)**
(c)	 RMST weight = 26 + 14 + 17 + 20 = 77 (km) Lower bound = 77 + 15 + 30 = 122 (km)	M1 A1 A1 **(3)**
(d)	$122 \leq$ length ≤ 170	B2,1,0 **(2)** **10 marks**

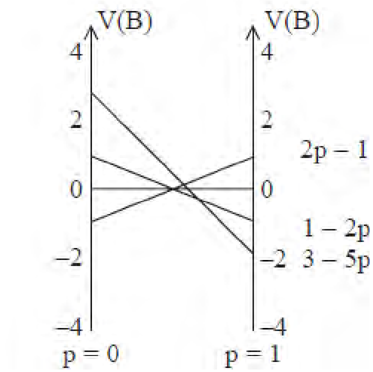
Notes for Question 2

a1B1: Understands the difference is connected to the number of times each vertex may be visited.
a2DB1: Correctly identifies which is classical and which is practical and correctly states the difference.
b1M1: Nearest neighbour A – D – E – F – B – C – or accept 145623 across top of table (condone lack of return to start).
b1A1: Route correctly stated, must return to A, accept link back to A.
b2A1: Length correctly stated. Do not ISW if candidates then go on to double the route length.
c1M1: Finding RST (maybe implicit) and using the correct two least lengths. Their RST must have only four arcs of which none are incident to A.
c1A1: RMST correct **or** list of arcs **or** 77 **or** 26 + 14 + 17 + 20 seen.
c2A1: CAO 122
d1B1ft: Their correct numbers correctly used (their upper bound must be a cycle and their lower bound must have scored M1 in (c)), accept any inequalities or any indication of interval from their 122 to their 170.
d2B2: CAO including correct inequalities (but condone $122 <$ length ≤ 170).

Question Number	Scheme	Marks
3.(a)	Column 3 dominates column 1, so delete column 1	B1

	B2	B3
A1	2	-3
A2	1	-1
A3	- 1	1

Question Number	Scheme	Marks
	Let B play 2 with probability p and 3 with probability $1 - p$	B1
	If A plays 1 B's expected winnings are $-\{2p - 3(1-p)\} = 3 - 5p$	
	If A plays 2 B's expected winnings are $-\{p - (1-p)\} = 1 - 2p$	M1 A1
	If A plays 3 B's expected winnings are $-\{-p + (1-p)\} = 2p - 1$	



Question Number	Scheme	Marks
	(graph)	M1 A1
	$2p - 1 = 1 - 2p$	DM1
	$p = \frac{1}{2}$	A1
	B should play column 2 and column 3 each with probability $\frac{1}{2}$ and never play column 1.	A1 **(9)**
(b)	V(B) = 0	B1 **(1)**
		10 marks

Notes for Question 3

a1B1: CAO Col 3 dominates Col 1
a2B1: Defines p – allow those who define B play 2 with prob. p but no incorrect statements.
a1M1: Setting up three probability equations, implicit definition of p.
a1A1: CAO (condone incorrect simplification).
a2M1: Three lines drawn, accept $p > 1$ or $p < 0$ here. Must be functions of p.
a2A1: CAO $0 \le p \le 1$, scale correct and clear (or 1 line = 1), condone lack of labels. Rulers used.
a3DM1: Must have drawn 3 lines. Finding their correct optimal point, must have three lines and set up an equation to find $0 \le p \le 1$. Dependent on previous M mark. Must have three intersection points. If solving each pair of SE's must clearly select the correct one or M0, but allow recovery if their choice is clear.
a3A1: CAO – dependent on all, but a2B1, being awarded in this part.
a4A1: CAO
bDB1: CAO – dependent on all previous M marks in (a).

Question Number	Scheme	Marks
4.(a)		

b.v	x	y	z	r	s	t	value	θ values
r	4	3	$\frac{5}{2}$	1	0	0	50	16.67
s	1	$\boxed{2}$	1	0	1	0	30	15
t	0	5	1	0	0	1	80	16
P	–25	–40	–35	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row ops
r	$\frac{5}{2}$	0	$\boxed{1}$	1	$-\frac{3}{2}$	0	5	R1 – 3R2
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	15	R2 ÷ 2
t	$-\frac{5}{2}$	0	$-\frac{3}{2}$	0	$-\frac{5}{2}$	1	5	R3 – 5R2
P	–5	0	–15	0	20	0	600	R4 + 40R2

M1 A1
B1
M1 A1

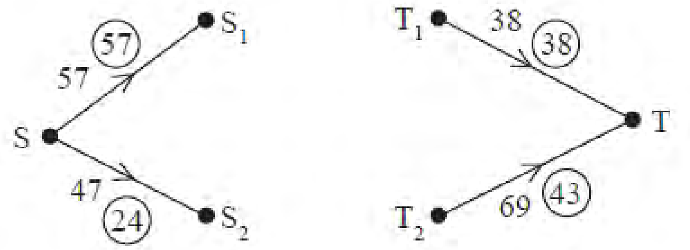
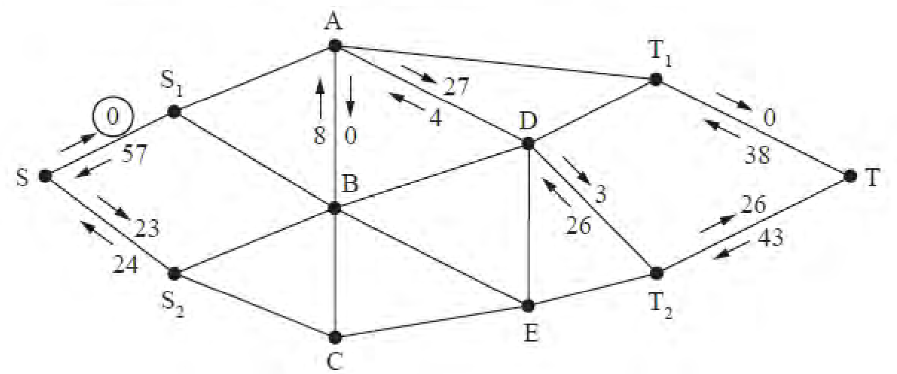
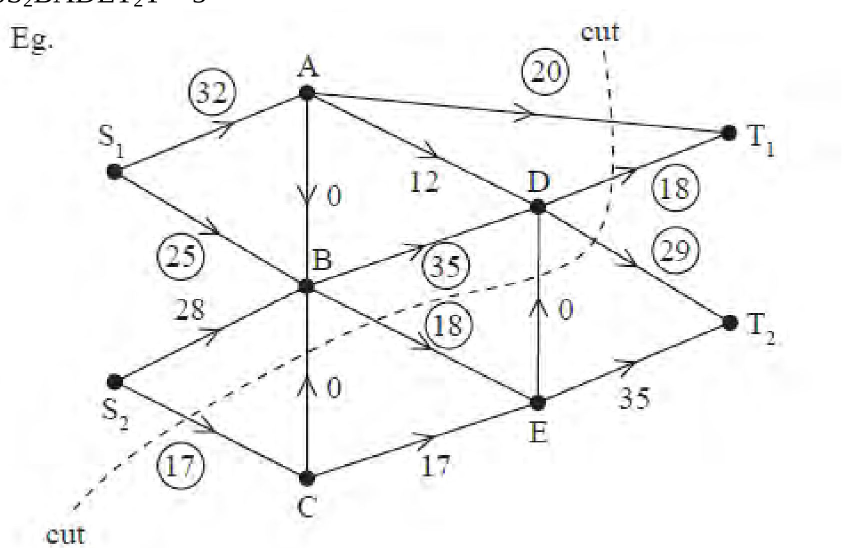
b.v.	x	y	z	r	s	t	value	Row ops
z	$\frac{5}{2}$	0	1	1	$-\frac{3}{2}$	0	5	R1stet
y	$-\frac{3}{4}$	1	0	$-\frac{1}{2}$	$\frac{5}{4}$	0	12.5	R2 – $\frac{1}{2}$R1
t	$\frac{5}{4}$	0	0	$\frac{3}{2}$	$-\frac{19}{4}$	1	12.5	R3 + $\frac{3}{2}$R1
P	32.5	0	0	15	–2.5	0	675	R4 + 15R1

B1
B1
M1 A1 **(9)**

(b)	$P + 32.5x + 15r - 2.5s = 675$	B1 **(1)**
(c)	$P = 675 - 32.5x - 15r + 2.5s$, so can increase profit by increasing s, hence not optimal.	B2,1,0 **(2)**
		12 marks

Notes for Question 4

a1M1: Correct pivot located, attempt to divide row. If choosing negative pivot no marks.
a1A1: Pivot row correct including change of b.v.
a1B1: Row operations CAO – allow if given in terms of old row 2.
a2M1: (ft) Correct row operations used at least once, column x, z, s or value correct.
a2A1: CAO on numbers (ignore row operations and b.v.).
a2B1: Correct pivot located and b.v. changed. If choosing negative pivot 2B0 3M0.
a3B1: Row operations CAO.
a3M1: (ft) Correct row operations used at least once, column x, r, s or value correct.
a3A1: CAO on numbers (ignore row operations and b.v.).
b1B1: CAO
c1B1ft: **Explanation**. Must have gained at least 2 M marks in (a) must refer to increasing x, r and s, (condone no ref to $y = z = t = 0$), must have correct signs in equation in (b). Do not accept 'negatives in profit row' o.e. alone.
c2DB1: CAO – dependent on correct equation in (b). Specifically identifies s as the next variable that could be increased.

Question Number	Scheme	Marks
5. (a)		M1 A1 A1 **(3)**
(b)		M1 A1 **(2)**
(c)	E.g. SS_2BET_2T – 13 SS_2BADT_2T – 3 SS_2BADET_2T – 5	 M1 A1 A1 A1 **(4)**
(d)		M1 A1 **(2)**
(e)	The cut through AT_1, DT_1, DT_2, DE, BE, CB and S_2C has value 102 Value of the flow is 102 so by max flow – min cut theorem flow is maximal	DB1 DB1 **(2)** **13 marks**

Notes for Question 5
a1M1: Four arcs added, SS_1, SS_2, T_1T, T_2T and 2 numbers on each. a1A1: CAO for arcs a2A1: CAO for flow values and capacities b1M1: Two numbers on each arc and at least three arcs or six numbers correct. b1A1: CAO do give bod since they might well cross these numbers out. c1M1: One valid flow augmenting route found and a value stated. c1A1: Flow increased by at least 3. c2A1: A second correct flow route of value at least 5 and value correct. c3A1: CSO Flow increased by 21 and no more. d1M1: Consistent flow pattern ≥ 84 (check each node). One number only per arc. No unnumbered arcs. d1A1: CAO, showing flow of 102, must follow from their routes. e1DB1: Must have attempted (d) - at least one number on all but one arc, and made an attempt at a cut, either drawn or stated. e2DB1: CSO - (d) fully correct (showing a correct flow of 102) and a correct cut. Must refer to max flow-min cut theorem – all four words.

Question Number	Scheme	Marks
6.	Let x_{ij} be 0 or 1 $\begin{cases} 1 \text{ if worker } (i) \text{ does task } (j) \\ 0 \text{ otherwise} \end{cases}$ where $i \in \{\text{A, B, C, D}\}$ and $j \in \{1, 2, 3, 4\}$	B1
	minimise $C = 29x_{A1} + 15x_{A2} + 32x_{A3} + 30x_{A4}$ $+ 34x_{B1} + 26x_{B2} + 40x_{B3} + 32x_{B4}$ $+ 28x_{C1} + 27x_{C2} + 35x_{C3} + '100'x_{C4}$ $+ '100'x_{D1} + 21x_{D2} + 33x_{D3} + 31x_{D4}$	M1 A1
	Subject to $x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1$ or $\sum x_{Aj} = 1$ $x_{B1} + x_{B2} + x_{B3} + x_{B4} = 1$ or $\sum x_{Bj} = 1$ $x_{C1} + x_{C2} + x_{C3} + x_{C4} = 1$ or $\sum x_{Cj} = 1$ $x_{D1} + x_{D2} + x_{D3} + x_{D4} = 1$ or $\sum x_{Dj} = 1$	M1 A1
	$x_{A1} + x_{B1} + x_{C1} + x_{D1} = 1$ or $\sum x_{i1} = 1$ $x_{A2} + x_{B2} + x_{C2} + x_{D2} = 1$ or $\sum x_{i2} = 1$ $x_{A3} + x_{B3} + x_{C3} + x_{D3} = 1$ or $\sum x_{i3} = 1$ $x_{A4} + x_{B4} + x_{C4} + x_{D4} = 1$ or $\sum x_{i4} = 1$	M1 A1 **7 marks**

Notes for Question 6

1B1: Defining variables fully both ‘bits’ values and subscripts. Penalise poor variable choice, (AP etc.) here.
1M1: Attempt at a 16 term expression, coefficients ‘correct’, 2 ‘large’ values included, condone 2 slips.
1A1: CAO + minimise. Penalise reversed subscripts once only per question.
2M1: Four equations, each in four variables, unit coefficients, all 16 variables included, = 1, accept $\leq 1, \geq 1$ here for this M only
2A1: Any 4 CAO.
3M1: All 8 equations, each in four variables, unit coefficients, all 16 variables included = 1.
3A1: CAO.

Question Number	Scheme	Marks
7. (a)	E.g.	

Stage	State	Action	Dest	Value	Marks
Bicycle	4	4	0	350	1M1 1A1 **(2)**
	3	3	0	260	
	2	2	0	170	
	1	1	0	80	
	(0	0	0	0)	
Dolls house	4	0	4	0 + 350 = 350	
		1	3	95 + 260 = 355*	
		2	2	165 + 170 = 335	2M1 2A1 3A1 **(3)**
		3	1	245 + 80 = 325	States 4 + 3
		4	0	335 + 0 = 335	
	3	0	3	0 + 260 = 260	
		1	2	95 + 170 = 265*	
		2	1	165 + 80 = 245	
		3	0	245 + 0 = 245	
	2	0	2	0 + 170 = 170	3M1 4A1 5A1 **(3)**
		1	1	95 + 80 = 175*	
		2	0	165 + 0 = 165	States 2 + 1
	1	0	1	0 + 80 = 80	
		1	0	95 + 0 = 95*	
	(0	0	0	0 + 0 = 0)	
Train set	4	0	4	0 + 355 = 355	
		1	3	100 + 265 = 365*	4M1 6A1ft 7A1 **(3)**
		2	2	180 + 175 = 355	
		3	1	260 + 95 = 355	
		4	0	340 + 0 = 340	

Toy	Bicycle	Dolls House	Train Set
Number of workers	2	1	1

1B1 **(1)**

(b) Total number of toys is 365. 1B1 **(1)**

13 marks

Notes for Question 7
• **ALL M marks - Must bring earlier optimal results into calculations. Ignore extra rows. Must have right 'ingredients' (– number of workers) at least once per stage.** • **Penalise inconsistency/errors with the state/destination columns with the first two A marks earned only.** • **Penalise empty/errors in stage column with first A mark earned only.** a1M1: First stage (Bicycle) completed – bod something in each cell. Must have columns for stage, state, value and one of either action or destination. a1A1: CAO condone missing * here. Condone missing zero row. a2M1: Second stage (Dolls house) completed for at least states 4 and 3. Bod something in each cell. a2A1: Any one of these states correct. No missing rows. (Penalise * errors only once in the question). a3A1: CAO both states 4 and 3 correct. No missing rows. (Penalise * errors only once in the question). a3M1: Second stage (Dolls house) fully completed, condone missing zero row. Bod something in each cell. a4A1: States 2 and 1 correct. No missing rows. (Penalise * errors only once in the question). a5A1: CAO for stage 2. No missing rows. (Penalise * errors only once in the question). a4M1: Third stage (Train set) completed. Bod something in each cell. a6A1ft: Any three rows of third stage correct. Ft on * values only. No missing rows. (Penalise * errors only once in the question). a7A1: CAO for the third stage. No missing rows. (Penalise * errors only once in the question). a1B1: CAO. Must have attempted algorithm, getting all previous M marks. b1B1: CAO. Must have attempted algorithm, getting all M marks in (a).

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Decision Mathematics 2 (6690/01)

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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Summer 2014
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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.	Since maximising, subtract all elements from some n ≥ 30 and insert large numbers in cells A4 and B2 e.g. $$\begin{bmatrix} 21 & 24 & 17 & 100 \\ 16 & 100 & 10 & 17 \\ 22 & 23 & 15 & 22 \\ 16 & 16 & 14 & 16 \end{bmatrix}$$	M1 M1
	Reduce rows $\begin{bmatrix} 4 & 7 & 0 & 83 \\ 6 & 90 & 0 & 7 \\ 7 & 8 & 0 & 7 \\ 2 & 2 & 0 & 2 \end{bmatrix}$ then columns $\begin{bmatrix} 2 & 5 & 0 & 81 \\ 4 & 88 & 0 & 5 \\ 5 & 6 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	M1 A1
	$$\begin{bmatrix} 0 & 3 & 0 & 79 \\ 2 & 86 & 0 & 3 \\ 3 & 4 & 0 & 3 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$	M1 A1ft
	either $\begin{bmatrix} 0* & 0 & 0 & 76 \\ 2 & 83 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 3 & 0* & 5 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0* & 3 & 2 & 79 \\ 0 & 84 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 0* & 4 & 0 \end{bmatrix}$ then $\begin{bmatrix} 0* & 2 & 2 & 78 \\ 0 & 83 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0* & 5 & 0 \end{bmatrix}$	M1 A1ft A1
	Two optimal allocations: A: 1, 1 B: 3, 4 C: 4, 3 D: 2, 2	A1
		10 marks

Notes for Question 1

1M1: Subtracting from some $n \geq 30$, condone up to 2 errors.
2M1: Dealing with the A4 and B2 entries.
3M1: Reducing rows **and then** columns.
1A1: CAO
4M1: Double covered + e; one uncovered – e; and one single covered unchanged. 2 lines needed to 3 lines needed.
2A1ft: follow through on their previous table - no errors
5M1: One double covered + e; one uncovered – e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to optimal table).
3A1ft: Follow through on their previous table - no errors.
4A1: CSO on final table.
5A1: CAO – either one – this mark is dependent on all M marks being awarded.

Special Cases: Minimising (can score a max. of 5)

1M0 2M1 3M1 1A1 4M0 2A0 5M1 3A1ft 4A0 5A0

E.g.

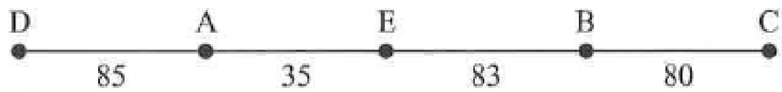
19	16	23	30		3	0	7	14		3	0	5	14
24	30	30	23	rows	1	7	7	0	columns	1	7	5	0
18	17	25	18		1	0	8	1		1	0	6	1
24	24	26	24		0	0	2	0		0	0	0	0

Then either

2	0*	4	14	or	2	0*	4	13
0	7	4	0*		1	8	5	0*
0*	0	5	1		0*	0	5	0
0	1	0*	1		0	1	0*	0

Not dealing with the – (can score a max. of 6)

1M1 2M0 3M1 1A0 4M1 2A1ft 5M1 3A1ft 4A0 5A0

Question Number	Scheme	Marks
2. (a)	A E F B C D A and A E F D B C A 35+75+88+80+108+85 = 471 35+75+88+100+80+130 = 508	M1 A1 A1 A1 **(4)**
(b)		
	RMST weight = 85 + 35 + 83 + 80 = 283 (seconds) Lower bound = 283 + 75 + 88 = 446 (seconds)	M1 A1 A1 **(3)**
(c)	446 ≤ time ≤ 471 [accept 446 < time ≤471]	B3,2,1,0 **(3)**
		10 marks

Notes for Question 2

a1M1: Nearest neighbour either A – E – F – B – C – D – or A – E – F – D – B – C – , condone lack of return to start. Accept 145623 or 156423 across top of table (numbers must be from NN **not** Prim).
a1A1: One route correctly stated, must return to A, accept link back to A.
a2A1: One route length correctly stated. Do not ISW if candidates then go on to double the route length in (a).
a3A1: Second route and its length correctly stated. Do not ISW if candidates then go on to double the route length in (a).
b1M1: Finding RST (maybe implicit) and using the correct two least lengths. Their RST must have only four arcs none of which are incident to F.
b1A1: RMST correct **or** list of arcs **or** 283 **or** 85 + 35 + 83 + 80 seen.
b2A1: CAO 446
c1B1ft: their 471 (must be a cycle) as an upper bound – allow recovery in this part.
c2B1ft: any indication of interval from their 446 (must come from six arcs) to their 471.
c3B1: 446 ≤ time ≤ 471 **or** 446 < time ≤471

Question Number	Scheme	Marks
3. (a)	see tableaux below	

b.v	x	y	z	r	s	t	value	θ values
r	5	3	$-\frac{1}{2}$	1	0	0	2500	833.3
s	3	[2]	1	0	1	0	1650	825
t	$\frac{1}{2}$	-1	2	0	0	1	800	n/a
P	-40	-50	-35	0	0	0	0	

b.v.	x	y	z	r	s	t	value	Row ops
r	$\frac{1}{2}$	0	-2	1	$-\frac{3}{2}$	0	25	R1-3R2
y	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	825	R2 ÷ 2
t	2	0	[$\frac{5}{2}$]	0	$\frac{1}{2}$	1	1625	R3 + R2
P	35	0	-10	0	25	0	41250	R4 + 50R2

M1 A1
B1
M1 A1
(5)

b.v.	x	y	z	r	S	t	value	Row ops
r	$\frac{21}{10}$	0	0	1	$-\frac{11}{10}$	$\frac{4}{5}$	1325	R1 + 2R3
y	$\frac{11}{10}$	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	500	R2 - $\frac{1}{2}$R3
z	$\frac{4}{5}$	0	1	0	$\frac{1}{5}$	$\frac{2}{5}$	650	R3 ÷ $\frac{5}{2}$
P	43	0	0	0	27	4	47750	R4 + 10R3

M1 A1ft
B1
M1 A1
(5)

(b) $P = 47750 \quad x = 0 \quad y = 500 \quad z = 650 \quad r = 1325 \quad s = t = 0$ B1ft B1 **(2)**

12 marks

Notes for Question 3

a1M1: Correct pivot located, attempt to divide row. If choosing negative pivot no marks.
a1A1: CAO pivot row correct **including change of b.v.**
a1B1: All row operations CAO – allow if given in terms of old row 2.
a2M1: (ft) The correct row operations used correctly at least once from their pivot, column *x*, *z*, *s* or value 'correct'.
a2A1: CAO on numbers (ignore row operations and b.v.)

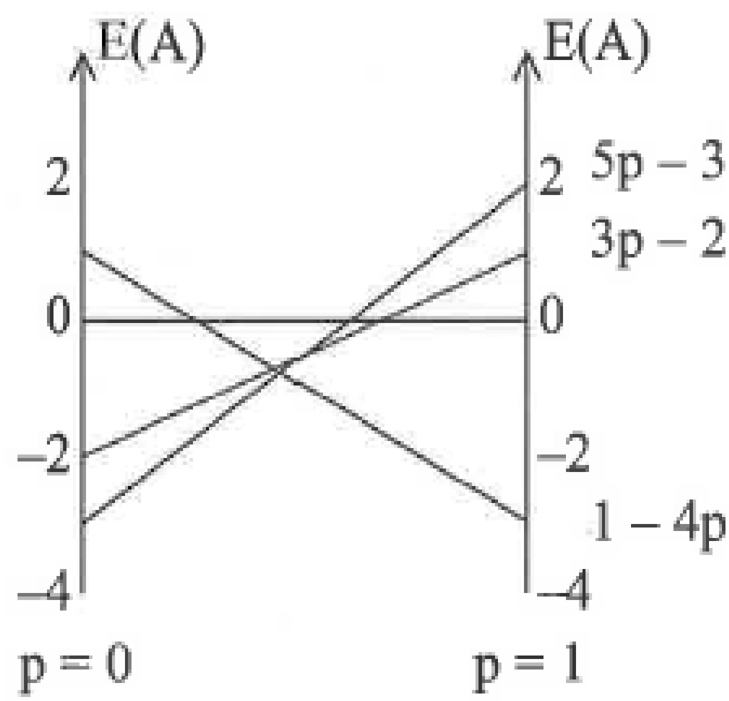
a3M1: Their correct pivot located, attempt to divide row. If choosing negative pivot M0M0.
a3A1ft: Pivot row correct on follow through **including change of b.v**.
a2B1: All row operations CAO – allow if given in terms of old row 3.
a4M1: (ft) The correct row operations used correctly at least once from their pivot, column *x*, *s*, *t* or value 'correct'.
a4A1: CAO on numbers (ignore row operations and b.v.)

b1B1ft: Their correct values stated for at least *P, x, y, z* from their 'optimal' iteration. No negatives. Two M marks in (a) must have been awarded.
Allow implicit stating of *P* e.g. $P+43x+27s+4t = 47750$ with $x, s, t = 0$.
b2B1: CAO For all 7 variables correct and given explicitly.

Question Number	Scheme	Marks
4. (a)	Row mins {-3, -3} Column max {2, 2, 1, 1}	M1
	Row maximin (-3) $\neq$ column minmax (1) so not stable	A1
		(2)
(b)	Column 4 dominates column 2 so delete column 2 **or** if B plays 2 A's expected winnings are $-p + 2(1-p)$ (= $2-3p$)	B1

	B1	B3	B4
A1	2	1	-3
A2	-3	-2	1

	Let A play 1 with probability p and 2 with probability $1-p$	B1
	If B plays 1 A's expected winnings are $2p - 3(1-p) = 5p-3$	
	If B plays 3 A's expected winnings are $p - 2(1-p) = 3p-2$	M1 A1
	If B plays 4 A's expected winnings are $-3p + (1-p) = 1-4p$	



	(graph)	M1 A1
	$5p - 3 = 1 - 4p$	M1
	$p = \frac{4}{9}$	A1
	A should play row 1 with probability $\frac{4}{9}$ and row 2 with probability $\frac{5}{9}$	A1
		(9)
		11 marks

Notes for Question 4

a1M1: Finding row minimums and column maximums – condone one error.
a1A1: CAO states -3 $\neq$ 1 (or row (maximin) $\neq$ col (minimax)) and draws the conclusion.
b1B1: CAO Col 4 dominates Col 2 (maybe implied by later working) or correctly stating the expression for A's expected winnings if B plays 2 (*2 – 3p)*.
b2B1: Defines *p*. Allow those who only define that A plays 1 with prob. p – no incorrect statements be generous.
b1M1: Setting up three probability equations, implicit definition of p.
b1A1: CAO (condone incorrect simplification).
b2M1: Either attempt at three lines (correct slant direction and relative intersection with 'axes') or four lines if no earlier domination, accept $p > 1$ or $p < 0$ here. Must be functions of p.
b2A1: CAO $0 \leq p \leq 1$, scaling correct and clear (or 1 line = 1), condone lack of labels. Rulers used.
b3DM1: Finding their correct optimal point, must have three (or four) lines and set up an equation to find $0 \leq p \leq 1$. Dependent on previous M mark. Must have at least three intersection points. Solving all three simultaneous equations and stating incorrect p is M0.
b3A1: CAO (must have scored all marks except b2B1 (define p mark) in this part).
b4A1: CAO

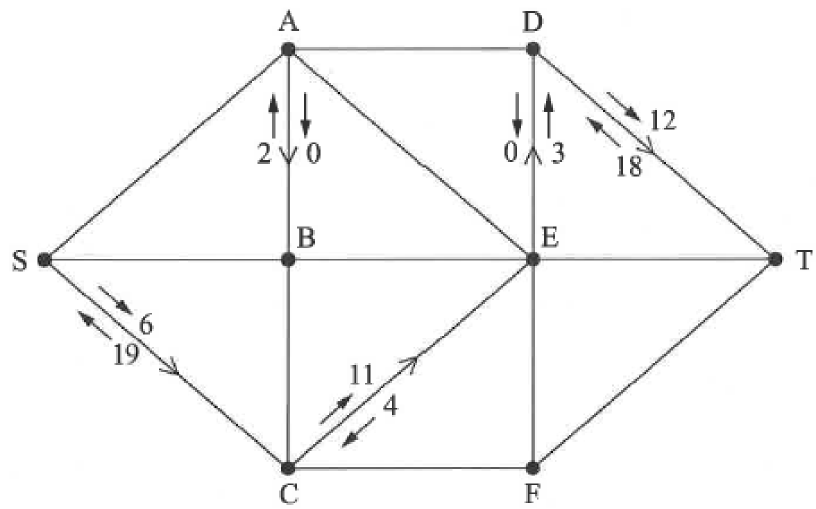
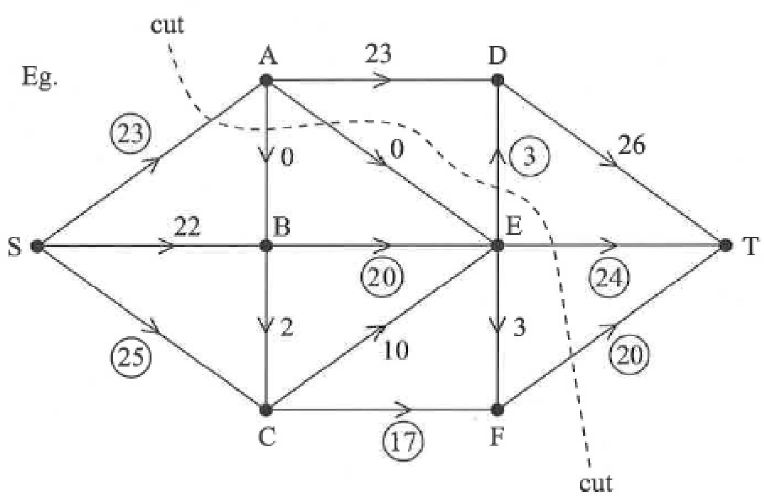
SC1: If column 4 is deleted in (b) candidates can earn a **maximum** of

B0 B1 M1 A0 M1 A0 M1 A0 A1 (max. of 5 out of 9 in part b)

The final A mark is for 'A should play row 1 with prob. 2/3 and row 2 with prob. 1/3.

SC2: If column 1 or 3 is deleted in (b), candidates can earn a **maximum** of

B0 B1 M1 A0 M1 A0 M0 A0 A0 (max. of 3 out of 9 in part b)

Question Number	Scheme	Marks
5.(a)	Initial flow = 62	B1 **(1)**
(b)		M1 A1 **(2)**
(c)	E.g. SCEDT - 3 · SCEADT – 3 · SBADT – 2 SCBADT 2 · SBCEDT 3 · SCEADT 3 SCEADT 2 · SCEDT 3 · SBCEADT 1 · SBADT 2 SBADT 2 · SBCEDT 1 · SCEDT 2 · SCEADT 3	M1 A1 A1 A1 **(4)**
(d)		M1 A1 **(2)**
(e)	The cut through SA, AB, AE, DE, ET and FT has value 70 Value of the flow is 70 so by max flow – min cut theorem flow is maximal	DB1 DB1 **(2)**
		11 marks

Notes for Question 5
a1B1: CAO b1M1: Two numbers on each arc **and** at least two arcs **or** four numbers correct (so correct numbers with the correct arrows). b1A1: CAO do give bod since they might well cross these number out. c1M1: One valid flow augmenting route found and a value stated. c1A1: Flow increased by at least 2. c2A1: A second correct flow route and value correct. c3A1: CSO Flow increased by 8 and no more. d1M1: Consistent flow pattern ≥ 64 (check each node). One number only per arc. No unnumbered arcs. d1A1: CAO, showing flow of 70, must follow from their routes. e1DB1: Must have attempted (d) - at least one number on all but one arc, and either drawn or stated a cut. Cut may be drawn on any diagram. e2DB1: CSO – (d) fully correct (showing a correct flow of 70) and a correct cut. Must refer to max flow-min cut theorem – all four words.

Question Number	Scheme	Marks
6.	Let x_{ij} be the number of washing machines transported from i to j where $i \in \{P,Q,R\}$ and $j \in \{A,B,C,D\}$	B1
	The objective is to minimise C = $11x_{PA} + 22x_{PB} + 13x_{PC} + 17x_{PD}$ $+ 21x_{QA} + 8x_{QB} + 19x_{QC} + 14x_{QD}$ $+15x_{RA} + 10x_{RB} + 9x_{RC} + 12x_{RD}$	B1 B1
	Subject to $x_{PA} + x_{PB} + x_{PC} + x_{PD} = 25$ or $\sum x_{Pj} = 25$	M1
	$x_{QA} + x_{QB} + x_{QC} + x_{QD} = 27$ or $\sum x_{Qj} = 27$	A1
	$x_{RA} + x_{RB} + x_{RC} + x_{RD} = 28$ or $\sum x_{Rj} = 28$ $x_{PA} + x_{QA} + x_{RA} = 18$ or $\sum x_{iA} = 18$ $x_{PB} + x_{QB} + x_{RB} = 16$ or $\sum x_{iB} = 16$ $x_{PC} + x_{QC} + x_{RC} = 20$ or $\sum x_{iC} = 20$	A1
	$x_{PD} + x_{QD} + x_{RD} = 26$ or $\sum x_{iD} = 26$ $x_{ij} \geq 0$	A1
		7 marks

Notes for Question 6

1B1: Variables defined correctly – withhold this mark if definition of x_{ij} or the elements of sets *i* and *j* are inconsistent with their later use in the objective function and constraints. Penalise poor variable choice, (AP etc.) here.
2B1: Minimise + an attempt at an objective with at least 5 correct terms.
3B1: Objective function correct (minimised not required for this mark).
1M1: At least 3 ‘correct’ constraints listed with **unit** coefficients (accept = or any inequality for the M mark) – rhs values must be correct.
1A1: At least 3 correct constraints (accept consistent use of = or ≤ on at least 3).
2A1: At least 6 correct constraints (accept consistent use of = or ≤ on at least 6).
3A1: All 8 constraints correct (first seven constraints consistently either = or ≤ but final constraint must be ≥ 0).

Question Number	Scheme	Marks
7.	E.g.	

Stage	State	Action	Dest	Value
July	2	1	0	1000 + 2000 = 3000*
(3)	1	2	0	500 + 2000 = 2500*
	0	3	0	2000 = 2000*
June	2	2	0	1000 + 2000 + 2000 = 5000*
(4)		3	1	1000 + 2000 + 2500 = 5500
		4	2	1000 + 2000 + 1000 + 3000 = 7000
	1	3	0	500 + 2000 + 2000 = 4500*
		4	1	500 + 2000 +1000 + 2500 = 6000
	0	4	0	2000 + 1000 + 2000 = 5000*
May	2	0	0	1000 + 5000 = 6000*
(2)		1	1	1000 + 2000 + 4500 = 7500
		2	2	1000 + 2000 + 5000 = 8000
	1	1	0	500 + 2000 + 5000 = 7500
		2	1	500 + 2000 + 4500 = 7000*
		3	2	500 + 2000 + 5000 = 7500
	0	2	0	2000 + 5000 = 7000
		3	1	2000 + 4500 = 6500*
		4	2	2000 + 1000 + 5000 = 8000
April	2	2	0	1000 + 2000 + 6500 = 9500*
(4)		3	1	1000 + 2000 + 7000 =10000
		4	2	1000 + 2000 + 1000 + 6000 =10000
	1	3	0	500 + 2000 + 6500 = 9000*
		4	1	500 + 2000 + 1000 + 7000 =10500
	0	4	0	2000 + 1000 + 6500 = 9500*
March	0	3	0	2000 + 9500 =11500*
(3)		4	1	2000 + 1000 + 9000 =12000

Marks: 1M1 1A1 **(2)**; 2M1 2A1ft 3A1 **(3)**; 3M1 4A1ft 5A1 **(3)**; 4M1 6A1 **(2)**; 5M1 7A1 **(2)**

Month	March	April	May	June	July
Number made	3	4	3	3	3

1B1

Total cost: £11500 — 2B1 **(2)**

14 marks

Notes for Question 7

ALL M marks - Must bring earlier optimal results into calculations. Ignore extra rows. Must have right 'ingredients' (– storage costs, overheads, additional space costs) at least once per stage.

Penalise lack of * only once per question.

1M1: First stage completed. 3 rows, something in each cell.
1A1: CAO condone missing * here. No extra rows.
2M1: Second stage completed with 3 states and at least 6 rows. Bod if something in each cell.
2A1ft: Any 2 states correct. Ft for their * values or the correct * values.
3A1: CAO All 3 states correct. No missing/extra rows.
3M1: 3rd stage completed with 3 states and at least 9 rows. Bod if something in each cell.
4A1ft: Any state correct. Ft on their * values or the correct * values.
5A1: CAO All 3 states correct. No missing/extra rows.
4M1: 4th stage completed with 3 states and at least 6 rows. Bod if something in each cell.
6A1: CAO All 3 states correct. No missing/extra rows.
5M1: 5th stage completed with at least 2 rows. Bod if something in each cell.
7A1: CAO Final, state correct. No missing/extra rows.
1B1: CAO Must have earned all previous M marks.
2B1: CAO Must have earned all previous M marks.

Alt correct solution – adding the storage costs at start of each month

Stage	State	Action	Dest	Value	
July	2	1	0	2000 = 2000*	1M1
(3)	1	2	0	2000 = 2000*	1A1
	0	3	0	2000 = 2000*	
June	2	2	0	2000 + 2000 = 4000*	2M1
(4)		3	1	500 + 2000 + 2000 = 4500	
		4	2	1000 + 2000 + 1000 + 2000 = 6000	
	1	3	0	2000 + 2000 = 4000*	2A1ft
		4	1	500 + 2000 + 1000 + 2000 = 5500	
	0	4	0	2000 + 1000 + 2000 = 5000*	3A1
May	2	0	0	5000 = 5000*	3M1
(2)		1	1	500 + 2000 + 4000 = 6500	
		2	2	1000 + 2000 + 4000 = 7000	
	1	1	0	2000 + 5000 = 7000	
		2	1	500 + 2000 + 4000 = 6500*	4A1ft
		3	2	1000 + 2000 + 4000 = 7000	
	0	2	0	2000 + 5000 = 7000	
		3	1	500 + 2000 + 4000 = 6500*	5A1
		4	2	1000 + 2000 + 1000 + 4000 = 8000	
April	2	2	0	2000 + 6500 = 8500*	4M1
(4)		3	1	500 + 2000 + 6500 = 9000	
		4	2	1000 + 2000 + 1000 + 5000 = 9000	
	1	3	0	2000 + 6500 = 8500*	6A1
		4	1	500 + 2000 + 1000 + 6500 = 10000	
	0	4	0	2000 + 1000 + 6500 = 9500*	
March	0	3	0	2000 + 9500 = 11500*	5M1
(3)		4	1	500 + 2000 + 1000 + 8500 = 12000	7A1

Month	March	April	May	June	July	
Number made	3	4	3	3	3	1B1

Total cost: £11500 2B1

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Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Decision Mathematics 2 (6690/01)

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Publications Code UA041216

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- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.(a)	see table below	M1 A1 M1 A1ft A1 **(5)**
(b)	$P+\frac{7}{2}y+z+\frac{3}{4}s=15$ $r=5,\ \ s=0,\ \ t=3$	B1ft B1 **(2)** **7 marks**

b.v.	x	y	z	r	s	t	Value	Row ops
r	0	-5	5	1	$-\frac{1}{2}$	0	5	R_1-2R_2
x	1	$\frac{1}{2}$	-2	0	$\frac{1}{4}$	0	5	$R_2\div 4$
t	0	$-\frac{3}{2}$	6	0	$-\frac{1}{4}$	1	3	R_3-R_2
P	0	$\frac{7}{2}$	1	0	$\frac{3}{4}$	0	15	R_4+3R_2

Notes for Question 1

a1M1: Correct pivot located (4 in column *x*), attempt to divide row
a1A1: Pivot row correct **including change of b.v.**
a2M1: **All** values in one of the non-pivot rows correct **or** one of the non zero and one columns (*y*, *z*, *s* or value) correct following through their choice of pivot from column *x*
a2A1ft: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (*y*, *z*, *s* or value) correct following through their choice of pivot from column *x*
a3A1: CAO – no follow through – all values and row operations correctly stated – allow if row operations given in terms of old row 2 – **ignore b.v. column for this mark**

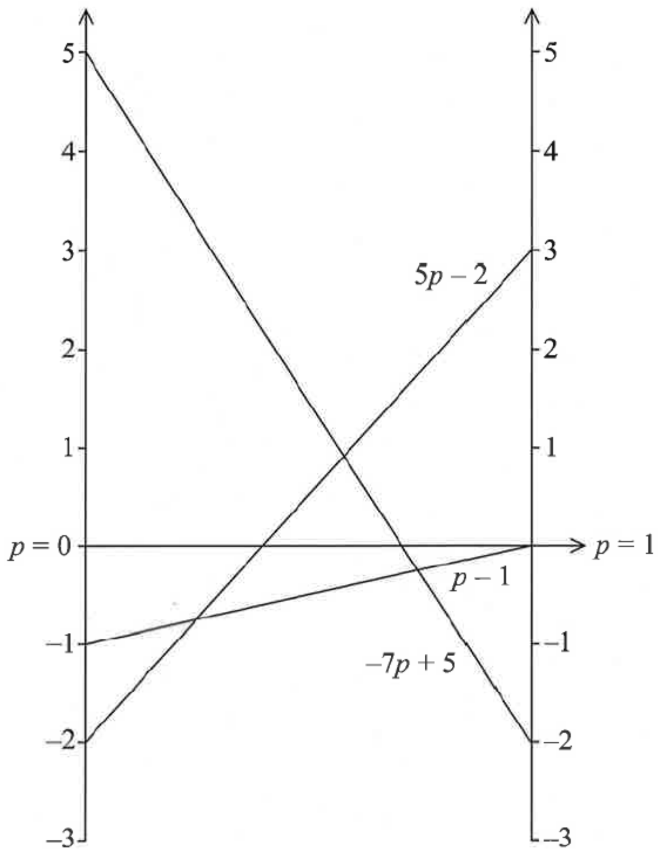
b1B1ft: Follow their profit equation from (a) dependent on scoring **both** M marks in (a)
b2B1: CAO (no follow through) for slack variables (r = 5, s = 0, t = 3)

Pivoting on the 1 in the *x*-column

b.v.	*x*	*y*	*z*	*r*	*S*	*t*	V
r	0	-2	-7	1	0	-2	-1
s	0	6	-24	0	1	-4	-12
x	1	-1	4	0	0	1	8
P	0	-1	19	0	0	3	24

Pivoting on the 2 in the *x*-column

b.v.	*x*	*y*	*z*	*r*	*s*	*t*	V
x	1	-2	0.5	0.5	0	0	7.5
s	0	10	-10	-2	1	0	-10
t	0	1	3.5	-0.5	0	1	0.5
P	0	-4	8.5	1.5	0	0	22.5

Question Number	Scheme	Marks
2.(a)	The gains (or losses) made by one player are exactly balanced by the losses (or gains) made by the other player.	B1 **(1)**
(b)	5	B1 **(1)**
(c)	Row minimum {-3,0,-5} Row maximin = 0 Column maximum {2,4,2} Column minimax = 2 $0 \neq 2$ so no stable solution	M1 A1 A1 **(3)**
(d)	Column 1 dominates column 2 so remove column 2 $\begin{pmatrix} 3 & 0 & -2 \\ -2 & -1 & 5 \end{pmatrix}$	B1 B1ft B1 **(3)**
(e)	(Let p = probability that Greg plays new row 1) If R plays 1: G's expected winnings = $3p - 2(1-p)$ $(= 5p - 2)$ If R plays 2: G's expected winnings = $0p - 1(1-p)$ $(= p - 1)$ If R plays 3: G's expected winnings = $-2p + 5(1-p)$ $(= -7p + 5)$  $p - 1 = -7p + 5$ $8p = 6$ $p = \frac{3}{4}$ G should play 1 with probability $\frac{3}{4}$, 2 never and play 3 with probability $\frac{1}{4}$ The value of the game to G is $-\frac{1}{4}$	M1 A1 B2, 1ft, 0 DM1 A1 A1ft A1 **(8)** **16 marks**

Question Number	Scheme	Marks
Notes for Question 2		

a1B1: CAO (indication that **either** the losses of one (player) are balanced by the gains of the other (player) **or** that the total points scored by both (players) is zero)

b1B1: CAO (5)

c1M1: Clear attempt to find the Row maximin and Column minimax (either the Row minimums or Column maximums correct **or** at least four (of the six) values stated correctly)
c1A1: Correct Row maximin **and** Column minimax (dependent on all row mins **and** column maxs correct)
c2A1: CAO (so both previous marks must have been awarded) states $0 \neq 2$ (or row (maximin) $\neq$ col (minimax) as long as 0 is clearly identified as the row maximin and 2 as the column minimax) **and** draws the correct conclusion

d1B1: CAO (accept reduced matrix or 'column 1 dominates column 2' or column crossed out). Allow recovery later (**seeing the correct 2 × 3 matrix implies all three marks in this part**)
d2B1ft: Either 3 × 2 matrix with correct values **for G** (so all signs changed correctly) or 2 × 3 matrix with correct values **for G** (condone incorrect signs). If incorrect column deleted (so B0 for first mark in this part) then allow this mark on the ft for their 3 × 2 matrix transposed 'correctly' **for G** (both values **and** signs 'correct')
d3B1: CAO

e1M1: Setting up all three probability expressions (allow p – 1), implicit definition of 'p'
e1A1: CAO (condone incorrect simplification)
e1B1ft: Attempt at three lines (correct slant direction and relative intersection with 'axes'), accept $p > 1$ or $p < 0$ here but must go from 'axis' to 'axis' (give bod if close). Must be functions of p
e2B1: CAO $0 \leq p \leq 1$, scaling correct and clear (expect to see 1 line = 1, although other scalings are acceptable eg 1 line = 2), condone lack of labels. Rulers used
e2DM1: Finding their correct optimal point, must have three lines and set up an equation to find $0 \leq p \leq 1$. Dependent on first B mark in this part. Must have three intersection points. Solving all three simultaneous equations and stating incorrect p is M0
e2A1: CSO (must have scored all previous marks in (e))
e3A1ft: All three options listed must ft from their p ($0 \leq p \leq 1$), check page 1 for G should never play 2. Dependent on both previous M marks in this part
e4A1: CAO $\left(-\frac{1}{4}\right)$

SC1: If column 1 is deleted in (d) candidates can earn a maximum in (e) of

M1 A0 B1 B0 M1 A0 A1 A1 (max. of 5) – the penultimate A mark is for G should play 1 never, play 2 and 3 with probability $\frac{1}{2}$, final A mark is for the value of the game being $-\frac{3}{2}$

SC2: If column 3 is deleted in (d) candidates can earn a maximum in (e) of

M1 A0 B1 B0 M0 A0 A0 A0 (max. of 2)

Question Number	Scheme	Marks
3.(a)	Prim: AF, EF, BE, BC, CD, DG	M1 A1 **(2)**
(b)	$2 \times 136 = 272$ (km)	B1 **(1)**
(c)	A F E B C D G A 21 20 19 27 24 25 30 = 166 (km)	B1 B1 **(2)**
(d)	Starting at F route length is 153 + x With $x > 21$, 153 + x is greater than 166 so the better upper bound is the one starting at A	B1 DB1 **(2)**
(e)	Length of RMST = 115 115 + 21 + x = 159 $\therefore x = 23$ (km)	B1 M1 A1 **(3)**
(f)	159 ≤ optimal ≤ 166 [accept 159 < optimal ≤ 166]	B2,1,0 **(2)** **12 marks**

Notes for Question 3

a1M1: Must be using Prim's algorithm not NNA – if any arc creates a cycle then M0. First four arcs (or all 7 nodes / or numbers across the top of the matrix) selected correctly. Award M1 only for a correct tree with no working. Award M1 only for the first four arcs (oe) selected correctly if starting at a different node than A
a1A1: CAO (order of arc selection clear)

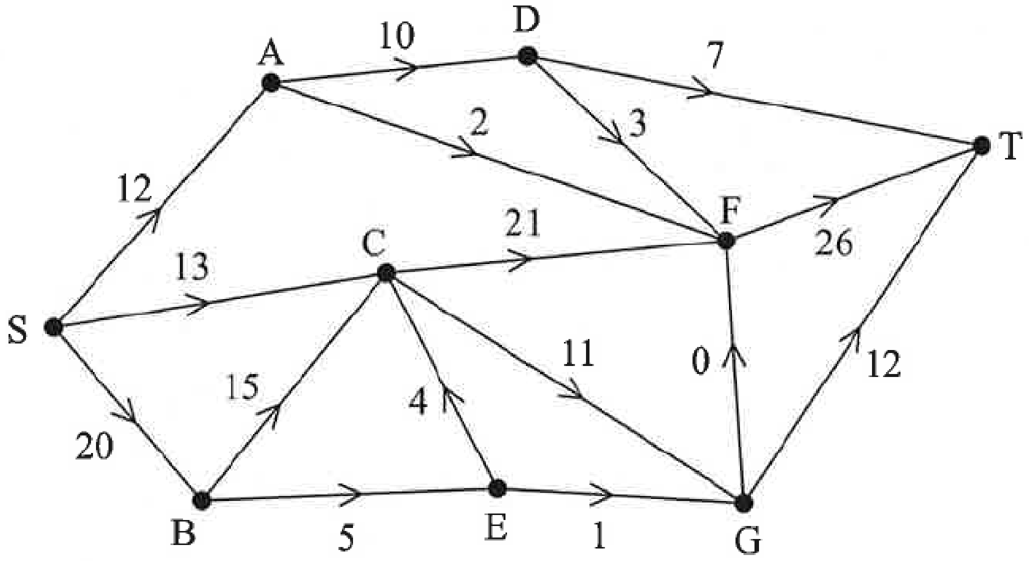
b1B1: CAO (272)

c1B1: CAO – must be either in terms of nodes or arcs (not weights)
c2B1: CAO (166)

d1B1: Either 153 + x **or** states a value in the interval 174 < value < 180 **or** considers one of the intervals 174 < value < 180 **or** 175 ≤ value ≤ 179
d2DB1: Correct argument that A gives the better upper bound. Must be considering either $x > 21$ or $x \geq 22$ with 153 (so expect to see as a minimum the mention of > 174 or ≥ 175) – must be clear that the upper bound starting at A is the better upper bound. This mark is dependent on the previous B mark in (d)

e1B1: CAO (length of RMST) – the length (115 or 19 + 20 + 27 + 24 + 25) must be either explicitly stated **or** seen in their working (not just implied by their working)
e1M1: Adding the **correct** two least values (21 and x) to **their** RMST length (their RMST may be incorrect but must contain only 5 arcs) and equating to 159. Accept, for example, 136 + x = 159 or 136 + 23 = 159 or 115 + 21 + 23 = 159 or equivalent calculations using the length of their RMST
e1A1: CAO (must be clear that (x =) 23 not just embedded in a calculation)

f1B1: Any indication of an interval containing 159 (as a lower bound) and **their** stated better upper bound from (d)
f2B1: CAO either 159 ≤ optimal (oe) ≤ 166 or 159 < optimal (oe) ≤ 166

Question Number	Scheme	Marks
4.(a)	$C_1 = 45,\ C_2 = 73$	B1 B1 **(2)**
(b)	45	B1ft **(1)**
(c)	20	B1 **(1)**
(d)	The maximum capacity of the arcs flowing into G is 21 and so both GF and GT cannot be full to capacity as the capacity of the arcs flowing out of G is 26	B1 **(1)**
(e)		M1 A1 **(2)** **7 marks**

Notes for Question 4

a1B1: CAO for C_1(45)
a2B1: CAO for C_2 (73)

b1B1ft: 45 or the value of their smallest cut from (a)

c1B1: CAO (20)

d1B1: CAO – argument must be numerical in nature (as a minimum accept 26 > 21 (oe))

e1M1: Consistent flow pattern – check each node, must have exactly 1 number per arc (arc EC must be 4, AD – 10 and DF – 3 but all other arcs may have over-capacatiated values)
e1A1: CAO

Question Number	Scheme	Marks

5.(a)

	P	Q	R
A	$74-\theta$	θ	
B			
C	$13+\theta$	$50-\theta$	
D			

giving

	P	Q	R
A	24	50	
B	58		
C	63		
D		7	78

M1 A1 **(2)**

(b)

Shadow costs		20	5	−1
		P	Q	R
0	A	X	X	14
−13	B	X	23	22
−11	C	X	20	33
11	D	−9	X	X

M1 A1

	P	Q	R
A	$24-\theta$	$50+\theta$	
B			
C			
D	θ	$7-\theta$	

giving

	P	Q	R
A	17	57	
B	58		
C	63		
D	7		78

Entering cell DP, exiting cell DQ

M1 A1 **(4)**

(c)

Shadow costs		20	5	8
		P	Q	R
0	A	X	X	5
−13	B	X	23	13
−11	C	X	20	24
2	D	X	9	X

Optimal since no negative improvement indices

M1 A1 A1 **(3)**

(d) (£) 2532 — B1 **(1)**

(e)

Let x_{ij} be the number of units transported from i to j — B1

where $i \in \{A, B, C, D\}$, $j \in \{P, Q, R\}$ and $x_{ij} \geq 0$ — B1

Minimise (C =) $20x_{AP} + 5x_{AQ} + 13x_{AR} + 7x_{BP} + 15x_{BQ} + 8x_{BR} + 9x_{CP} + 14x_{CQ} + 21x_{CR} + 22x_{DP} + 16x_{DQ} + 10x_{DR}$ — M1 A1

Subject to $x_{AP} + x_{AQ} + x_{AR} \leq 74$ or $\sum x_{Aj} \leq 74$

$x_{BP} + x_{BQ} + x_{BR} \leq 58$ or $\sum x_{Bj} \leq 58$

$x_{CP} + x_{CQ} + x_{CR} \leq 63$ or $\sum x_{Cj} \leq 63$ — M1

$x_{DP} + x_{DQ} + x_{DR} \leq 85$ or $\sum x_{Dj} \leq 85$ — A1

$x_{AP} + x_{BP} + x_{CP} + x_{DP} \leq 145$ $\sum x_{iP} \leq 145$ — A1 **(7)**

$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} \leq 57$or $\sum x_{iQ} \leq 57$

$x_{AR} + x_{BR} + x_{CR} + x_{DR} \leq 78$ or $\sum x_{iR} \leq 78$

17 marks

Question Number	Scheme	Marks
	Notes for Question 5	

a1M1: A valid route, only one empty square, AQ used, θ's balance
a1A1: Correct route, up to an improved solution (six numbers no zeros)

b1M1: Finding 7 shadow costs and 6 Improvement indices
b1A1: Shadow costs [Alt: A(20), B(7), C(9), D(31), P (0), Q(-15), R(-21)] and improvement indices CAO
b2M1: A valid route, their most negative II chosen, only one empty square used, θ's balance
b2A1: CSO (for part (b)) (entering DP, and exiting DQ clearly stated)

c1M1: Finding 7 shadow costs **and** all 6 IIs **or** at least 1 negative II found
c1A1: CAO for the shadow costs [Alt: A(20), B(7), C(9), D(22), P(0), Q(-15), R(-12)] and 6 positive II
c2A1: CSO (for part (c)) + reason + optimal

d1B1: CAO (2532)

e1B1: x_{ij} (not just *x*) defined correctly (must include 'number of' (oe) and 'from *i* to *j*' (oe)). Withold this mark if $x_{ij}$is further defined as taking the values of either 0 or 1
e2B1: Defining the set of values for *i* and *j* **including** non-negativity constraint - withold this mark if definition is inconsistent with their later use in the objective function and constraints (eg A, B,… in the definition but 1, 2,… used in constraints and objective)
e1M1: Objective function (allow one error either in coefficient **or** variable) – minimise **not** required for this mark
e1A1: CAO – Correct objective function and minimise
e2M1: At least 3 constraints listed with unit coefficients (accept = or any inequality for the M mark) – rhs values must be correct
e2A1: At least 5 correct constraints (accept consistent use of = or ≤ on at least 5)
e3A1: All 7 constraint correct (accept consistent use of = or ≤ on all 7)

Note: if there are inconsistencies between the constraints and the objective function then mark to the benefit of the candidate. For example, a candidate who correctly defines $x_{ij}$and its set of values and writes down the constraints correctly (based on their definition of x_{ij}) but in the objective function omits the *x* (so uses, for example, AP, AQ, etc.) then this would scored B1B1M0A0M1A1A1

Question Number	Scheme	Marks
6.(a)	Maximin	B1 **(1)**
(b)	(see table below)	M1 A1 M1 A1 A1 M1 A1ft A1 M1 A1 **(10)**
(c)	Maximum weight = 8 (tonnes)	B1 **(1)**
(d)	Route: S – B – E – G – T	B1 **(1)**
(e)(i) **(ii)**	Increase HT (by 5) to 10 Maximum weight = 10 (tonnes) New route: S – C – D – H – T	B1 B1 B1 **(3)** **16 marks**

Stage	State	Action	Destination	Value
3	G	GT	T	8*
	H	HT	T	5*
	J	JT	T	6*
2	D	DH	H	min (10, 5) = 5*
	E	EG	G	min (9, 8) = 8*
		EH	H	min (8, 5) = 5
		EJ	J	min (7, 6) = 6
	F	FH	H	min (8, 5) = 5*
		FJ	J	min (5, 6) = 5*
1	A	AD	D	min (8, 5) = 5
		AE	E	min (6, 8) = 6*
	B	BE	E	min (17, 8) = 8*
		BF	F	min (9, 5) = 5
	C	CD	D	min (10, 5) = 5*
		CF	F	min (10, 5) = 5*
0	S	SA	A	min (11, 6) = 6
		SB	B	min (8, 8) = 8*
		SC	C	min (12, 5) = 5

Question Number	Scheme	Marks
Notes for Question 6		

a1B1: CAO

Throughout (b):
• Condone lack of destination column and/or reversed stage numbers throughout
• Only penalise incorrect result in value – ie ignore working values
• Penalise absence of state or action column with first two A marks earned only
• Penalise empty/errors in stage column with first A mark earned only

2nd, 3rd and 4th M marks - must bring earlier optimal results into calculations at least once

Penalise lack of * only once

b1M1: First stage completed. 3 rows, something in each cell
b1A1: CAO condone missing * here
b2M1: Second stage completed with 3 states and at least 6 rows. Bod if something in each cell
b2A1: Second stage any 2 states correct
b3A1: CAO all 3 states correct (no missing/extra rows)
b3M1: Third stage completed with 3 states and at least 6 rows. Bod if something in each cell
b4A1ft: Third stage any two states correct. Follow through their * values or the correct * values
b5A1: CAO all 3 states correct (no missing/extra rows)
b4M1: Fourth stage completed with 1 state and at least 3 rows. Bod if something in each cell
b6A1: CAO final state correct (no missing/extra rows)

c1B1: CAO weight (8) (dependent on scoring **all** M marks in (b))

d1B1: CAO route (S – B – E – G – T) (dependent on scoring **all** M marks in (b))

e1B1: Indication of either increasing HT by 5 or increasing HT to 10
e2B1: CAO (10)
e3B1: CAO (S – C – D – H – T)

Special Cases for (b), (c) and (d)

SC1 Minimax: M1 A1 M1 A0 A0 M1 A1 A0 M1 A0 B1 B1 (Max 8/12)
SC2 Maximum: M1 A1 M1 A0 A0 M1 A0 A0 M1 A0 B0 B1 (Max 6/12)
SC3 Minimum: As above (SC2)
SC4 Maximax: M1 A1 M1 A0 A0 M1 A0 A0 M1 A0 B0 B0 (Max 5/12)
SC5 Minimin: As above (SC4)
SC6 Working forwards: M1 A0 M1 A0 A0 M1 A0 A0 M1 A0 B0 B0 (Max 4/12)
SC7 Reversed states: M1 A0 M1 A0 A0 M1 A0 A0 M1 A1 B1 B1 (Max 7/12)

Question Number	Scheme	Marks

SC1 Minimax:

Stage	State	Action	Destination	Value
3	G	GT	T	8*
	H	HT	T	5*
	J	JT	T	6*
2	D	DH	H	max (10, 5) = 10*
	E	EG EH EJ	G H J	max (9, 8) = 9 max (8, 5) = 8 max (7, 6) = 7*
	F	FH FJ	H J	max (8, 5) = 8 max (5, 6) = 6*
1	A	AD AE	D E	max (8, 10) = 10 max (6, 7) = 7*
	B	BE BF	E F	max (17, 7) = 17 max (9, 6) = 9*
	C	CD CF	D F	max (10, 10)=10* max (10, 6) = 10*
0	S	SA SB SC	A B C	max (11, 7) = 11 max (8, 9) = 9* max (12, 10) = 12

Weight: 9 Route: S – B – F – J – T

SC2 Maximum:

Stage	State	Action	Destination	Value
3	G	GT	T	8*
	H	HT	T	5*
	J	JT	T	6*
2	D	DH	H	10 + 5 = 15*
	E	EG EH EJ	G H J	9 + 8 = 17* 8 + 5 = 13 7 + 6 = 13
	F	FH FJ	H J	8 + 5 = 13* 5 + 6 = 11
1	A	AD AE	D E	8 + 15 = 23* 6 + 17 = 23*
	B	BE BF	E F	17 + 17 = 34* 9 + 13 = 22
	C	CD CF	D F	10 + 15 = 25* 10 + 13 = 23
0	S	SA SB SC	A B C	11 + 23 = 34 8 + 34 = 42* 12 + 25 = 37

Route: S – B – E – G – T

Question Number	Scheme	Marks

SC3 Minimum:

Stage	State	Action	Destination	Value
3	G	GT	T	8*
	H	HT	T	5*
	J	JT	T	6*
2	D	DH	H	10 + 5 = 15*
	E	EG EH EJ	G H J	9 + 8 = 17 8 + 5 = 13* 7 + 6 = 13*
	F	FH FJ	H J	8 + 5 = 13 5 + 6 = 11*
1	A	AD AE	D E	8 + 15 = 23 6 + 13 = 19*
	B	BE BF	E F	17 + 13 = 30 9 + 11 = 20*
	C	CD CF	D F	10 + 15 = 25 10 + 11 = 21*
0	S	SA SB SC	A B C	11 + 19 = 30 8 + 20 = 28* 12 + 21 = 33

Route: S – B – F – J – T

SC4 Maximax:

Stage	State	Action	Destination	Value
3	G	GT	T	8*
	H	HT	T	5*
	J	JT	T	6*
2	D	DH	H	max (10, 5) = 10*
	E	EG EH EJ	G H J	max (9, 8) = 9* max (8, 5) = 8 max (7, 6) = 7
	F	FH FJ	H J	max (8, 5) = 8* max (5, 6) = 6
1	A	AD AE	D E	max (8, 10) = 10* max (6, 9) = 9
	B	BE BF	E F	max (17, 9) = 17* max (9, 8) = 9
	C	CD CF	D F	max (10, 10)=10* max (10, 8) = 10*
0	S	SA SB SC	A B C	max (11, 10) = 11 max (8, 17) = 17* max(12, 10) =12

Question Number	Scheme	Marks

SC5 Minimin:

Stage	State	Action	Destination	Value
3	G	GT	T	8*
	H	HT	T	5*
	J	JT	T	6*
2	D	DH	H	min (10, 5) = 5*
	E	EG	G	min (9, 8) = 8
		EH	H	min (8, 5) = 5*
		EJ	J	min (7, 6) = 6
	F	FH	H	min (8, 5) = 5*
		FJ	J	min (5, 6) = 5*
1	A	AD	D	min (8, 5) = 5*
		AE	E	min (6, 5) = 5*
	B	BE	E	min (17, 5) = 5*
		BF	F	min (9, 5) = 5*
	C	CD	D	min (10, 5) = 5*
		CF	F	min (10, 5) = 5*
0	S	SA	A	min (11, 5) = 5*
		SB	B	min (8, 5) = 5*
		SC	C	min (12, 5) = 5*

SC6 Working forwards S to T:

Stage	State	Action	Destination	Value
3	A	AS	S	11*
	B	BS	S	8*
	C	CS	S	12*
2	D	DA	A	min (8, 11) = 8
		DC	C	min (10, 12) =10*
	E	EA	A	min (6, 11) = 6
		EB	B	min (17, 8) = 8*
	F	FB	B	min (9, 8) = 8
		FC	C	min (10, 12) =10*
1	G	GE	E	min (9, 8) = 8*
	H	HD	D	min (10, 12) =10*
		HE	E	min (8, 8) = 8
		HF	F	min (8, 10) = 8
	J	JE	E	min (7, 8) = 7*
		JF	F	min (5, 10) = 5
0	T	TG	G	min (8, 8) = 8*
		TH	H	min (5, 10) = 5
		TJ	J	min (6, 7) = 6

Question Number	Scheme	Marks

SC7 Reversed States:

Stage	State	Action	Destination	Value
3	T	TG	G	8*
		TH	H	5*
		TJ	J	6*
2	G	GE	E	min (9, 8) = 8*
	H	HD	D	min (10, 5) = 5*
		HE	E	min (8, 5) = 5
		HF	F	min (8, 5) = 5*
	J	JE	E	min(7, 6) = 6
		JF	F	min (5, 6) = 5*
1	D	DA	A	min (8, 5) = 5
		DC	C	min (10, 5) = 5*
	E	EA	A	min (6, 8) = 6*
		EB	B	min (17, 8) = 8*
	F	FB	B	min (9, 5) = 5
		FC	C	min (10, 5) = 5*
0	A	AS	S	min (11, 6) = 6
	B	BS	S	min (8, 8) = 8*
	C	CS	S	min (12, 5) = 5

Weight: 8 Route: S – B – E – G – T

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