Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MD02

Unit Decision 2

Thursday 13 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

P61924/Jun13/MD02 6/6/6/

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Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** opposite shows an activity diagram for a project. The duration required for each activity is given in hours. The project is to be completed in the minimum time.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical path. *(1 mark)*

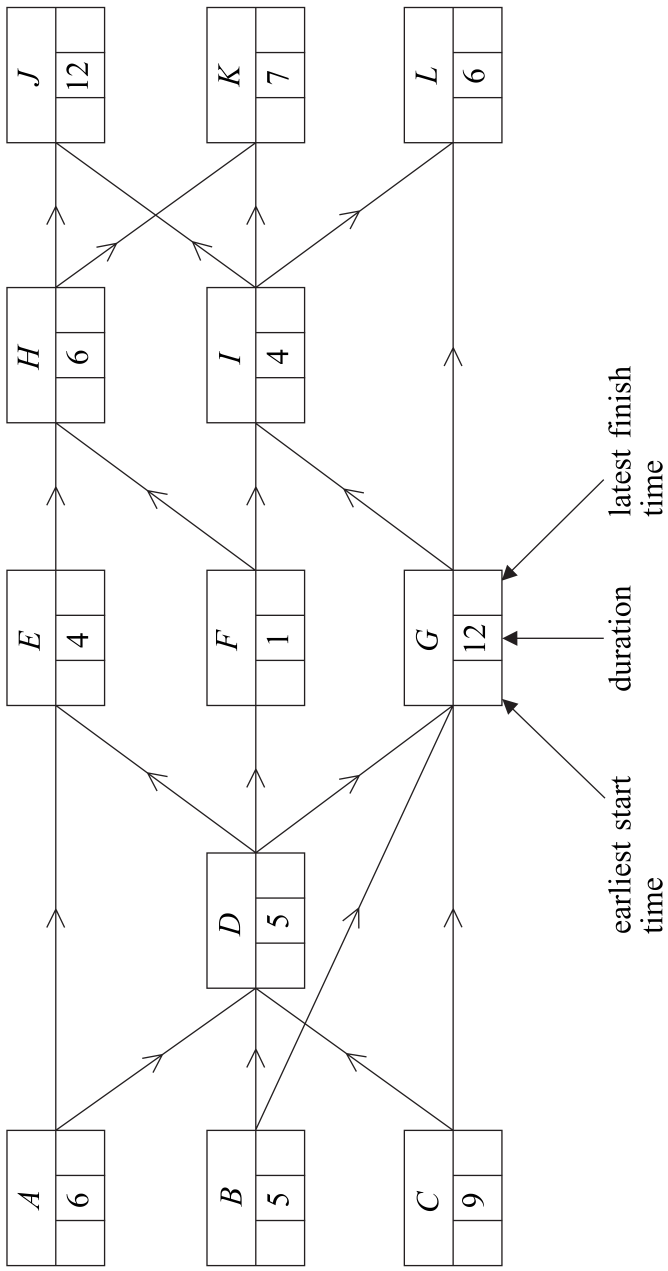
(c) Find the float time of activity E. *(1 mark)*

(d) Given that activities H and K will both overrun by 10 hours, find the new minimum completion time for the project. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 1

Figure 1


A
6
B
5
C
9
D
5
E
4
F
1
G
12
H
6
I
4
J
12
K
7
L
6
earliest start time
duration
latest finish time

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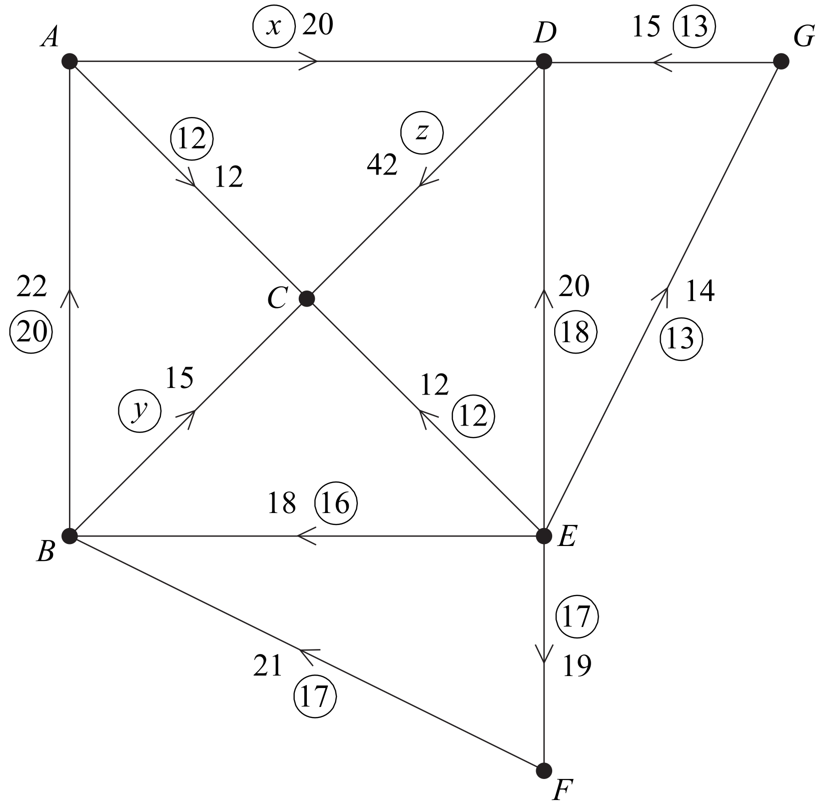
2 The network below represents a system of pipes. The number **not** circled on each edge represents the capacity of each pipe in litres per second. The number or letter in each circle represents an initial flow in litres per second.



(a) Write down the capacity of edge *EF*. *(1 mark)*

(b) State the source vertex. *(1 mark)*

(c) State the sink vertex. *(1 mark)*

(d) Find the values of x, y and z. *(3 marks)*

(e) Find the value of the initial flow. *(1 mark)*

(f) Find the value of a cut through the edges *EB*, *EC, ED*, *EF* and *EG*. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 The table shows the times taken, in minutes, by five people, A, B, C, D and E, to carry out the tasks V, W, X, Y and Z.

	A	***B***	***C***	***D***	***E***
Task *V*	100	110	112	102	95
Task *W*	125	130	110	120	115
Task *X*	105	110	101	108	120
Task *Y*	115	115	120	135	110
Task *Z*	100	98	99	100	102

Each of the five tasks is to be given to a different one of the five people so that the total time for the five tasks is minimised. The Hungarian algorithm is to be used.

(a) By reducing the **columns first**, and then the rows, show that the new table of values is

0	12	13	2	0
14	21	0	k	9
3	10	0	6	23
0	2	6	20	0
0	0	0	0	7

and state the value of the constant k. *(3 marks)*

(b) Show that the zeros in the table in part **(a)** can be covered with four lines. Use augmentation **twice** to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the possible ways of allocating the five tasks to the five people to achieve the minimum total time. *(3 marks)*

(d) Find the minimum total time. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

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QUESTION PART REFERENCE

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QUESTION PART REFERENCE

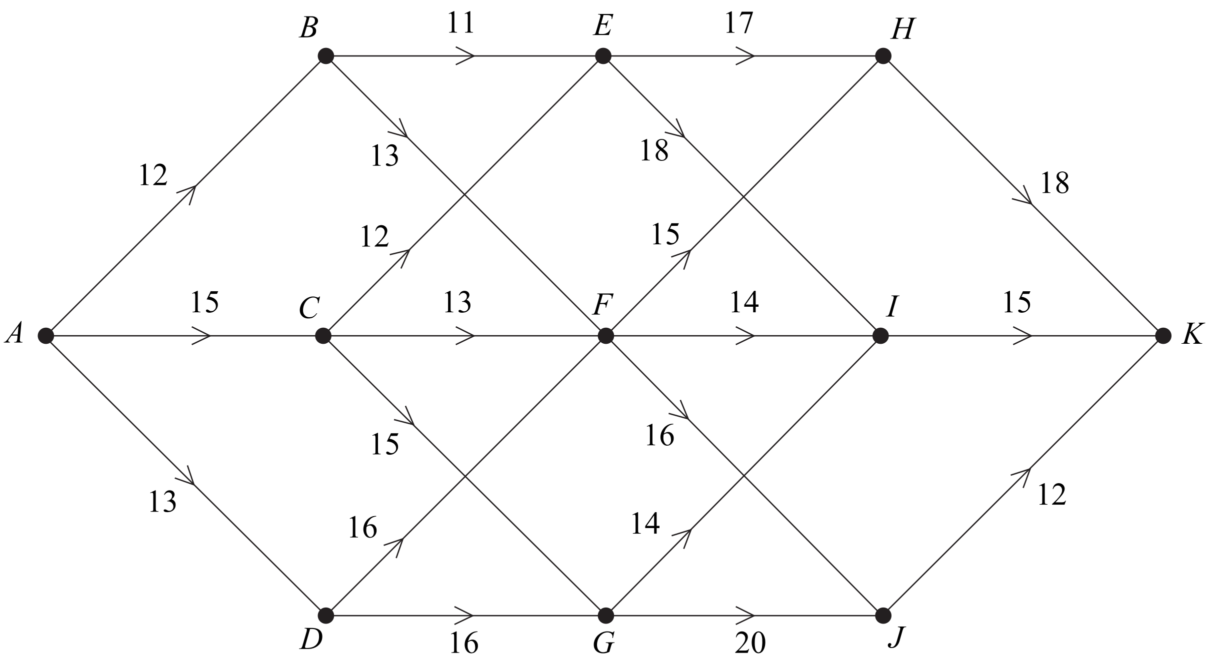
Answer space for question 3

Turn over ▶

4 A haulage company, based in town A, is to deliver a tall statue to town K. The statue is being delivered on the back of a lorry.

The network below shows a system of roads. The number on each edge represents the height, in feet, of the lowest bridge on that road.

The company wants to ensure that the height of the lowest bridge along the route from A to K is maximised.



Working backwards from *K*, use dynamic programming to find the optimal route when driving from A to K.

You must complete the table opposite as your solution. *(9 marks)*

QUESTION PART REFERENCE

Answer space for question 4

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Stage	State	From	Value
1	*H*	*K*	
	I	*K*	
	J	*K*	
2			

Optimal route is ....

Turn over ▶

5 Romeo and Juliet play a zero-sum game. The game is represented by the following pay-off matrix for Romeo.

Juliet

Romeo

Strategy	**D**	**E**	**F**
A	4	−4	0
B	−2	−5	3
C	2	1	−2

(a) Find the play-safe strategy for each player. *(3 marks)*

(b) Show that there is no stable solution. *(1 mark)*

(c) Explain why Juliet should never play strategy D. *(1 mark)*

(d) (i) Explain why the following is a suitable pay-off matrix **for Juliet**.

4	5	−1
0	−3	2

(2 marks)

(ii) Hence find the optimal strategy for Juliet. *(7 marks)*

(iii) Find the value of the game for Juliet. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

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Turn over ▶

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 5

6 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 4x + 3y + z$

subject to

$$2x + y + z \leqslant 25$$
$$x + 2y + z \leqslant 40$$
$$x + y + 2z \leqslant 30$$

and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$. *(2 marks)*

(b) The first pivot to be chosen is from the x-column.

Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret your final tableau and state the values of your slack variables. *(3 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 6

Turn over ▶

7 **Figure 2** shows a network of pipes.

Water from two reservoirs, R_1 and R_2, is used to supply three towns, T_1, T_2 and T_3.

In **Figure 2**, the capacity of each pipe is given by the number **not** circled on each edge. The numbers in circles represent an initial flow.

(a) Add a supersource, supersink and appropriate weighted edges to **Figure 2**. *(2 marks)*

(b) (i) Use the initial flow and the labelling procedure on **Figure 3** to find the maximum flow through the network.

You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut. *(2 marks)*

Figure 2

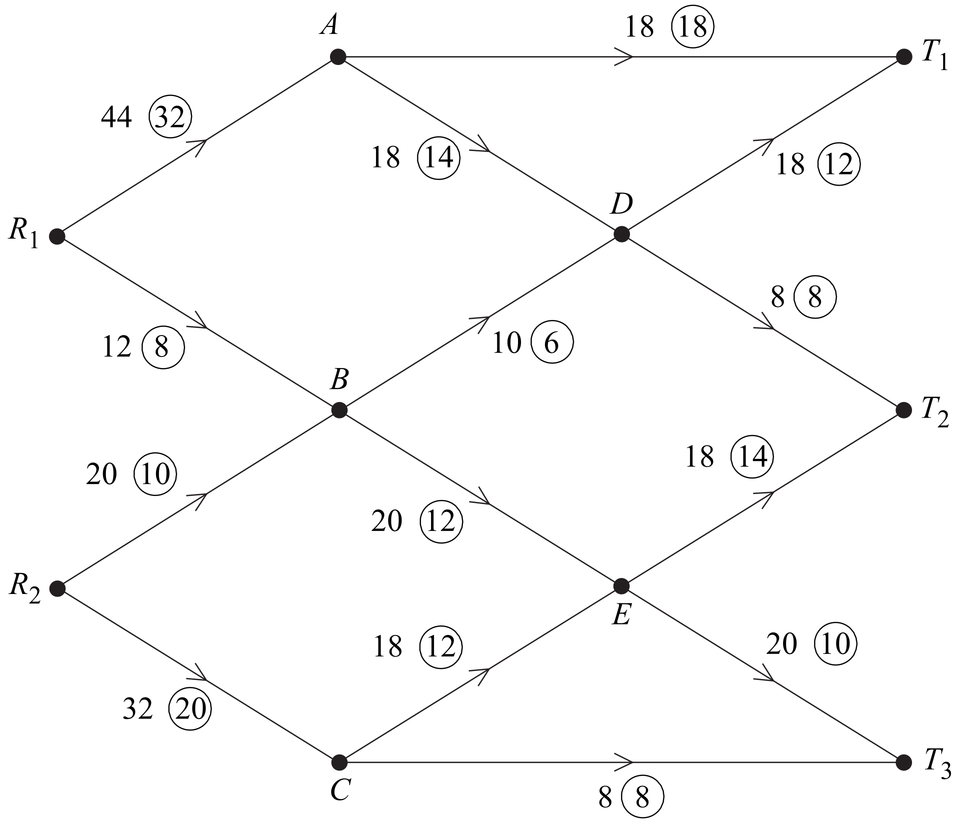
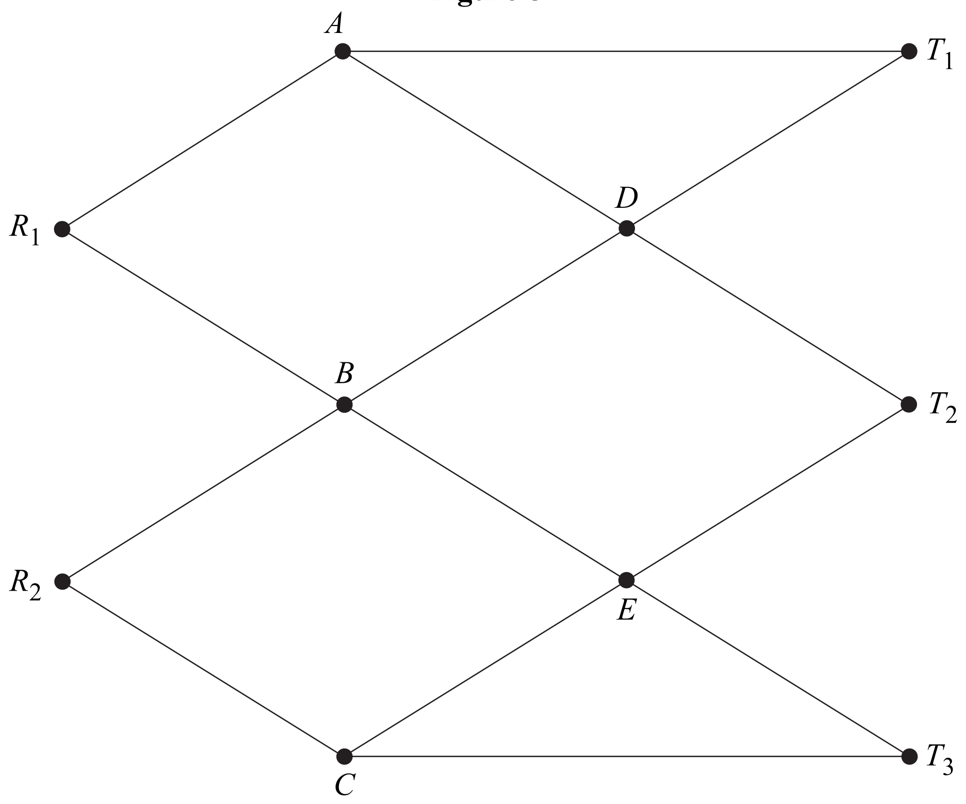
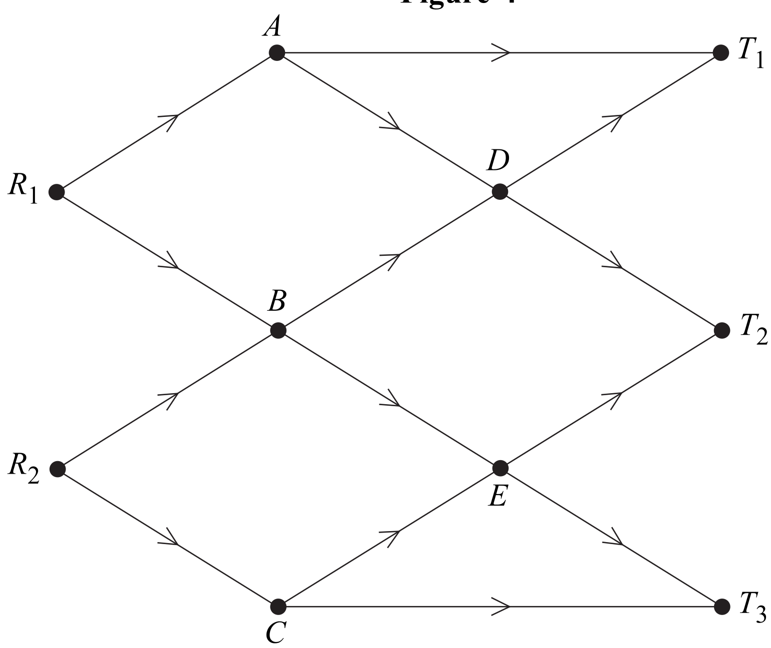


Figure 3



Route	Flow

Figure 4



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QUESTION PART REFERENCE

Answer space for question 7

END OF QUESTIONS