

Decision Mathematics 1

Graphs

June 2008 – Question 2 – 6 Marks

A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

- (i) Draw an Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is not simply connected. [3]
- (ii) Draw a non-Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is non-Eulerian even though its vertices are all of even order. [3]

January 2008 – Question 2 – 5 Marks

A puzzle involves a 3 by 3 grid of squares, numbered 1 to 9, as shown in Fig. 1a below. Eight of the squares are covered by blank tiles. Fig. 1b shows the puzzle with all of the squares covered except for square 4. This arrangement of tiles will be called position 4.

1	2	3
4	5	6
7	8	9

Fig. 1a

4		

Fig. 1b

A move consists of sliding a tile into the empty space. From position 4, the next move will result in position 1, position 5 or position 7.

- (i) Draw a graph with nine vertices to represent the nine positions and arcs that show which positions can be reached from one another in one move. What is the least number of moves needed to get from position 1 to position 9? [3]
- (ii) State whether the graph from part (i) is Eulerian, semi-Eulerian or neither. Explain how you know which it is. [2]

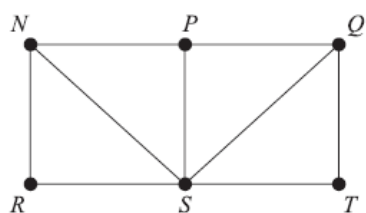
Specimen Paper – Question 1 – 4 Marks

The graph K_5 has five nodes, A, B, C, D and E , and there is an arc joining every node to every other node.

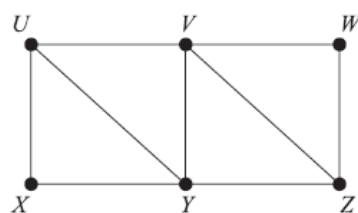
- (i) Draw the graph K_5 and state how you know that it is Eulerian. [2]
- (ii) By listing the arcs involved, give an example of a **path** in K_5 . (Your path must include more than one arc.) [1]
- (iii) By listing the arcs involved, give an example of a **cycle** in K_5 . [1]

June 2007 – Question 1 – 9 Marks

Two graphs A and B are shown below.



Graph A



Graph B

- (i) Write down an example of a cycle on graph A . [1]
- (ii) Why is $U-Y-V-Z-Y-X$ not a path on graph B ? [1]
- (iii) How many arcs would there be in a spanning tree for graph A ? [1]
- (iv) For each graph state whether it is Eulerian, semi-Eulerian or neither. [2]
- (v) The graphs show designs to be etched on metal plates. The etching tool is positioned at a starting point and follows a route without repeating any arcs. It may be lifted off and positioned at a new starting point. What is the smallest number of times that the etching tool must be positioned, including the initial position, to draw each graph? [2]

An arc is drawn connecting Q to U , so that the two graphs become one. The resulting graph is not Eulerian.

- (vi) Extra arcs are then added to make an Eulerian graph. What is the smallest number of extra arcs that need to be added? [2]

January 2007 – Question 3 – 9 Marks

A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is connected, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

- (i) A simply connected graph is drawn with 6 vertices and 9 arcs.
 - (a) What is the sum of the orders of the vertices? [1]
 - (b) Explain why if the graph has two vertices of order 5 it cannot have any vertices of order 1. [2]
 - (c) Explain why the graph cannot have three vertices of order 5. [2]
- (ii) Draw an example of a simply connected graph with 6 vertices and 9 arcs in which one of the vertices has order 5 and all the orders of the vertices are odd numbers. [2]
- (iii) Draw an example of a simply connected graph with 6 vertices and 9 arcs that is also Eulerian. [2]