

Version



**General Certificate of Education (A-level)
January 2013**

Mathematics

MD02

(Specification 6360)

Decision 2

Final

Mark Scheme

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Key to mark scheme abbreviations

| | |
|--------------|--|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| ✓ or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| -x EE | deduct x marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q | Solution | Marks | Total | Comments | | | | | | | | | | | | | | | | | | | | | | | | |
|----------|--|----------------------|-------------|--|---|---|---|---|---|---|------|---|---|---|------|---|------|---|------|---|------|---|------|---|------|----------|---|---------------------------|
| 1(a) | | M1 A1 M1 A1 | 4 | Forward pass, correct at two of <i>D, E, F</i> All correct Backward pass, correct at <i>G AND H</i> ft All correct | | | | | | | | | | | | | | | | | | | | | | | | |
| (b) | <table border="1"> <thead> <tr> <th>Activity</th> <th>Predecessor</th> </tr> </thead> <tbody> <tr><td>A</td><td>–</td></tr> <tr><td>B</td><td>–</td></tr> <tr><td>C</td><td>B</td></tr> <tr><td>D</td><td>A, C</td></tr> <tr><td>E</td><td>C</td></tr> <tr><td>F</td><td>B, C</td></tr> <tr><td>G</td><td>D, E</td></tr> <tr><td>H</td><td>E, F</td></tr> <tr><td>I</td><td>G, H</td></tr> <tr><td>J</td><td>G, H</td></tr> <tr><td>K</td><td>I, J</td></tr> </tbody> </table> | Activity | Predecessor | A | – | B | – | C | B | D | A, C | E | C | F | B, C | G | D, E | H | E, F | I | G, H | J | G, H | K | I, J | B1 B1 | 2 | 6+ correct All correct |
| Activity | Predecessor | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | – | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B | – | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| C | B | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| D | A, C | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| E | C | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| F | B, C | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| G | D, E | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| H | E, F | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| I | G, H | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| J | G, H | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| K | I, J | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (c) | (Critical) <i>B C F H I K</i> | B1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| (d) | (Float <i>E</i>) 6 (hrs) | B1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| (e) | | M1 A1 A1 B1 | 3 | Their critical activities and 3 others shown Critical activities and 3 others correct All correct, condone floats seen | | | | | | | | | | | | | | | | | | | | | | | | |
| (f) | 34 (hrs) | B1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| (g) | 62 (hrs) | B1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | Total | | 13 | | | | | | | | | | | | | | | | | | | | | | | | | |

MD02

| Q | Solution | Marks | Total | Comments |
|------|--|-----------------------------|----------|--|
| 2(a) | $\begin{array}{cccc} & & & \text{Min} \\ \left(\begin{array}{cccc} 4 & -1 & 2 & 3 \end{array} \right) & -1 \\ \left(\begin{array}{cccc} 4 & 6 & 3 & 7 \end{array} \right) & 3 \\ \left(\begin{array}{cccc} 1 & 3 & -2 & 4 \end{array} \right) & -2 \\ \text{Max } 4 & 6 & 3 & 7 \end{array}$ <p>Maximin (row) = 3</p> <p>Minimax (col) = 3</p> <p>As Maximin (row) = Minimax (col) There is a stable solution</p> $\left. \begin{array}{l} \text{(Play safe) (H)} \quad B \\ \text{(Play safe) (W)} \quad F \end{array} \right\}$ | M1 A1 CSO E1 B1 | 4 | <p>Either correct, including correct values</p> <p>Both correct, written as equations PI by next line</p> <p>Must have equation and statement and scored first 2 marks</p> <p>Both correct</p> |
| (b) | Saddle point (B, F) | B1 | 1 | |
| | Total | | 5 | |

MD02

| Q | Solution | Marks | Total | Comments |
|------|---|-------|----------|---|
| 3(a) | $\begin{pmatrix} 8 & 5 & 0 & 9 & 6 \\ 5 & 6 & 5 & 9 & 7 \\ 11 & 10 & 12 & 12 & 11 \\ 9 & 5 & 8 & 12 & 9 \end{pmatrix}$ | B1 | 1 | |
| (b) | <p>Add an extra row ≥ 12</p> $\begin{pmatrix} 8 & 5 & 0 & 9 & 6 \\ 5 & 6 & 5 & 9 & 7 \\ 11 & 10 & 12 & 12 & 11 \\ 9 & 5 & 8 & 12 & 9 \\ 12 & 12 & 12 & 12 & 12 \end{pmatrix} \begin{matrix} (0) \\ (5) \\ (10) \\ (5) \\ (12) \end{matrix}$ | B1 | | |
| | $\begin{matrix} 8 & 5 & 0 & 9 & 6 \\ 0 & 1 & 0 & 4 & 2 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 0 & 3 & 7 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$ | M1 | | 3 rows correct from row reduction |
| | | A1 | | All correct |
| | $\begin{pmatrix} 8 & 5 & 0 & 9 & 6 \\ 0 & 1 & 0 & 4 & 2 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 0 & 3 & 7 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ <p>(Zeros correctly covered by 4 lines)</p> | B1F | | <p>Alternatives</p> $\begin{pmatrix} 8 & 5 & 0 & 9 & 6 \\ 0 & 1 & 0 & 4 & 2 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 0 & 3 & 7 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ |
| | <p>Covered in 4 lines, not optimal (reduce by 1)</p> | E1 | | * |
| | $\begin{matrix} 8 & 5 & 0 & 8 & 5 \\ 0 & 1 & 0 & 3 & 1 \\ 1 & 0 & 2 & 1 & 0 \\ 4 & 0 & 3 & 6 & 3 \\ 1 & 1 & 1 & 0 & 0 \end{matrix}$ | B1 | | $\begin{matrix} 7 & 5 & 0 & 8 & 5 \\ 0 & 2 & 1 & 4 & 2 \\ 0 & 0 & 2 & 1 & 0 \\ 3 & 0 & 3 & 6 & 3 \\ 0 & 1 & 1 & 0 & 0 \end{matrix} \begin{matrix} (8) & (6) & (0) & (9) & (6) \\ (0) & (2) & (1) & (4) & (2) \\ (0) & (0) & (2) & (1) & (0) \\ (3) & (0) & (3) & (6) & (3) \\ (0) & (1) & (1) & (0) & (0) \end{matrix}$ |
| | <p>5 lines needed, optimal</p> | (E1) | | *or earned here |
| | <p>Match WC, XA, YE, ZB, (-D)</p> | B1 | | |
| | <p>Value = 151</p> | B1 | 8 | |
| | Total | | 9 | |

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| Q | Solution | Marks | Total | Comments |
|-----------|--|----------|----------|------------------------------------|
| (4)(a)(i) | Max Flow = 50 (Min cut = 50) | E1 | | Either statement |
| (ii) | $35 \leq \text{max flow} \leq 50$ (or min cut) | E1, E1 | | E1 for strict inequalities |
| (iii) | Error or contradiction | E1 | 4 | oe |
| (b) | At F , $\left. \begin{array}{l} \text{flow in} \geq 8 \\ \text{flow out} \leq 7 \end{array} \right\}$ | M1 A1 | 2 | Stating F and one of the 'flows' |
| | Total | | 6 | |

MD02

| Q | Solution | Marks | Total | Comments |
|---------------------------------|--|--------|-----------|-------------------------------------|
| 5(a) | P x y z r s t value | | | |
| | 1 -1 2 -3 0 0 0 0 | | | |
| | 0 1 1 1 1 0 0 16 | B2,1,0 | 2 | All correct, 3 rows correct |
| | 0 1 -2 2 0 1 0 17 | | | |
| 0 2 -1 2 0 0 1 19 | | | | |
| (b)(i) | z -col: $\frac{16}{1}, \frac{17}{2}, \frac{19}{2}$ | M1 | | |
| | Min, R_3 as pivot | A1 | 2 | |
| (ii) | 1 $\frac{1}{2}$ -1 0 0 $1\frac{1}{2}$ 0 $\frac{51}{2}$ | M1 | | Row operations |
| | 0 $\frac{1}{2}$ 2 0 1 $-\frac{1}{2}$ 0 $\frac{15}{2}$ | A1 | | One row (other than R_3) correct |
| | 0 $\frac{1}{2}$ -1 1 0 $\frac{1}{2}$ 0 $\frac{17}{2}$ | | | |
| | 0 1 1 0 0 -1 1 2 | A1 | 3 | All correct |
| | Alternative | | | |
| | 2 1 -2 0 0 3 0 51 | (M1) | | |
| | 0 1 4 0 2 -1 0 15 | (A1) | | |
| | 0 1 -2 2 0 1 0 17 | (A1) | | |
| | 0 1 1 0 0 -1 1 2 | | | |
| (c)(i) | y col $\frac{15}{4}, \left(-\frac{17}{2}\right), \frac{2}{1}$ R_4 as pivot | B1 | | Fully correct description |
| | | | | |
| | 1 $1\frac{1}{2}$ 0 0 0 $\frac{1}{2}$ 1 $\frac{55}{2}$ | M1 | | Row operations |
| | 0 $-1\frac{1}{2}$ 0 0 1 $1\frac{1}{2}$ -2 $\frac{7}{2}$ | | | |
| | 0 $1\frac{1}{2}$ 0 1 0 $-\frac{1}{2}$ 1 $\frac{21}{2}$ | | | |
| | 0 1 1 0 0 -1 1 2 | A1 | 3 | All correct |
| | Alternative | | | |
| | 2 3 0 0 0 1 2 55 | (M1) | | |
| | 0 -3 0 0 2 3 -4 7 | | | |
| | 0 3 0 2 0 -1 2 21 | (A1) | | |
| | 0 1 1 0 0 -1 1 2 | | | |
| (c)(ii) | Optimal | | | |
| | $P = \frac{55}{2}$ | B1 | | Both statement and value needed. OE |
| | $x=0, y=2, z=\frac{21}{2}$ | B1 | | |
| | $s=t=0, r=\frac{7}{2}$ | B1 | 3 | |
| | Total | | 13 | |

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| Q | Solution | Marks | Total | Comments |
|------|---|--|-------|---|
| 6(a) | $R_C > R_B$ | E1 | 1 | oe |
| (b) | $A \begin{pmatrix} -2 & 0 & 3 \\ 4 & 1 & -1 \end{pmatrix}$ $C \begin{pmatrix} 4 & 1 & -1 \end{pmatrix}$ <p>K plays A prob p C prob $1-p$</p> <p>P plays</p> $\left. \begin{array}{l} D, K \text{ wins } -2p + 4(1-p) \quad (= 4 - 6p) \\ E, K \text{ wins } 1-p \\ F, K \text{ wins } 3p - 1(1-p) \quad (= -1 + 4p) \end{array} \right\}$ <p>Max at $1-p = -1+4p$</p> $p = \frac{2}{5}$ <p>$(K \text{ plays}) A \text{ prob } \frac{2}{5}, C \text{ prob } \frac{3}{5}$</p> <p>Value of game = $\frac{3}{5}$</p> | E1 M1 A1 M1 A1 M1 A1 B1 | 7 | <p>Allow 2 expressions in unsimplified form All 3 correct</p> <p>Must have 3 lines</p> <p>With values shown</p> <p>Identifying correct maximum from their graph</p> <p>Both stated, coming from equating correct two equations and M2 scored</p> |

MD02

| Q | Solution | Marks | Total | Comments |
|------|---|--|-----------|---|
| 6(c) | <p>P plays D prob p E " q F " $1-p-q$</p> <p>K plays A, P loses $-2p + 3(1-p-q) = 3 - 5p - 3q$</p> <p>$K$ plays C, P loses $4p + q - 1(1-p-q) = -1 + 5p + 2q$</p> $\frac{3 - 5p - 3q}{-1 + 5p + 2q} = \frac{3}{5}$ $2 - q = \frac{6}{5}$ $q = \frac{4}{5}$ $5p + \frac{8}{5} - 1 = \frac{3}{5}$ $p = 0$ <p>P plays D prob 0 E, prob $\frac{4}{5}$ F, prob $\frac{1}{5}$</p> <p>Alternative method Probability of D is 0 $3(1-p) = \frac{3}{5}$ or $p - 1(1-p) = \frac{3}{5}$ $p = \frac{4}{5}$ E prob $\frac{4}{5}$ F prob $\frac{1}{5}$</p> | <p>M1</p> <p>m1</p> <p>A1 CSO</p> <p>E1</p> <p>(E1) (M1) (m1) (A1) CSO</p> | <p>4</p> | <p>Either (unsimplified) expression correct</p> <p>Equating BOTH of their expressions to value of their game</p> <p>Or for finding p</p> <p>All three needed, must have scored previous A mark</p> <p>OE, might be earned in final line</p> <p>Or equating the expressions</p> |
| | Total | | 12 | |

MD02

| Q | Solution | | | | Marks | Total | Comments |
|--------------|------------------------|--------------|-------------|--------------|----------------|----------|--|
| 7(a) | Stage | State | From | Value | | | |
| | 1 | <i>G</i> | <i>I</i> | 15 | | | |
| | | <i>H</i> | <i>I</i> | 12 | | | |
| | -- | -- | -- | -- | | | |
| | 2 | <i>E</i> | <i>G</i> | 15+15 = 30 ← | B1 | 7 | Stage 2 values correct |
| | | | <i>H</i> | 12+16 = 28 | | | |
| | | <i>F</i> | <i>G</i> | 15+13=28 | | | |
| | | | <i>H</i> | 12+17= 29 ← | | | |
| | | -- | -- | -- -- -- | | | |
| | 3 | <i>B</i> | <i>E</i> | 30+16 = 46 | M1 m1 | 7 | Calculating 4 values at stage 3 Using max values at <i>E</i> and <i>F</i> |
| | | <i>C</i> | <i>E</i> | 30+14 = 44 ← | | | |
| | | | <i>F</i> | 29+12 = 41 | | | |
| | | <i>D</i> | <i>F</i> | 29+15 = 44 | A1 | | All 4 values correct |
| | | -- | -- | -- -- -- | | | |
| | 4 | <i>A</i> | <i>B</i> | 46+12 = 58 | m1 A1 B1 | 7 | Using max at <i>C</i> All correct Identifying 64 as maximum value |
| | | | <i>C</i> | 44+20= 64 ← | | | |
| | | | <i>D</i> | 44+18 = 62 | | | |
| (b) | Route <i>A C E G I</i> | | | | B1 | 1 | |
| Total | | | | | | 8 | |

