General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS   **MD02**
Unit Decision 2

Wednesday 21 May 2008   1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
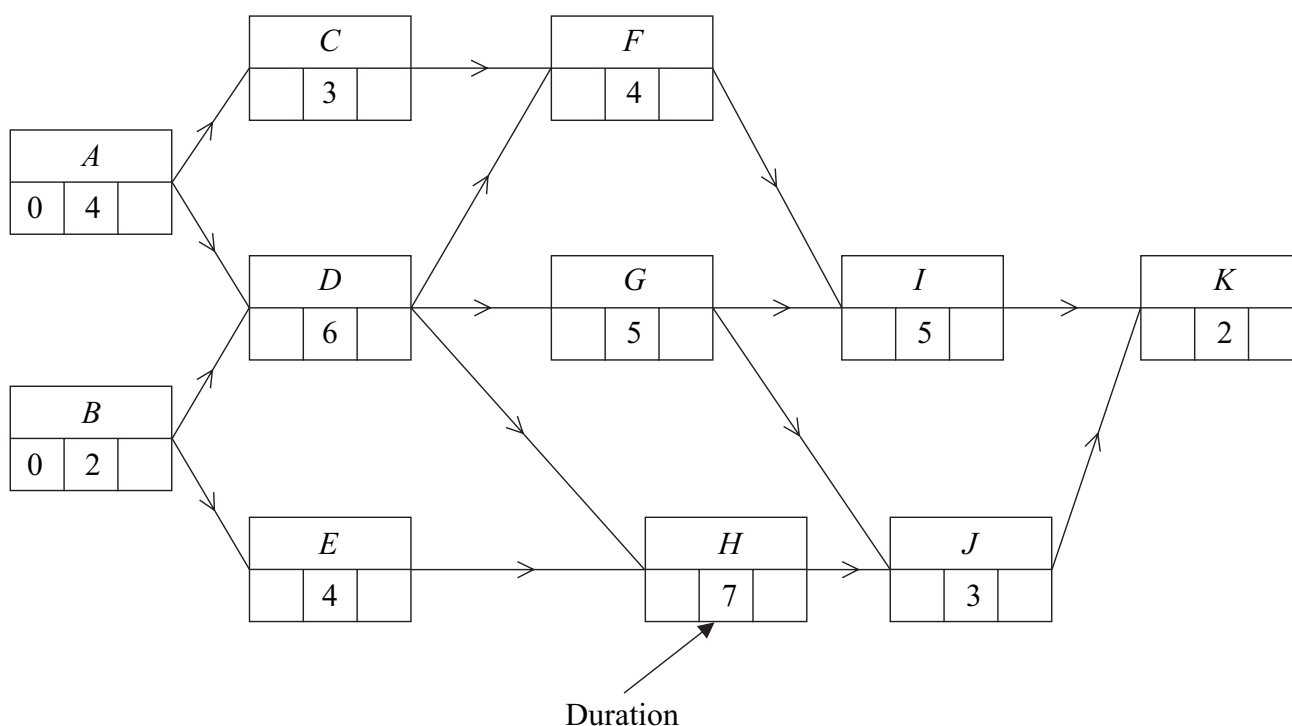- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

MD02

Answer **all** questions.

1   [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

The following diagram shows an activity network for a project. The time needed for each activity is given in days.

Duration

(a)   Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**.   *(4 marks)*

(b)   Find the critical paths and state the minimum time for completion.   *(3 marks)*

(c)   On **Figure 2**, draw a cascade diagram (Gantt chart) for the project, assuming each activity starts as early as possible.   *(3 marks)*

(d)   Activity *C* takes 5 days longer than first expected. Determine the effect on the earliest start time for other activities and the minimum completion time for the project.
                                                                                                                        (2 marks)

2 The following table shows the scores of five people, Alice, Baji, Cath, Dip and Ede, after playing five different computer games.

	Alice	Baji	Cath	Dip	Ede
Game 1	17	16	19	17	20
Game 2	20	13	15	16	18
Game 3	16	17	15	18	13
Game 4	13	14	18	15	17
Game 5	15	16	20	16	15

Each of the five games is to be assigned to one of the five people so that the total score is maximised. No person can be assigned to more than one game.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $20 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 20, and hence show that, by reducing **columns first** and then rows, the resulting table of values is as below.

3	1	1	1	0
0	4	5	2	2
4	0	5	0	7
5	1	0	1	1
5	1	0	2	5

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with one horizontal and three vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of games to the five people so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

3 Two people, Roseanne and Collette, play a zero-sum game. The game is represented by the following pay-off matrix for Roseanne.

	Strategy	Collette		
		C_1	C_2	C_3
Roseanne	R_1	-3	2	3
	R_2	2	-1	-4

(a) (i) Find the optimal mixed strategy for Roseanne. *(7 marks)*

  (ii) Show that the value of the game is -0.5. *(1 mark)*

(b) (i) Collette plays strategy C_1 with probability p and strategy C_2 with probability q. Write down, in terms of p and q, the probability that she plays strategy C_3. *(1 mark)*

  (ii) Hence, given that the value of the game is -0.5, find the optimal mixed strategy for Collette. *(4 marks)*

4 A linear programming problem consists of maximising an objective function P involving three variables x, y and z. Slack variables s, t, u and v are introduced and the Simplex method is used to solve the problem. Several iterations of the method lead to the following tableau.

P	x	y	z	s	t	u	v	*value*
1	0	-12	0	5	-3	0	0	37
0	1	-8	0	1	2	0	0	16
0	0	4	0	0	3	0	1	20
0	0	2	0	-3	2	1	0	14
0	0	1	1	2	5	0	0	8

(a) (i) The pivot for the next iteration is chosen from the **y-column**. State which value should be chosen and explain the reason for your choice. *(2 marks)*

  (ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) Explain why your new tableau solves the original problem. *(1 mark)*

(c) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(d) State the values of the slack variables at the optimum point. Hence determine how many of the original inequalities still have some slack when the optimum is reached. *(2 marks)*

5   [**Figure 3**, printed on the insert, is provided for use in this question.]

A small firm produces high quality cabinets.

It can produce up to 4 cabinets each month.

Whenever at least one cabinet is made during that month, the overhead costs for that month are £300.

It is possible to hold in stock a maximum of 2 cabinets during any month.

The cost of storage is £50 per cabinet per month.

The orders for cabinets are shown in the table below. There is no stock at the beginning of January and the firm plans to clear all stock after completing the April orders.

| Month	January	February	March	April
Number of cabinets required	3	3	5	2

(a)   Determine the total cost of storing 2 cabinets and producing 3 cabinets in a given month. *(2 marks)*

(b)   By completing the table of values on **Figure 3**, or otherwise, use dynamic programming, **working backwards from April**, to find the production schedule which minimises total costs. *(8 marks)*

(c)   Each cabinet is sold for £2000 but there is an additional cost of £300 for materials to make each cabinet and £2000 per month in wages. Determine the total profit for the four-month period. *(3 marks)*

Turn over for the next question

6 [**Figures 4**, **5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.
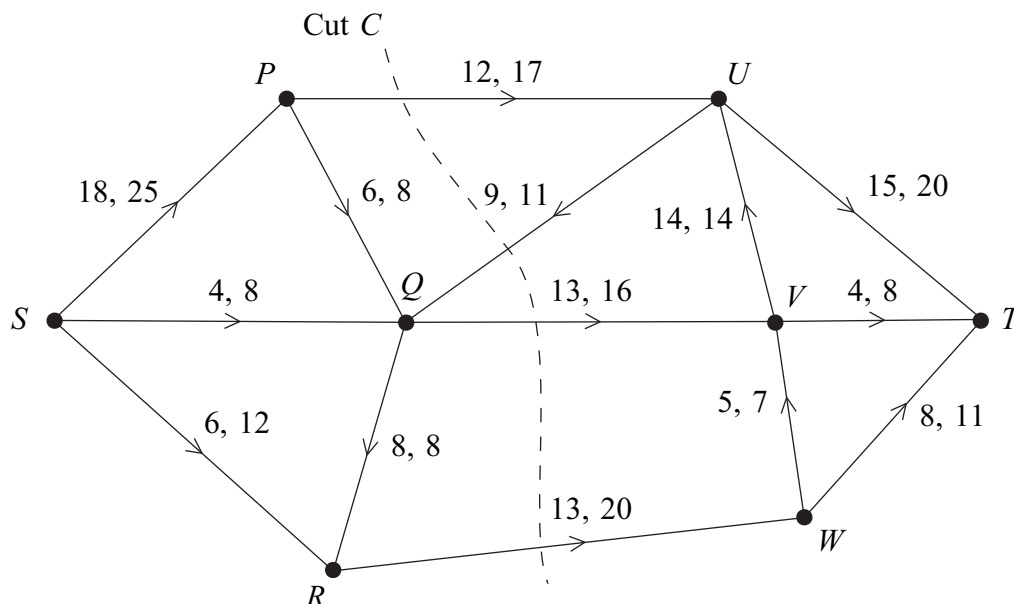
(a) (i) Find the value of the cut C. *(1 mark)*

(ii) Hence state what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 32 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges PQ, UQ and UT. *(3 marks)*

(c) (i) Taking your feasible flow from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(1 mark)*

END OF QUESTIONS

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There are no questions printed on this page

Surname					Other Names					
Centre Number					Candidate Number					
Candidate Signature										

General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

AQA

ASSESSMENT and
QUALIFICATIONS
ALLIANCE

Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

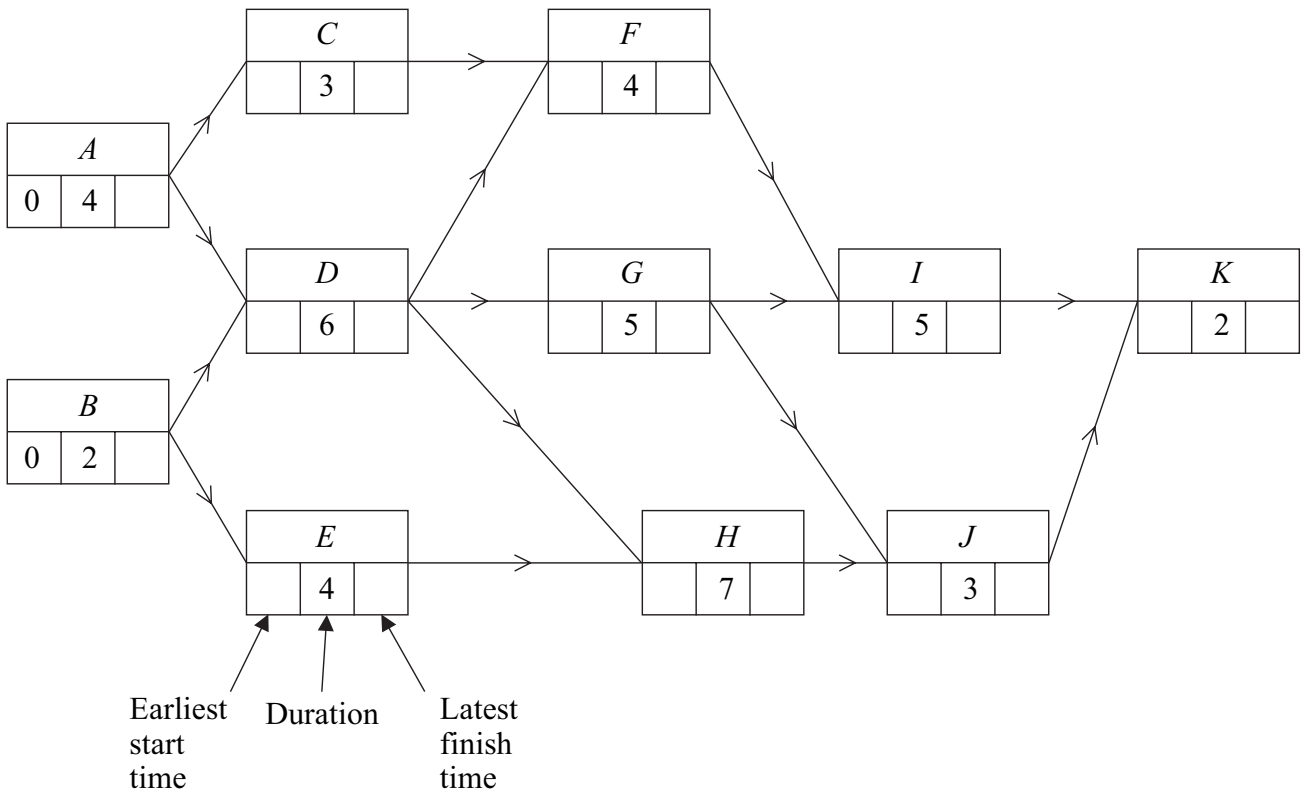**Turn over ▶**

Figure 1 (for use in Question 1)

	C	
	3	

	F	
	4	

A		
0	4	

	D	
	6	

	G	
	5	

	I	
	5	

	K	
	2	

B		
0	2	

	E	
	4	

	H	
	7	

	J	
	3	

Earliest start time   Duration   Latest finish time

Figure 2 (for use in Question 1)

Days
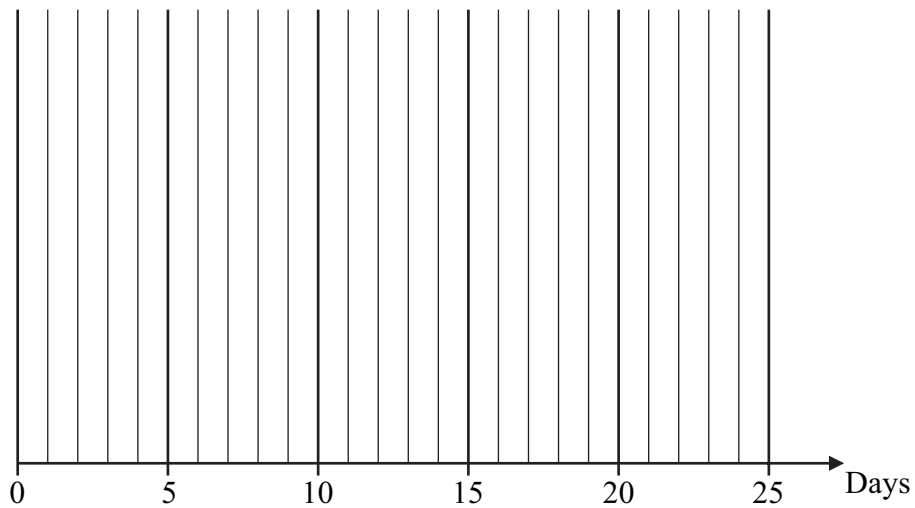
Figure 3 (for use in Question 5)

Initial State is the number of cabinets in stock at the beginning of that month.

Action is the number of cabinets made during that month.

Destination State is the number of cabinets in stock at the end of that month after the demand has been met.

The destination state for March becomes the initial state for April and so on.

Month & Demand	Initial State	Action	Destination State	Value
April	0	2	0	$300 + 0 = 300$
(demand 2)				
	1	1	0	$300 + 50 = 350$
	2	0	0	$0 + 100 = 100$
March	1	4	0	$300 + 50 + 300 = 650$
(demand 5)				
	2	3	0	
		4	1	
February	0	4	1	
(demand 3)				
	1	3	1	
		4	2	
	2	2	1	
		3	2	
January	0	3	0	
(demand 3)		4	1	

Production Schedule which minimises total costs

Month	January	February	March	April
Number of cabinets made				

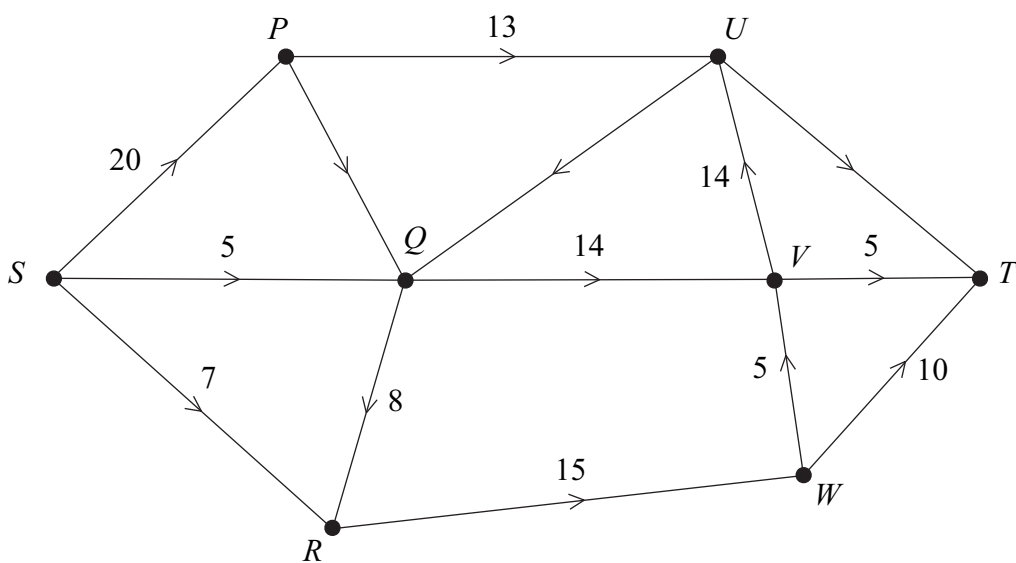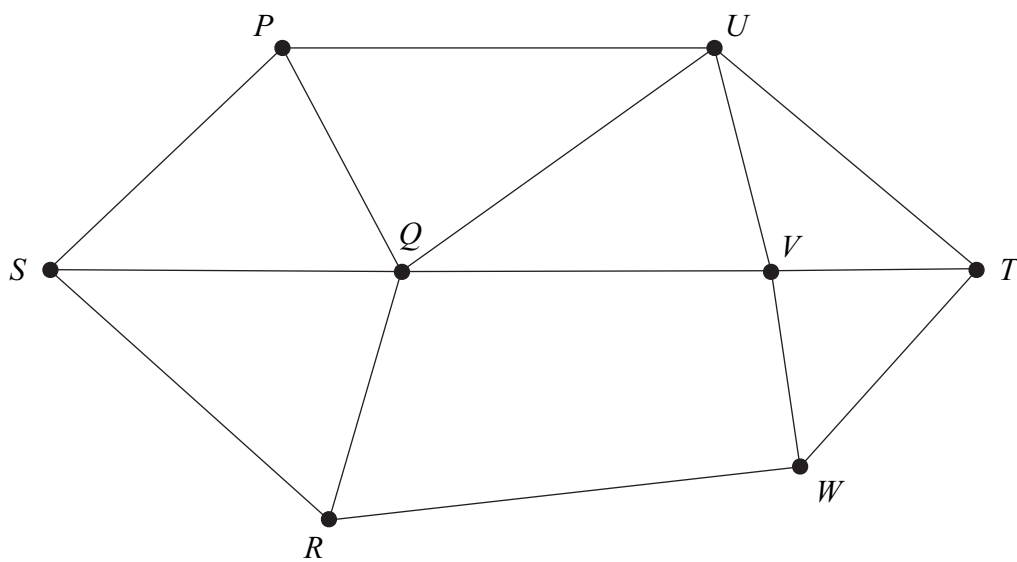**Turn over ▶**

Figure 4 (for use in Question 6)

Figure 5 (for use in Question 6)

Path	Additional Flow

Figure 6 (for use in Question 6)