

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Decision 2

MD02

Friday 22 June 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

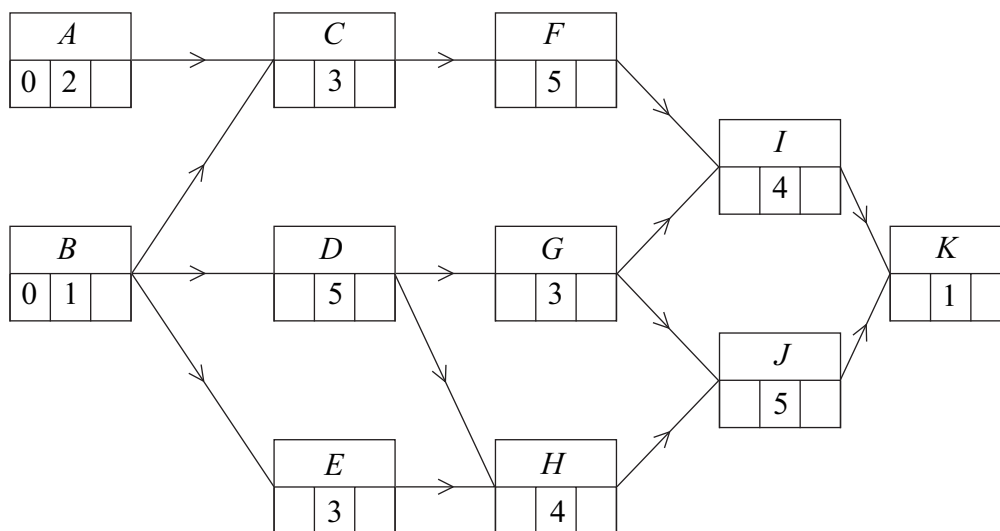
Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [Figures 1 and 2, printed on the insert, are provided for use in this question.]

The following diagram shows an activity diagram for a building project. The time needed for each activity is given in days.



- Complete the precedence table for the project on **Figure 1**. (2 marks)
- Find the earliest start times and latest finish times for each activity and insert their values on **Figure 2**. (4 marks)
- Find the critical path and state the minimum time for completion of the project. (2 marks)
- Find the activity with the greatest float time and state the value of its float time. (2 marks)

- 2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	A	B	C	D	E
Centre 1	10	11	8	12	5
Centre 2	11	5	11	6	7
Centre 3	12	8	7	11	4
Centre 4	10	9	14	10	6
Centre 5	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

- (a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

- (b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros. (5 marks)
- (c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost. (3 marks)
- (d) Find the value of this minimum daily total travel cost. (1 mark)

Turn over ►

- 3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

		Callum		
		C_1	C_2	C_3
Rose	R_1	5	2	-1
	R_2	-3	-1	5
	R_3	4	1	-2

- (a) (i) State the play-safe strategy for Rose and give a reason for your answer. *(2 marks)*
- (ii) Show that there is no stable solution for this game. *(2 marks)*
- (b) Explain why Rose should never play strategy R_3 . *(1 mark)*
- (c) Rose adopts a mixed strategy, choosing R_1 with probability p and R_2 with probability $1 - p$.
- (i) Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions. *(3 marks)*
- (ii) Illustrate graphically these expected gains for $0 \leq p \leq 1$. *(2 marks)*
- (iii) Hence determine the optimal mixed strategy for Rose. *(3 marks)*
- (iv) Find the value of the game. *(1 mark)*

- 4 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 3x + 5y$. The initial Simplex tableau is given below.

P	x	y	s	t	u	$value$
1	-3	-5	0	0	0	0
0	1	2	1	0	0	36
0	1	1	0	1	0	20
0	4	1	0	0	1	39

- (a) In addition to $x \geq 0$, $y \geq 0$, write down **three** inequalities involving x and y for this problem. *(2 marks)*
- (b) (i) By choosing the first pivot from the **y -column**, perform **one** iteration of the Simplex method. *(4 marks)*
- (ii) Explain how you know that the optimal value has not been reached. *(1 mark)*
- (c) (i) Perform one further iteration. *(4 marks)*
- (ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

Turn over for the next question

Turn over ►

5 [Figure 3, printed on the insert, is provided for use in this question.]

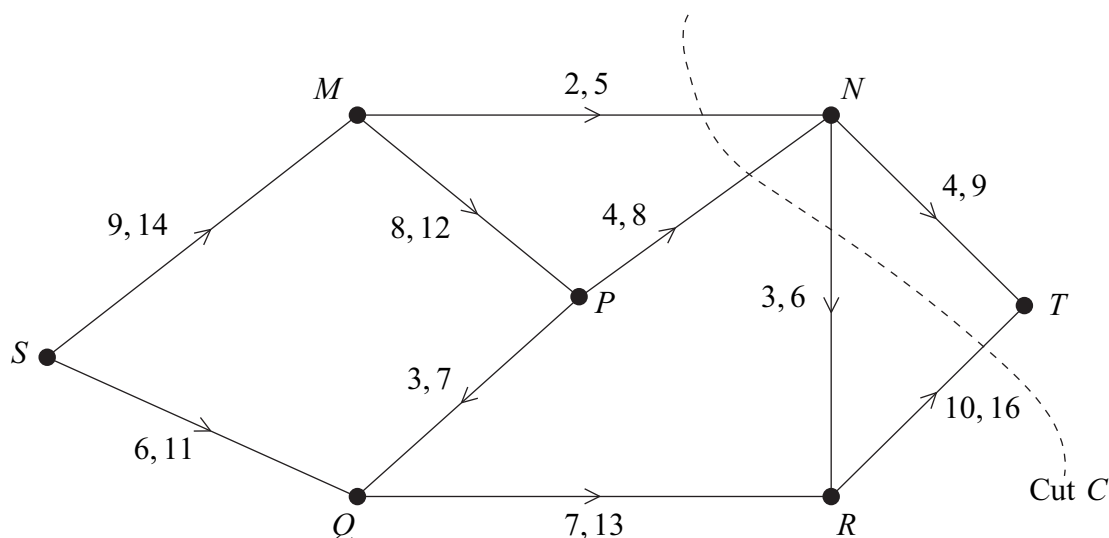
A maker of exclusive furniture is planning to build three cabinets *A*, *B* and *C* at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

Month	Already built	Cost		
		<i>A</i>	<i>B</i>	<i>C</i>
1	–	500	440	475
2	<i>A</i>	–	440	490
	<i>B</i>	510	–	500
	<i>C</i>	520	490	–
3	<i>A and B</i>	–	–	520
	<i>A and C</i>	–	500	–
	<i>B and C</i>	510	–	–

- (a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. (6 marks)
- (b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. (4 marks)

6 [Figures 4, 5 and 6, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



- (a) (i) Find the value of the cut C . (1 mark)
- (ii) State what can be deduced about the maximum flow from S to T . (1 mark)
- (b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T . Indicate, on **Figure 4**, the flows along the edges MP , PN , QR and NR . (4 marks)
- (c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. (2 marks)
- (ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T . You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. (5 marks)
- (iii) Illustrate the maximum flow on **Figure 6**. (2 marks)

END OF QUESTIONS

There are no questions printed on this page

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS
Unit Decision 2

MD02



Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 1)

Activity	Immediate Predecessors
<i>A</i>	–
<i>B</i>	–
<i>C</i>	
<i>D</i>	
<i>E</i>	
<i>F</i>	
<i>G</i>	
<i>H</i>	
<i>I</i>	
<i>J</i>	
<i>K</i>	

Figure 2 (for use in Question 1)

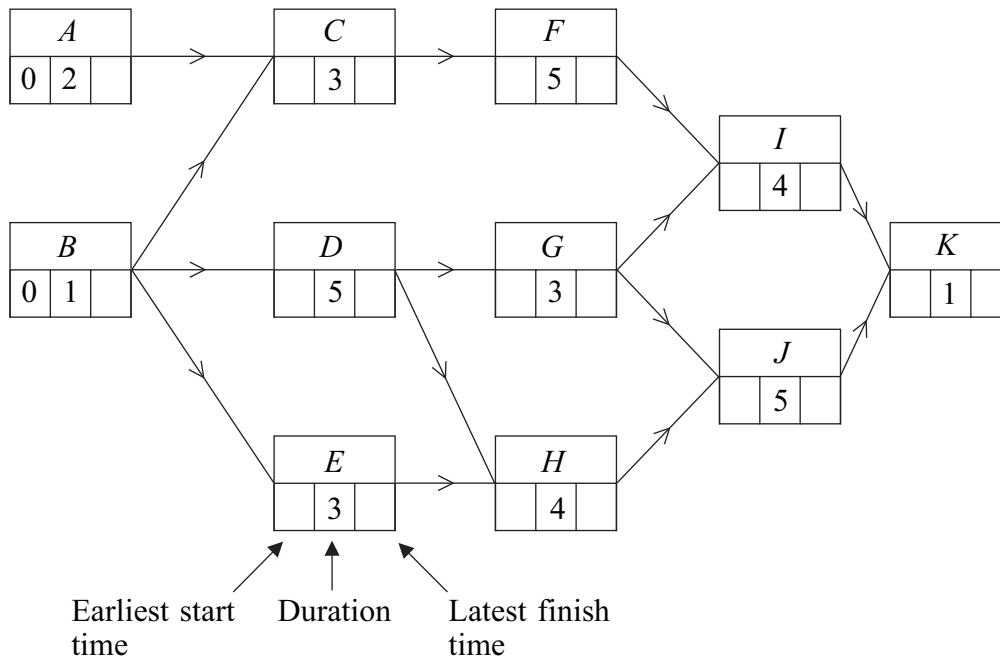


Figure 3 (for use in Question 5)

Month	Already built	Machine built	For use in part (a)	For use in part (b)
3	<i>A</i> and <i>B</i>	<i>C</i>		
	<i>A</i> and <i>C</i>	<i>B</i>		
	<i>B</i> and <i>C</i>	<i>A</i>		
2	<i>A</i>	<i>B</i>		
		<i>C</i>		
	<i>B</i>	<i>A</i>		
		<i>C</i>		
	<i>C</i>	<i>A</i>		
		<i>B</i>		

Turn over ►

Figure 4 (for use in Question 6)

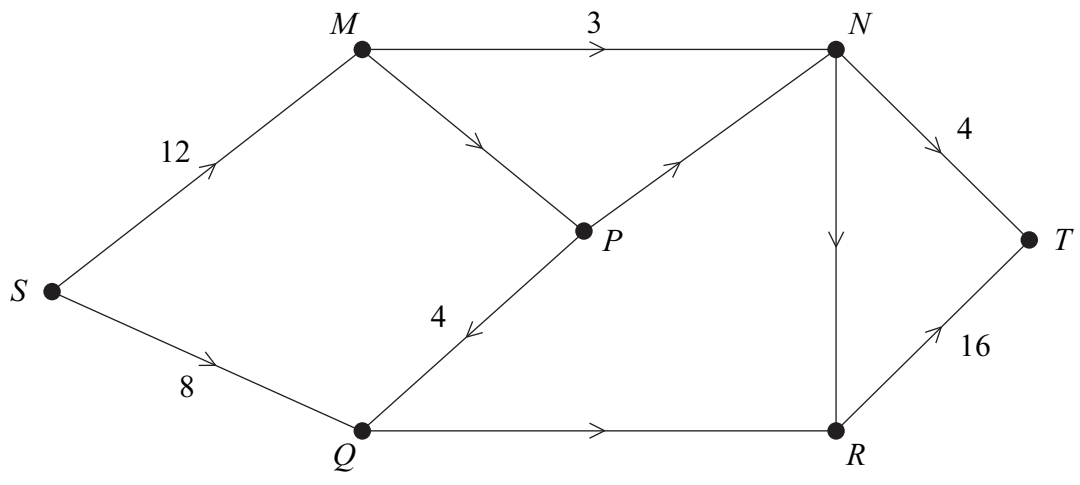
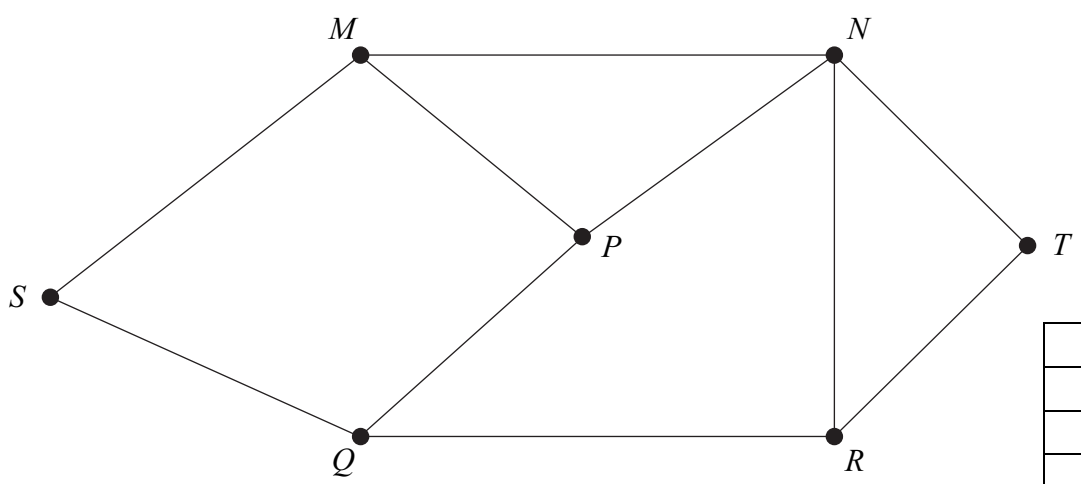


Figure 5 (for use in Question 6)



Path	Flow

Figure 6 (for use in Question 6)

