Version 1.0

Mathematics   **MDO2**

Decision 2

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MD02

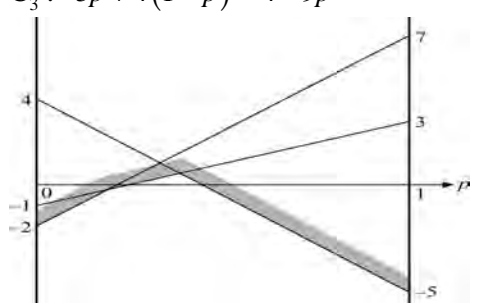| Q	Solution	Marks	Total	Comments
1(a)				
	Earliest start times	M1		one slip follow through
		A1		all correct
	Latest finish times	M1		one slip follow through
		A1	4	all correct
(b)	Critical paths are *AEHKL* and *BFHKL*	M1		one correct
		A1		both correct and no extras
	Minimum completion time = 21 days	B1	3	
(c)				$A(0 \rightarrow 4)$ $B(0 \rightarrow 3)$ $C(0 \rightarrow 2 \rightarrow 3)$ $D(4 \rightarrow 7 \rightarrow 9)$ $E(4 \rightarrow 8)$ $F(3 \rightarrow 8)$ $G(8 \rightarrow 16 \rightarrow 17$ $H(8 \rightarrow 14)$ $I(8 \rightarrow 10 \rightarrow 14$ $J(16 \rightarrow 18 \rightarrow 19$ $K(14 \rightarrow 19)$ $L(19 \rightarrow 21)$
		B1		*A*, *B*, *E*, *F*, *H*, *K*, *L* correct
		M1		*C*, *D*, *G*, *I*, *J* *(*4 with correct start and duration)
		A1	3	All 5 correct with correct slack indicated
(d)(i)	*K* now starts day 17	B1		or "delayed" b 3 days if 14 in network
	L now starts day 22	B1	2	or "delayed" b 3 days if 19 in network
(ii)	Overall delay 3 days	B1	1	
	Total		**13**	

MD02 (cont)

Q	Solution					Marks	Total	Comments
2(a)								
	2	4	0	5	5			
	4	2	0	4	3	M1		rows reduced (allow one slip)
	5	0	1	9	2			
	1	1	0	7	4			
	0	2	0	3	5			
	2	4	0	2	3			
	4	2	0	1	1	m1		columns reduced next
	5	0	1	6	0			Correct table
	1	1	0	4	2	A1	3	$k = 6$ stated or correct in table
	0	2	0	0	3			
(b)	3 lines needed to cover zeros shown					B1		middle column, middle and bottom rows
	Reduce each uncovered element by 1 and increase double covered by 1					M1		Condone one slip
	1	3	0	1	2			
	3	1	0	0	0			
	5	0	2	6(*k*)	0			
	0	0	0	3	1	A1	3	FT "their k". Condone k instead
	0	2	1	0	3			of 6
(c)	*A*3					M1		Or correct "rings" round elements for one complete solution
	(A3)	*B*4	*C*5	*D*2	*E*1	A1		first correct matching – must be stated
	(A3)	*B*5	*C*2	*D*1	*E*4	A1	3	second correct matching and no others
(d)	Minimum total penalty points = 22					B1	1	
				Total			**10**	

MD02 (cont)

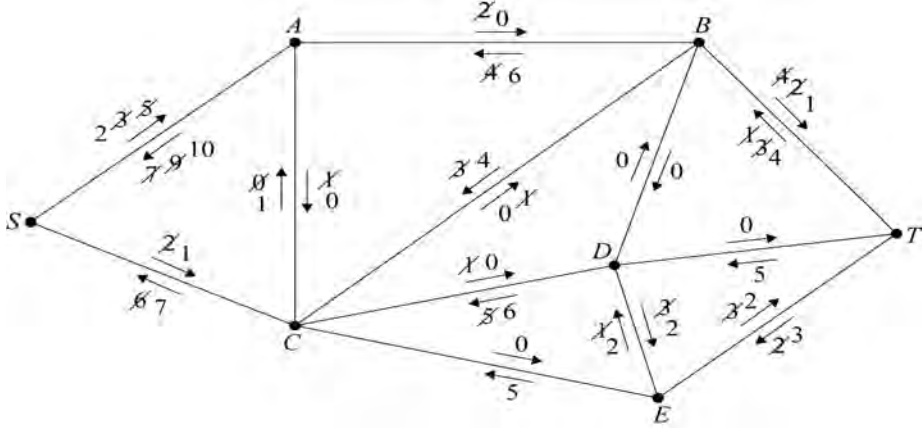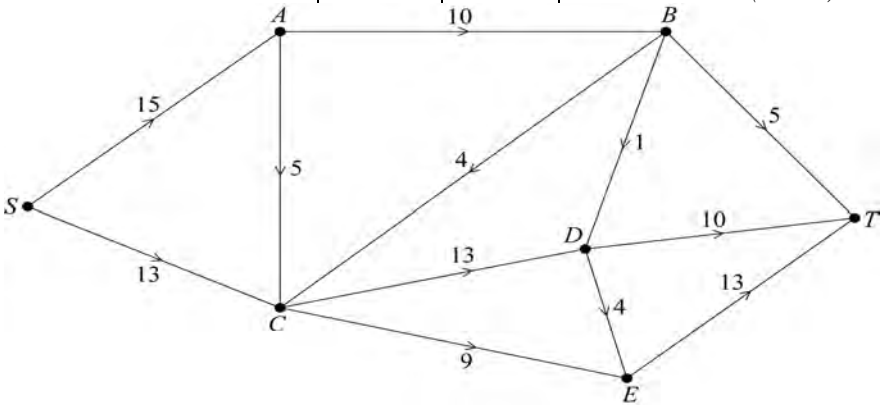| Q	Solution	Marks	Total	Comments
3(a)				
	P x y z s t value	M1		Two slack variables used correctly
	1 –6 –5 –3 0 0 0	A1		1 row correct
	0 ① 2 k 1 0 8	A1		all correct
	0 2 10 1 0 1 17		3	
(b)(i)	Pivot in x-column = 1	B1		May earn in (b)(i) May be implied by second row unchanged
	1 0 7 $6k$–3 6 0 48	M1		row operations (even with wrong pivot)
	0 1 2 k 1 0 8	A1		1st or 3rd row correct
	0 0 6 $1–2k$ –2 1 1	A1	4	all correct
(ii)	$6k - 3 < 0$	M1		"their" $6k - 3 < 0$
	$\Rightarrow k < \dfrac{1}{2}$	A1	2	
(c)	1 0 7 –9 6 0 48			
	0 1 2 –1 1 0 8			
	0 0 6 ③ –2 1 1	M1		new pivot correct from their tableau and row operations attempted
	1 0 25 0 0 3 51	A1		2 rows correct (may be multiples of rows) usually pivot row & 1 other
	0 1 4 0 $\frac{1}{3}$ $\frac{1}{3}$ $8\frac{1}{3}$	A1	3	all correct (condone multiples of rows) Condone FT from one slip in (b)(i)
	0 0 2 1 $-\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{3}$			
	Max P now achieved	E1		Or "optimum", "$P_{max} =$...."etc" Bur must have no negatives in top row
	$P = 51$	B1✓		FT their tableau
	$x = 8\frac{1}{3}$, $y = 0$, $z = \dfrac{1}{3}$ (all three)	B1	3	correct values from almost 'correct' tableau (condone one slip) condone 8.33 or better
	Total		**15**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	Let Roger play R_1 with probability p and R_2 with probability $1-p$ Expected gains: $C_1 : 7p - 2(1-p) = 9p - 2$ $C_2 : 3p - (1-p) = 4p - 1$ $C_3 : -5p + 4(1-p) = 4 - 9p$ C_2 and C_3 lines give optimum $4p - 1 = 4 - 9p$ $p = \dfrac{5}{13}$ Roger plays $R_1 \dfrac{5}{13}$ of time and $R_2 \dfrac{8}{13}$ of time	M1 A1 M1 A1 M1 A1 E1	 7	one correct unsimplified all correct unsimplified 2 of their lines drawn correctly all correct and accurate for $0 \le p \le 1$ Condone lines not quite to p = 1 if using "accurate" intersection points on p-axis i.e. $\frac{2}{9} < \frac{1}{4}$ and $\frac{4}{9} \approx twice \frac{2}{9}$ ft their max point of region Condone 0.385 or 0.3846(15…) must be correct rounding if 3sf used CAO
(ii)	Value of game $= 4 \times \dfrac{5}{13} - 1 = \dfrac{7}{13}$	B1	1	AG or $\left(4 - 9 \times \dfrac{5}{13}\right) = \dfrac{7}{13}$ must see correct calculation
(b)	Let Corrie play C_1 with prob p, C_2 with prob q, C_3 with prob $1-p-q$ $R_1 : 7p + 3q - 5(1-p-q)$ $R_2 : -2p - q + 4(1-p-q)$ $\Rightarrow 12p + 8q = 5\frac{7}{13}$ $6p + 5q = 3\frac{6}{13}$ $\Rightarrow \left.\begin{array}{c} q = \dfrac{9}{13} \\[2mm] p = 0 \end{array}\right\}$ $\Rightarrow$ Optimal mixed strategy is C_1 with prob 0 C_2 with prob $\dfrac{9}{13}$ C_3 with prob $\dfrac{4}{13}$	M1 A1 m1 A1CSO E1	 5	any correct expression either equation correctly with coefficients of p and q correctly simplified *may reason that $p(C_1) = 0$ from part(a)E1 with M1, A1, A1, E1 from 2 × 2 equations* $3r - 5s = \dfrac{7}{13}$ $-r + 4s = \dfrac{7}{13}$ Condone 0.692 CAO & 0.308
	Total		**13**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
5(a)	*PQSV* has longest journey 12 ⎫ *PQTV* has longest journey 13 ⎭	B1		Both of these
	Since 12 < 13, *PQSV* is better	E1	2	OE

(b)

Stage	State	Action	Calculation	Value			
1	*S*	*SV*	-	11	⎫		
	T	*TV*	-	9	⎬ B1		
	U	*UV*	-	12	⎭		
2	*Q*	*QS*	Max (12, 11)	12		M1	2 values correct
		QT	Max (13, 9)	13			
		QU	Max (7, 12)	12		A1	All correct with pairs of correct values compared in calculation column
	R	*RS*	Max (10, 11)	11		M1	2 values correct
		RT	Max (14, 9)	14			
		RU	Max (8, 12)	12		A1	All correct with pairs of correct values compared to calculation column
3	*P*	*PQ*	Max (9, 12)	12		A1	CSO; all table correct With word "MAX" seen at least once (or 12 >11 etc)
		PR	Max (11, 11)	11			

Using their minimum at stage 3	M1		Implied by route starting *PR* (Or PQ if that is their least value)
Minimax route from *P* to *V* is *PRSV*	A1	8	**SC** B1 for correct minimax route when several values in table are incorrect
Total		**10**	

Network approach: Use same mark scheme for 6 marks insisting on precisely these values, pairs of correct values **seen** and considered with maximum selected for first two A marks, and word 'Max' seen and all correct for final A mark

MD02 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Value of cut = 10 + 10 + 15 – 4 – 1	M1		condone one slip if working shown
	= 30	A1	2	
(b)	*BT* 2 , *DE* 3 , *ET* 12	B1		any 2 correct
		B1	2	all correct
(c)(i)	Initial flows forward and back or double	M1		Condone pairs of values, (coordinates) with single arrow
	Arc with arrows(at **least 6 pairs correct**)	A1	2	all correct (condone pairs with single arrow provided key indicated)
(ii)		M1		first correct path and flow

Path	Flow
SABT	2
SCDET	1
SACBT	1

A1 — another correct path and flow
A1 — all correct
(*other possibilities also*)

(or SCBT instead of SACBT with flow 1

(iii)	Must have forward and backward flows	M1		augmenting flows (6 pairs correct)
		A1	5	correct
				Alternative SA (3 & 9) SC (0&8)

May have
SA(14), *SC*(14)
and *AC*(4) using
alternative

	Maximum flow values	M1		at least 8 correctly interpreted from their Figure 4 but 24 < *their maxflow* < 29
		A1	2	
(d)	Cut through *AB, CB, CD* and *CE*	B1		**But** must have total flow of 28 in their network (condone one slip)
	May use {*S,A,C*} / {*B,D,E,T*}			
		B1	1	

9

	Total	14	
	TOTAL	75	