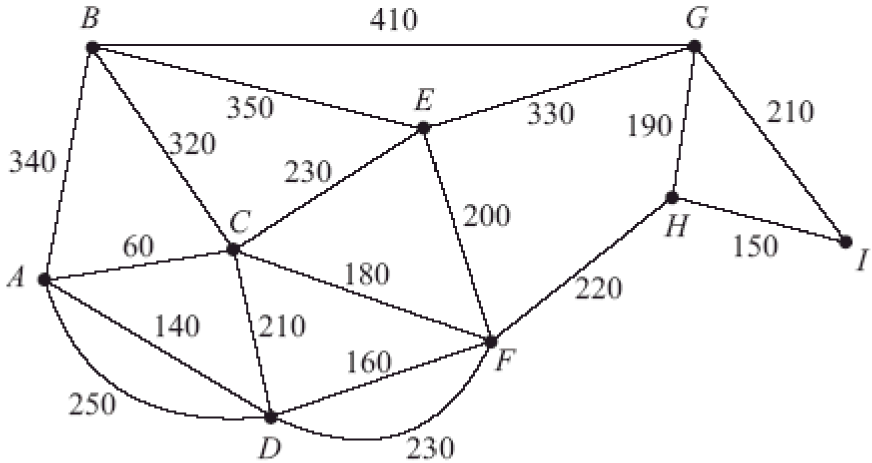
1. (a) Explain why a network cannot have an odd number of vertices of odd degree.

(2)



The figure above shows a network of paths in a public park. The number on each arc represents the length of that path in metres. Hamish needs to walk along each path at least once to check the paths for frost damage starting and finishing at *A*. He wishes to minimise the total distance he walks.

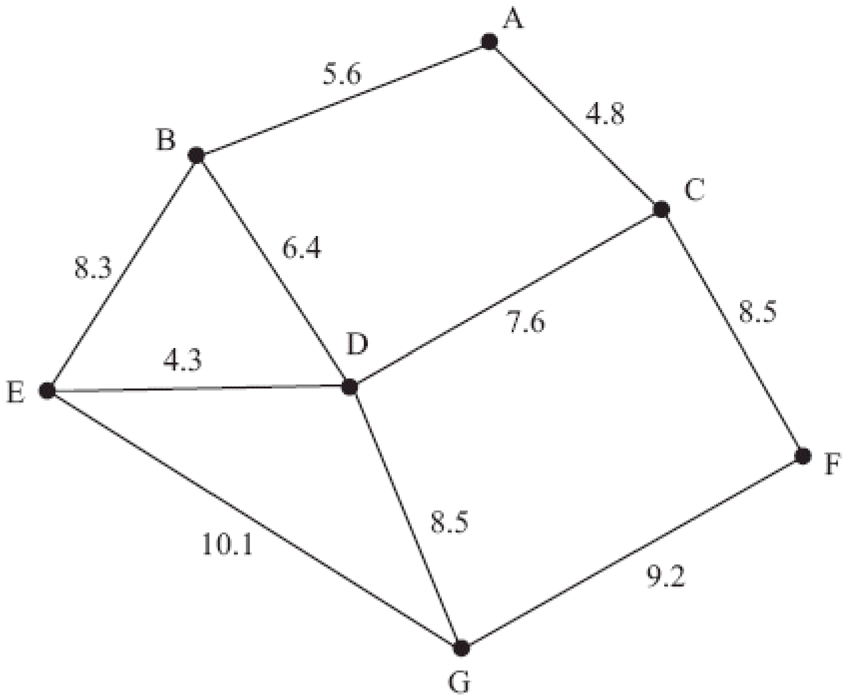
(b) Use the route inspection algorithm to find which paths, if any, need to be traversed twice.

(4)

(c) Find the length of Hamish's route.
[The total weight of the network in Figure 4 is 4180m.]

(1)
(Total 7 marks)

2.



[*The total weight of the network is 73.3 km*]

The diagram above models a network of tunnels that have to be inspected. The number on each arc represents the length, in km, of that tunnel.
Malcolm needs to travel through each tunnel at least once and wishes to minimise the length of his inspection route.
He must start and finish at A.

(a) Use the route inspection algorithm to find the tunnels that will need to be traversed twice. You should make your method and working clear.

(5)

(b) Find a route of minimum length, starting and finishing at A.

State the length of your route.

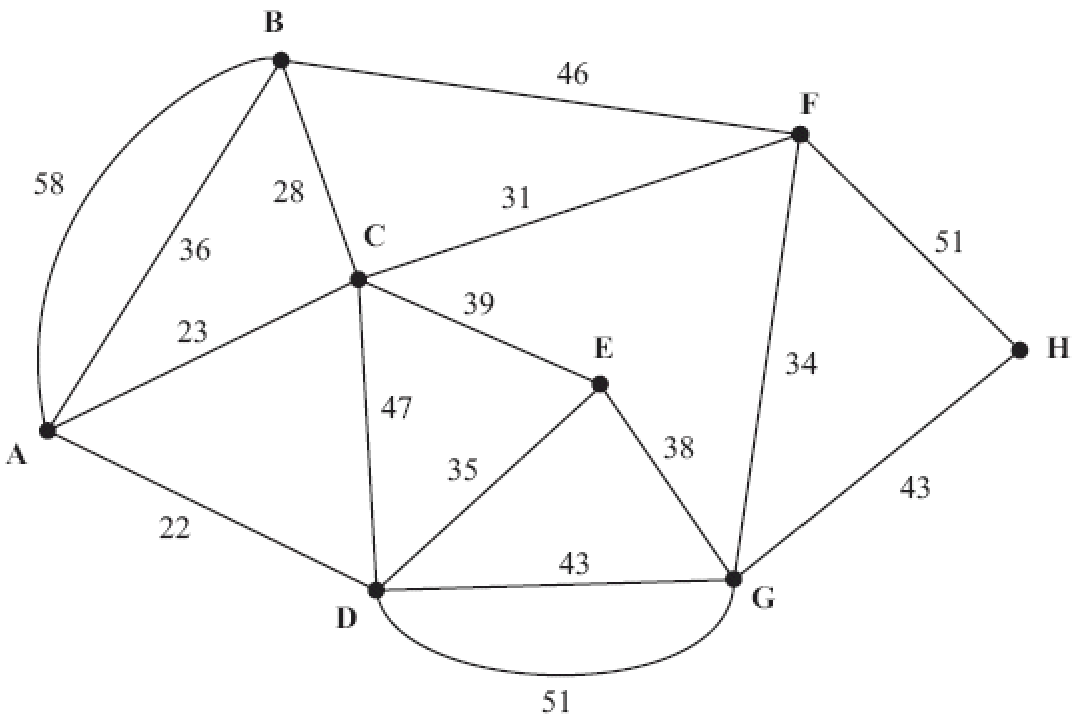
(3)

A new tunnel, CG, is under construction. It will be 10 km long.
Malcolm will have to include the new tunnel in his inspection route.

(c) What effect will the new tunnel have on the total length of his route?
Justify your answer.

(2)
(Total 10 marks)

3.



[The total weight of the network is 625 m]

The diagram above models a network of paths in a park. The number on each arc represents the length, in m, of that path.
Rob needs to travel along each path to inspect the surface. He wants to minimise the length of his route.

(a) Use the route inspection algorithm to find the length of his route. State the arcs that should be repeated. You should make your method and working clear.

(6)

The surface on each path is to be renewed. A machine will be hired to do this task and driven along each path.
The machine will be delivered to point G and will start from there, but it may be collected from any point once the task is complete.

(b) Given that each path must be traversed at least once, determine the finishing point so that the length of the route is minimised. Give a reason for your answer and state the length of your route.

(3)
(Total 9 marks)

4. (a) Draw the activity network described in this precedence table, using activity on arc and exactly two dummies.

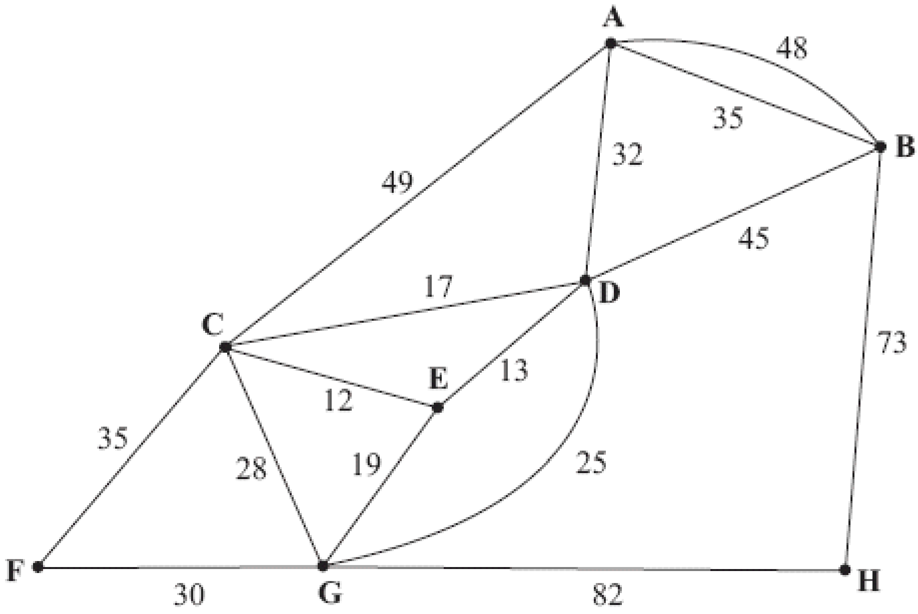
Activity	Immediately preceding activities
A	–
B	–
C	–
D	**B**
E	**B, C**
F	**B, C**
G	**F**
H	**F**
I	**G, H**
J	**I**

(5)

(b) Explain why each of the two dummies is necessary.

(2)
(Total 7 marks)

5.



(The total weight of the network above is 543 km.)

The diagram above models a network of railway tracks that have to be inspected. The number on each arc is the length, in km, of that section of railway track.
Each track must be traversed at least once and the length of the inspection route must be minimised.
The inspection route must start and finish at the same vertex.

(a) Use an appropriate algorithm to find the length of the shortest inspection route. You should make your method and working clear.

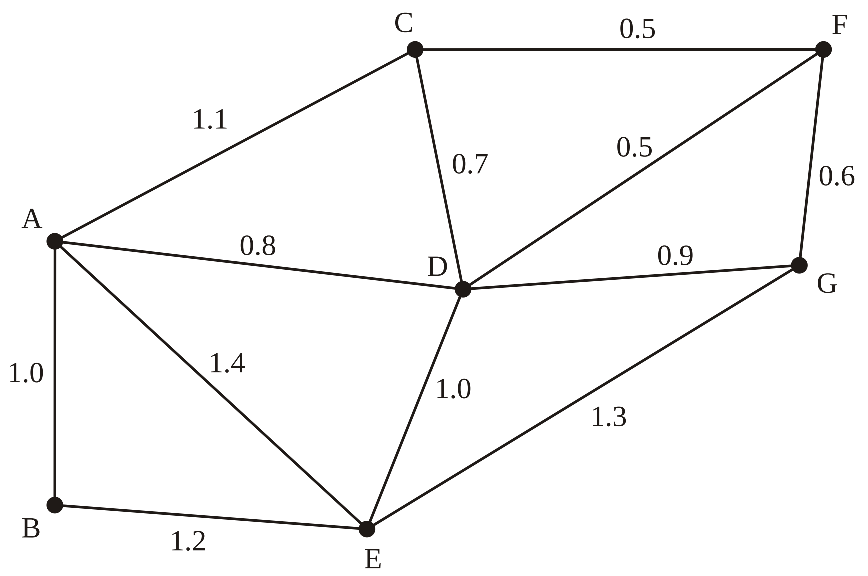
(5)

It is now permitted to start and finish the inspection at two distinct vertices.

(b) State which two vertices should be chosen to minimise the length of the new route. Give a reason for your answer.

(3)
(Total 8 marks)

6.



The diagram above models a network of roads in a housing estate. The number on each arc represents the length, in km, of the road.

The total weight of the network is 11 km.

A council worker needs to travel along each road once to inspect the road surface. He will start and finish at A and wishes to minimise the length of his route.

(a) Use an appropriate algorithm to find a route for the council worker. You should make your method and working clear. State your route and its length.

(6)

A postal worker needs to walk along each road twice, once on each side of the road. She must start and finish at A. The length of her route is to be minimised. You should ignore the width of the road.

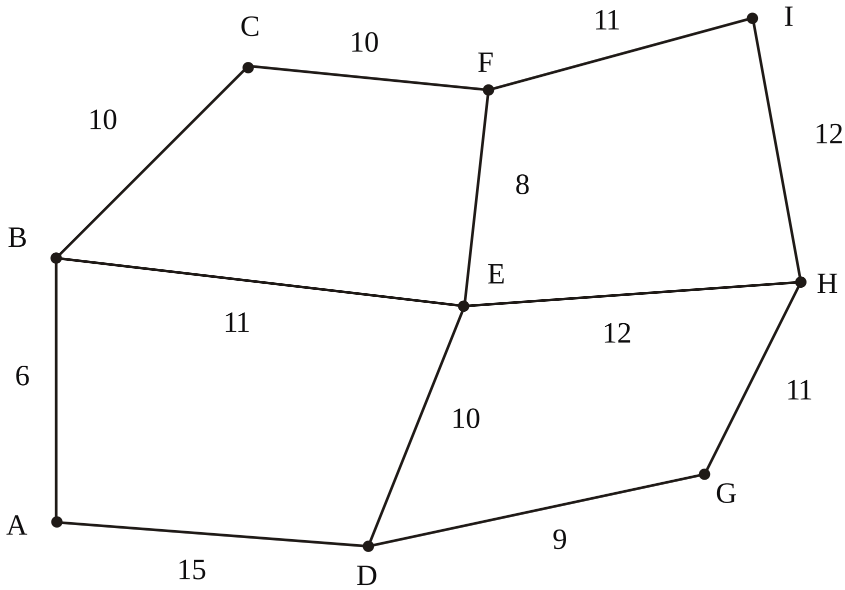
(b) (i) Explain how this differs from the standard route inspection problem.

(1)

(ii) Find the length of the shortest route for the postal worker.

(2)
(Total 9 marks)

7.



The diagram above models a network of underground tunnels that have to be inspected. The number on each arc represents the length, in km, of each tunnel.

Joe must travel along each tunnel at least once and the length of his inspection route must be minimised.

The total weight of the network is 125 km.

The inspection route must start and finish at A.

(a) Use an appropriate algorithm to find the length of the shortest inspection route. You should make your method and working clear.

(5)

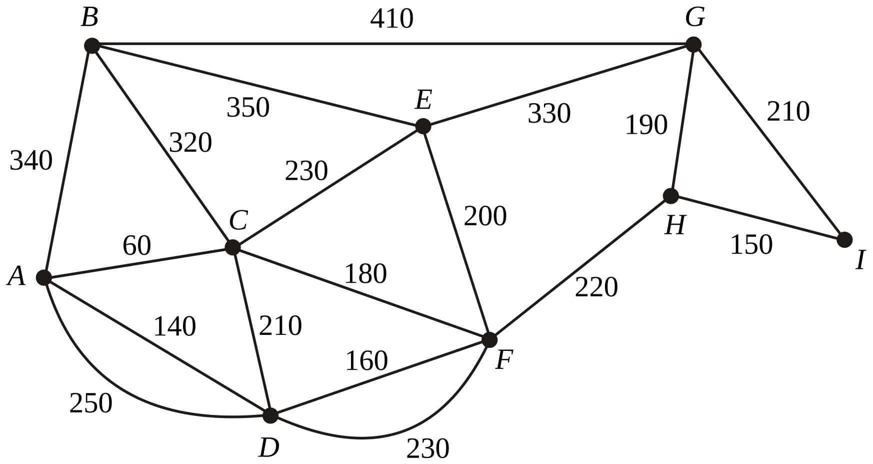
Given that it is now permitted to start and finish the inspection at two distinct vertices,

(b) state which two vertices should be chosen to minimise the length of the new route. Give a reason for your answer.

(2)
(Total 7 marks)

8. (a) Explain why a network cannot have an odd number of vertices of odd degree.

(2)



The diagram above shows a network of paths in a public park. The number on each arc represents the length of that path in metres. Hamish needs to walk along each path at least once to check the paths for frost damage starting and finishing at *A*. He wishes to minimise the total distance he walks.

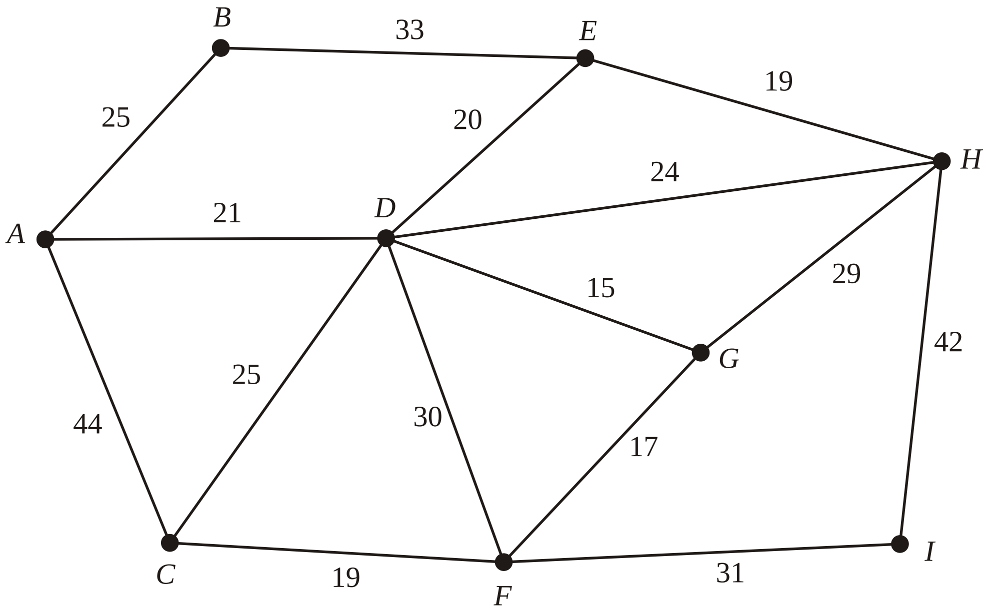
(b) Use the route inspection algorithm to find which paths, if any, need to be traversed twice.

(4)

(c) Find the length of Hamish's route.
[The total weight of the network in Figure 4 is 4180 m.]

(1)
(Total 7 marks)

9.



The figure above shows a network of pipes represented by arcs. The length of each pipe, in kilometres, is shown by the number on each arc. The network is to be inspected for leakages, using the shortest route and starting and finishing at *A*.

(a) Use the route inspection algorithm to find which arcs, if any, need to be traversed twice.

(4)

(b) State the length of the minimum route.
[The total **weight** of the network is 394 km]

(1)

It is now permitted to start and finish the inspection at two distinct vertices.

(c) State, with a reason, which two vertices should be chosen to minimise the length of the **new** route.

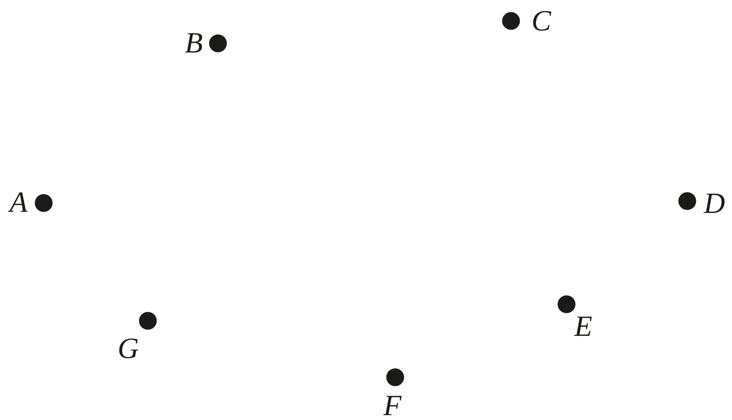
(2)
(Total 7 marks)

10.

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	—	48	117	92	—	—	—
B	48	—	—	—	—	63	55
C	117	—	—	28	—	—	85
D	92	—	28	—	58	132	—
E	—	—	—	58	—	124	—
F	—	63	—	132	124	—	—
G	—	55	85	—	—	—	—

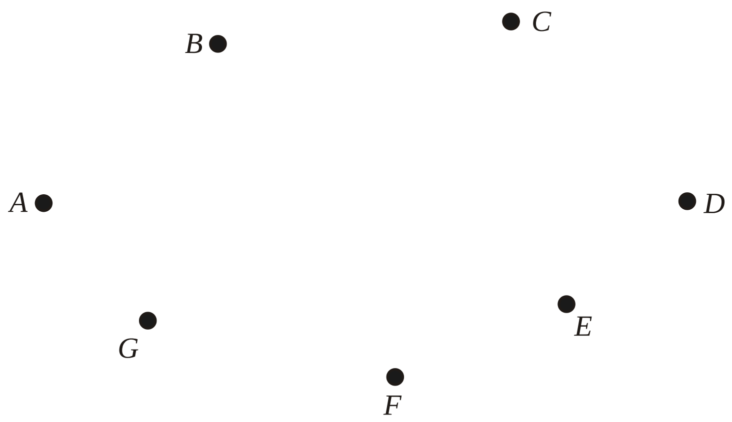
The table shows the lengths, in metres, of the paths between seven vertices *A, B, C, D, E, F* and *G* in a network N.

(a) Use Prim's algorithm, starting at *A*, to solve the minimum connector problem for this table of distances. You must clearly state the order in which you selected the edges of your tree, and the weight of your final tree. Draw your tree using the vertices given in the diagram below.



(5)

(b) Draw N using the vertices given in the diagram below.



(3)

(c) Solve the Route Inspection problem for N. You must make your method and working clear. State a shortest route and find its length.

(The weight of N is 802)

Shortest route:

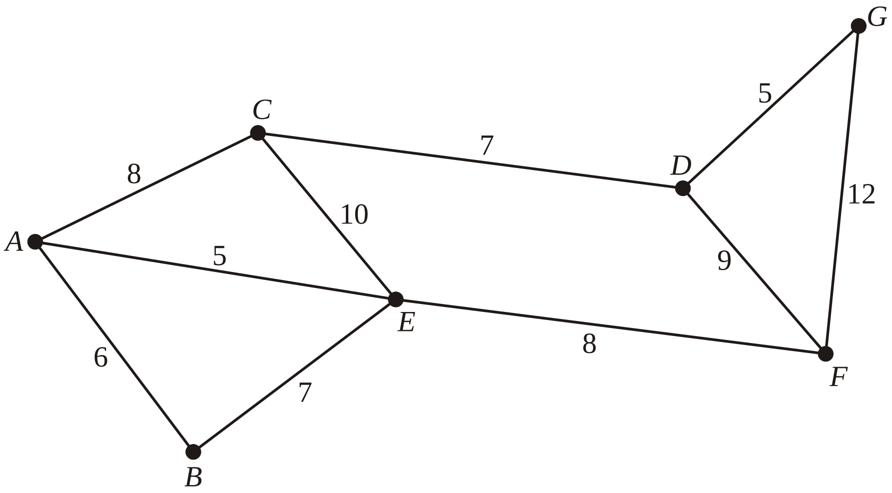
....

Length:

....

(7)
(Total 15 marks)

11.



This figure models a network of roads which need to be inspected to assess if they need to be resurfaced. The number on each arc represents the length, in km, of that road.

Each road must be traversed at least once and the length of the inspection route must be minimised.

(a) Starting and finishing at A, solve this route inspection problem. You should make your method and working clear. State the length of the shortest route.
(The weight of the network is 77 km.)

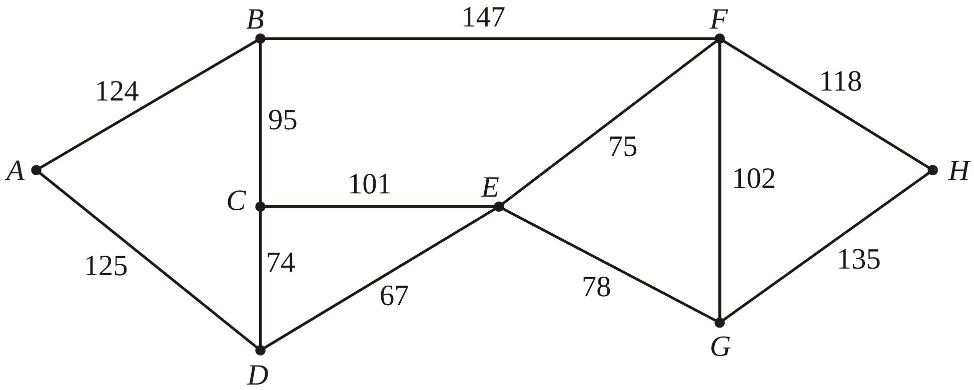
(5)

Given that it is now permitted to start and finish the inspection at two distinct vertices,

(b) state which two vertices you should choose to minimise the length of the route. Give a reason for your answer.

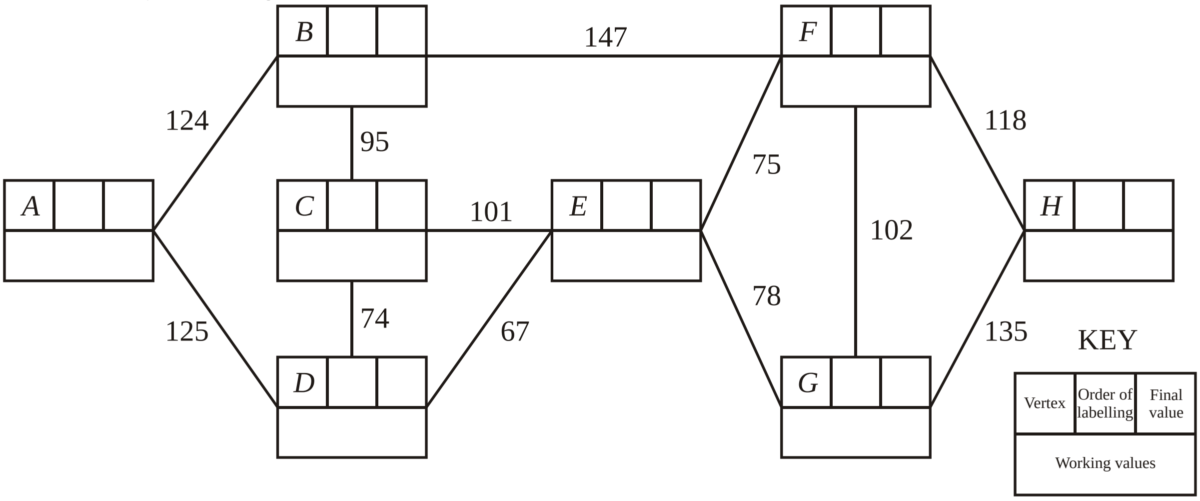
(2)
(Total 7 marks)

12.



The diagram above shows a network of paths. The number on each arc gives the distance, in metres, of that path.

(i) Use Dijkstra's algorithm to find the shortest distance from *A* to *H*.



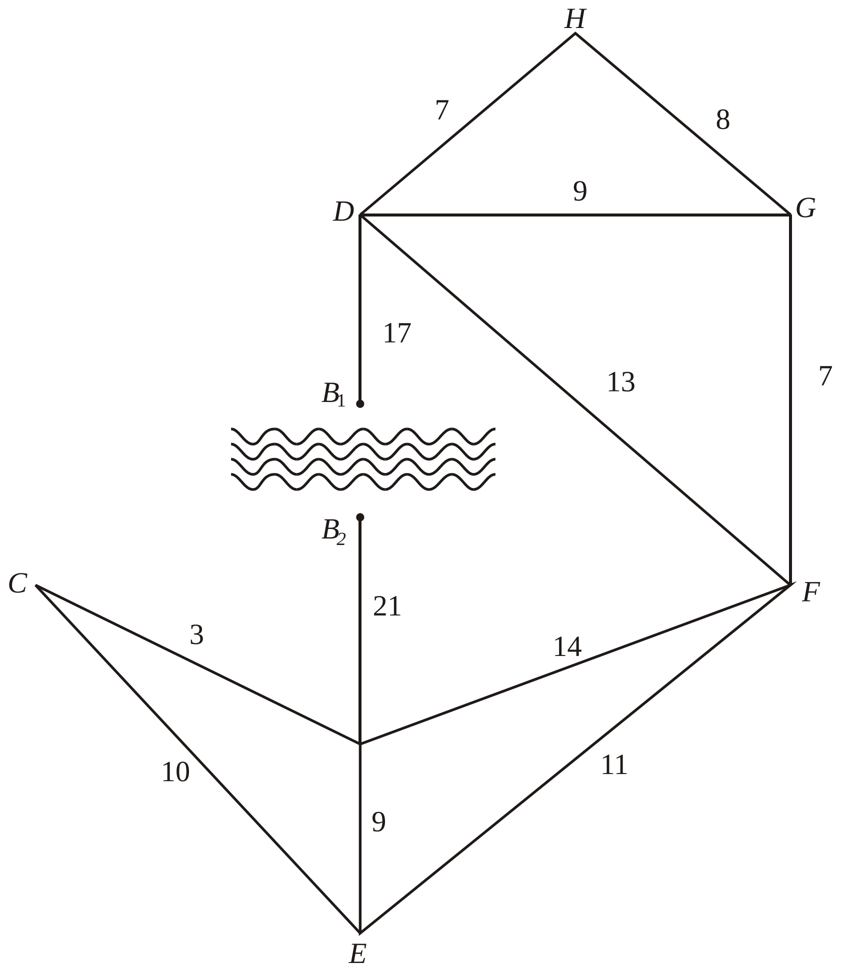
(5)

(ii) Solve the route inspection problem for the network shown in the diagram. You should make your method and working clear. State a shortest route, starting at *A*, and find its length.

[The total weight of the network is 1241]

(6)
(Total 11 marks)

13.



The diagram above shows a network of roads connecting villages. The length of each road, in km, is shown. Village *B* has only a small footbridge over the river which runs through the village. It can be accessed by two roads, from *A* and *D*.

The driver of a snowplough, based at *F*, is planning a route to enable her to clear all the roads of snow. The route should be of minimum length. Each road can be cleared by driving along it once. The snowplough cannot cross the footbridge.

Showing all your working and using an appropriate algorithm,

(a) find the route the driver should follow, starting and ending at *F*, to clear all the roads of snow. Give the length of this route.

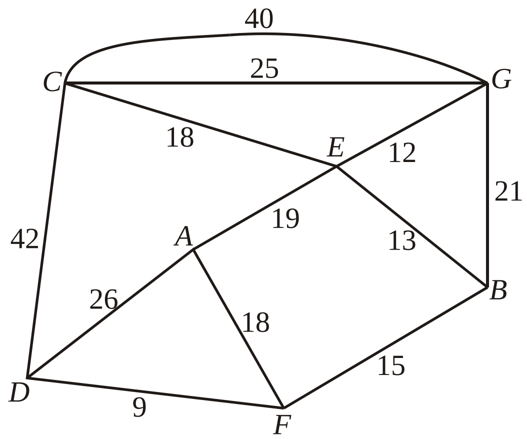
(7)

The local authority decides to build a road bridge over the river at *B*. The snowplough will be able to cross the road bridge.

(b) Reapply the algorithm to find the minimum distance the snowplough will have to travel (ignore the length of the new bridge).

(3)
(Total 10 marks)

14.



(a) Describe a practical problem that could be modelled using the network in the diagram above and solved using the route inspection algorithm.

(1)

(b) Use the route inspection algorithm to find which paths, if any, need to be traversed twice.

(4)

(c) State whether your answer to part (b) is unique. Give a reason for your answer.

(1)

(d) Find the length of the shortest inspection route that traverses each arc at least once and starts and finishes at the same vertex.

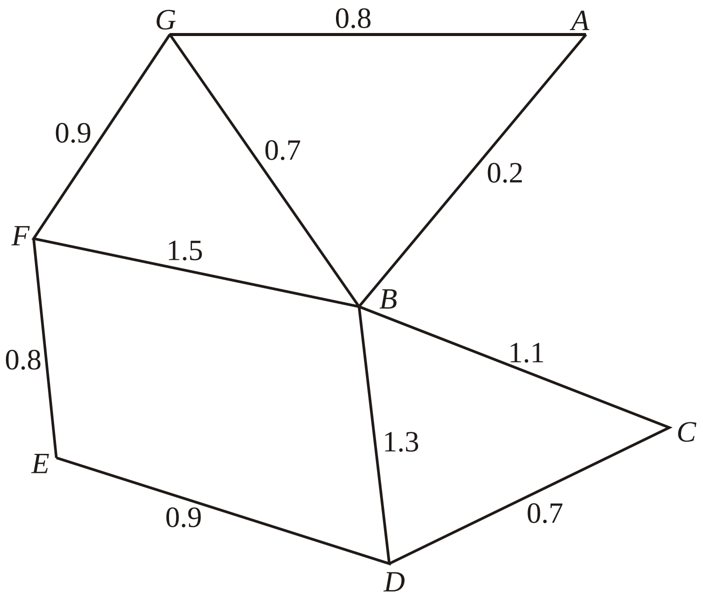
(1)

Given that it is permitted to start and finish the inspection at two distinct vertices,

(e) find which two vertices should be chosen to minimise the length of the route. Give a reason for your answer.

(2)
(Total 9 marks)

15.



An engineer needs to check the state of a number of roads to see whether they need resurfacing. The roads that need to be checked are represented by the arcs in the diagram above. The number on each arc represents the length of that road in km. To check all the roads, he needs to travel along each road at least once. He wishes to minimise the total distance travelled.

The engineer's office is at G, so he starts and ends his journey at G.

(a) Use an appropriate algorithm to find a route for the engineer to follow. State your route and its length.

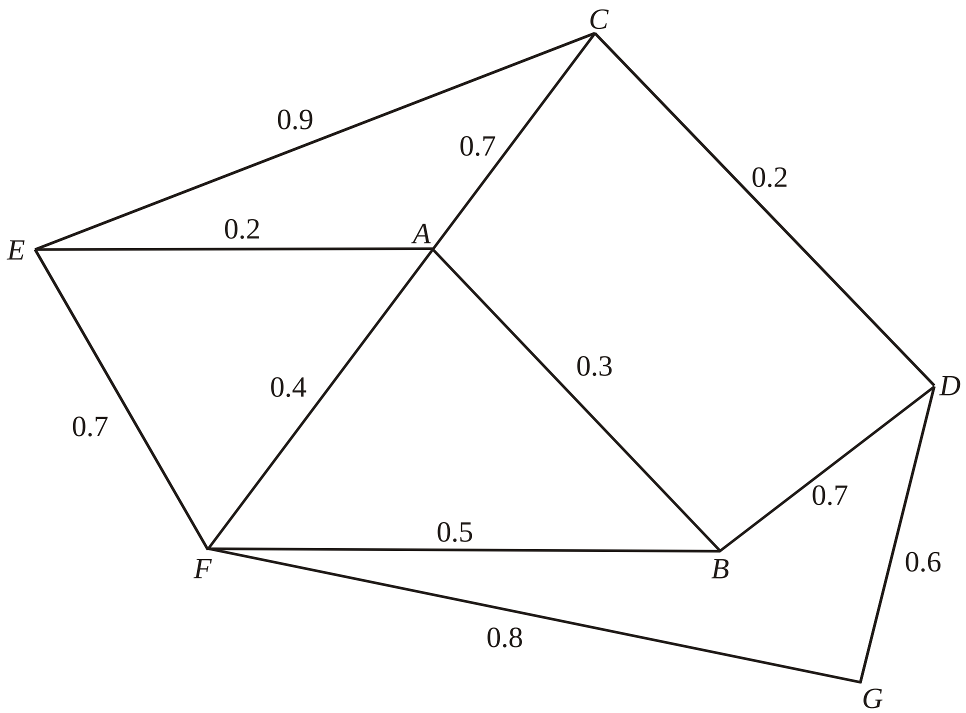
(6)

The engineer lives at D. He believes he can reduce the distance travelled by starting from home and inspecting all the roads on the way to his office at G.

(b) State whether the engineer is correct in his belief. If so, calculate how much shorter his new route is. If not, explain why not.

(3)
(Total 9 marks)

16.



A local council is responsible for maintaining pavements in a district. The roads for which it is responsible are represented by arcs in the diagram above.The junctions are labelled A, B, C, …, G. The number on each arc represents the length of that road in km.

The council has received a number of complaints about the condition of the pavements. In order to inspect the pavements, a council employee needs to walk along each road twice (once on each side of the road) starting and ending at the council offices at C. The length of the route is to be minimal. Ignore the widths of the roads.

(a) Explain how this situation differs from the standard Route Inspection problem.

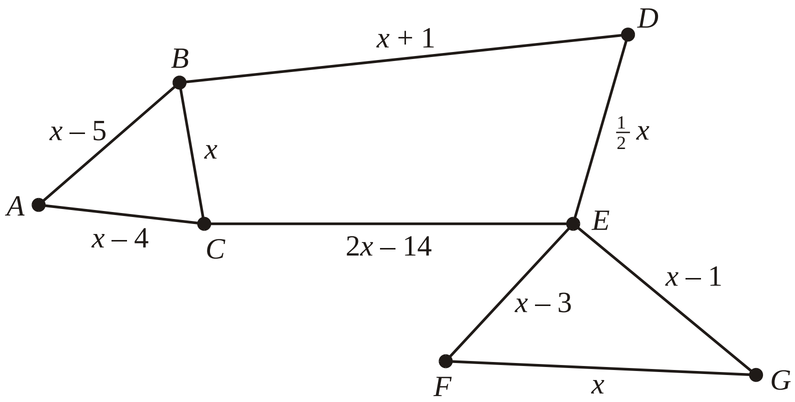
(1)

(b) Find a route of minimum length and state its length.

(3)
(Total 4 marks)

17. (a) Explain why it is impossible to draw a network with exactly three odd vertices.

(2)

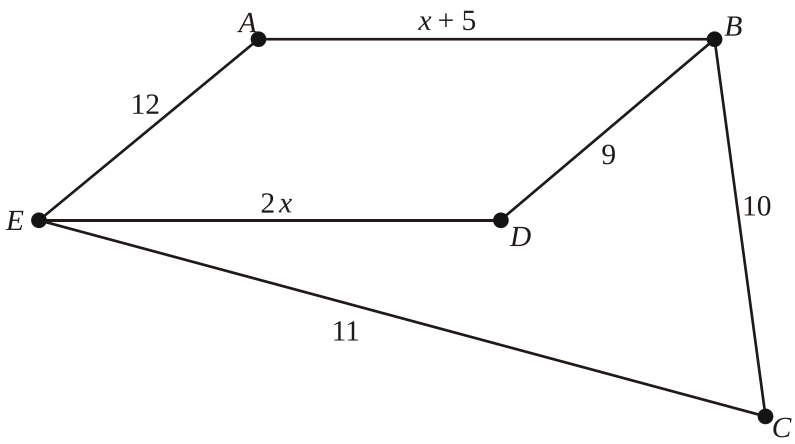


The Route Inspection problem is solved for the network in the diagram above and the length of the route is found to be 100.

(b) Determine the value of x, showing your working clearly.

(4)
(Total 6 marks)

18.



The arcs in the diagram above represent roads in a town. The weight on each arc gives the time, in minutes, taken to drive along that road. The times taken to drive along *AB* and *DE* vary depending upon the time of day.
A police officer wishes to drive along each road at least once, starting and finishing at *A*. The journey is to be completed in the least time.

(a) Briefly explain how you know that a route between *B* and *E* will have to be repeated.

(1)

(b) List the possible routes between *B* and *E*. State how long each would take, in terms of x where appropriate.

(2)

(c) Find the range of values that x must satisfy so that *DE* would be one of the repeated arcs.

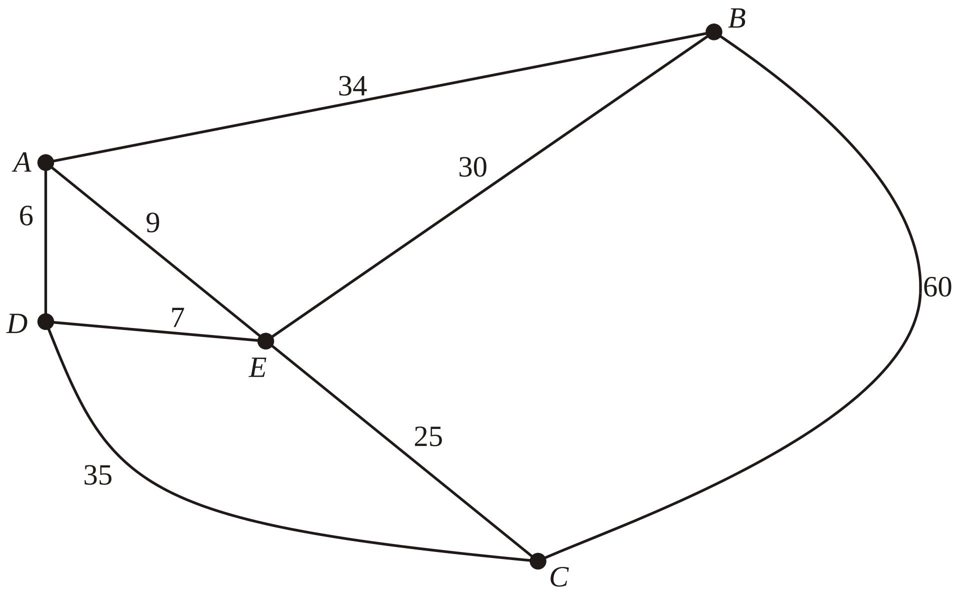
(3)

Given that $x = 7$,

(d) find the total time needed for the police officer to carry out this journey.

(3)
(Total 9 marks)

19.



The diagram above shows the paths in Bill's garden. The length of each path is given in metres. Each morning Bill likes to inspect the garden. He starts and finishes at *A* and traverses each path at least once.

(a) Use the route inspection algorithm to find the minimum distance he must walk. State which paths must be traversed more than once and state a route of minimum length.

(6)

Caroline, a friend of Bill's, wishes to look at the garden covering each path at least once. However, she wants to start at *B* and finish at a vertex other than *B*.

(b) In order for Caroline to walk a minimum distance, determine where she should end her walk and the distance she will cover. Explain your method carefully and give a possible route.

(7)
(Total 13 marks)

1.	(a)	e.g. Each edge contributes 2 to the sum of degree, hence this sum must be even.	B2,1,0		
		Therefore there must be an even (or zero) number of vertices of odd degree			
		Hence there cannot be an odd number of vertices of odd degree		2	
	(b)	$CD + FH = 200 + 220 = 420$	M1A1		
		$CF + DH = 180 + 380 = 560$	A1		
		$CH + DF = 400 + 160 = 560$			
		Repeat *CA*, *AD* and *FH*	A1	4	
	(c)	Length = 4180 + 420 = 4600 m	B1(ft)	1	
					[7]

2.	(a)	BC + EG = 10.4 + 10.1 = 20.5 smallest	M1 A1	
		BE + CG = 8.3 + 16.1 = 24.4	A1	
		BG + CE = 14.9 + 11.9 = 26.8	A1	
		So repeat tunnels BA, AC and EG	A1	5

Note

1M1: Three pairings of their four odd nodes
1A1: one row correct
2A1: two rows correct
3A1: all correct
4A1: correct **arcs** identified

(b)	Any route e.g. ACFGDCABDEGEBA	B1	
	Length = 73.3 + their 20.5 = 93.8km	M1 A1	3

Note

1B1: Any correct route (14 nodes)
1M1: 73.3 + ft their least, from a choice of at least two.
1A1: cao

(c)	The new tunnel would make C and G even.		
	So only BE would need to be repeated.	B1	
	Extra distance would be 10 + 8.3 = 18.3 < 20.5 [91.6 < 93.8]		
	So it would decrease the total distance.	DB1	2

Note

1B1: A correct explanation, referring to BE and relevant numbers (8.3, 12.2, 2.2, 18.3,81.3, 91.6) maybe confused, incomplete or lack conclusion –bod gets B1
2B1D: A correct, clear explanation all there + conclusion (ft on their numbers.)

[10]

3.	(a)	CD + EG = 45 + 38 = 83	M1 1A1	
		CE + DG = 39 + 43 = 82 ←	2A1	
		CG + DE = 65 + 35 = 100	3A1	
		Repeat CE and DG	4A1ft	
		Length 625 + 82 = 707 (m)	5A1ft	6

Note

1M1: Three pairings of their four odd nodes

1A1: one row correct

2A1: two rows correct

3A1: three rows correct

4A1ft: ft their least, but must be the correct shortest route arcs on network. (condone DG)

5A1ft: 625 + their least = a number. Condone lack of m

(b)	DE (or 35) is the **smallest**	M1	
	So finish at C.	A1ft	
	New route 625 + 35 = 660 (m)	A1ft=1B1	3

Note

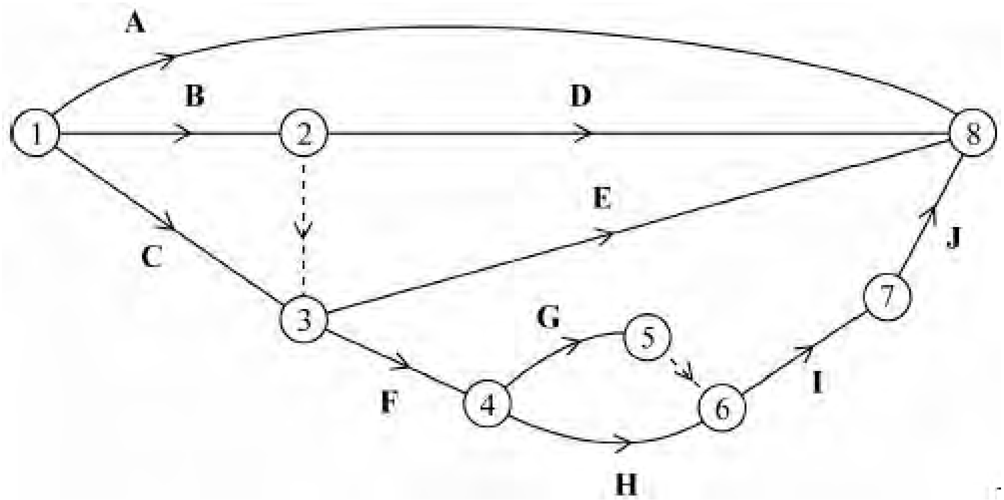
1M1: Identifies their shortest from a choice of at least 2 rows.

1A1ft: ft from their least or indicates C.

2A1ft = 1Bft: correct for their least. (Indept of M mark)

[9]

4. (a)



			M1A1A1A1A1	5

Note

1M1: one start and A to C and one of D, E or F drawn correctly
1A1: 1st dummy (+arrow) and D, E and F drawn correctly
2A1: G, H, I and J drawn in correct place
3A1: second dummy (+arrow) drawn in a correct place
4A1: cso. all arrows and one finish.

	(b)	1st dummy – D depends on B only, but E and F depend on B and C	B1	
		2nd dummy – G and H both must be able to be described uniquely in terms of the events at each end.	B1	2

Note

1B1: cao, but B, C, D, E and/or F referred to, generous
2B1: cao, but generous.

[7]

5.	(a)	Odd vertices C, D, E, G	B1	
		CD + EG = 17 + 19 = 36 ←	M1 A1	
		CE + DG = 12 + 25 = 37		
		CG + DE = 28 + 13 = 41	A1	
		Length = 543 + 36 = 579 (km)	A1ft	5

Note

1B1: cao (may be implicit)
1M1: Three pairings of their four odd nodes
1A1: one row correct
2A1: all correct
3A1ft: 543 + their least = a number. Condone lack of km

	(b)	CE (12) is the shortest	M1	
		So repeat CE (12)	A1ft	
		Start and finish at D and G	A1ft	3

Note

1M1ft: Identifies their shortest from a choice of at least 2 rows.
1A1ft: indicates their intent to repeat shortest.
2A1ft: correct for their least.

[8]

6.	(a)	CD + FG = 0.7 + 0.6 = 1.3*	M1A1	
		CF + DG = 0.5 + 0.9 = 1.4	A1	
		CG + DF = 1.1 + 0.5 = 1.6	A1	
		repeat CD and FG		
		A possible route e.g.		
		A C D C F G F D G E D A G B A	B1	
		length: 11 + 1.3 = 12.3 km	B1	6

	(b)	(i) Each arc has to be traversed twice	B1	1	
		(ii) 2 × 11 = 22 km	B2,0	2	
					[9]
7.	(a)	odd vertices *B*, *D*, *F*, *H*			
		BD + *FH* = 21 + 20 = 41	M1A1		
		BF + *DH* = 19 + 20 = 39 *	A1		
		BH + *DF* = 23 + 18 = 41	A1		
		{Repeat *BE*, *EF*, *DG* and *GH*]			
		Shortest route = 125 + 39 = 164 km	A1ft	5	
	(b)	Seek to keep the least pairing – *DF*/18	B1ft		
		Therefore start/finish at *B* and *H*.	B1ft	2	
					[7]
8.	(a)	e.g. Each edge contributes 2 to the sum of degrees, hence this sum must be even. Therefore there must be an even (or zero) number of vertices of odd degree Hence their cannot be an odd number of vertices of odd degree	B2,1,0	2	
	(b)	CD + FH = 200 + 220 = 420 (*)	M1A1		
		CF + DH = 180 + 380 = 560	A1		
		CH + DF = 400 + 160 = 560			
		repeat CA, AD and FH	A1	4	
	(c)	length = 4180 + 420 ft = 4600 m	B1ft	1	
					[7]
9.	(a)	AC + EG = 44 + 35 = 79	M1		
		AE + CG = 41 + 36 = 77	A1		
		AG + CE = 36 + 45 = 81	A1		
		Repeat AD, DE, CF and FG	A1ft	4	

M1 Three pairs of their odd vertices (different)

A1 One pairing and total correct – i.e. one line correct

A1 All three pairings and totals correct

A1ft Correct arcs identified – must be two pairings to choose from AD DE CF FG

(b) Length = 394 + 77 = 471km B1ft 1

B1 471 (km) 394 + their shortest – must be two pairings to choose from

(c) Since EG is the smallest, choose to repeat this. M1

Hence start and finish at A and C. A1ft 2

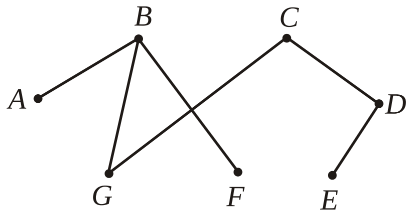
M1 Identifies {35 EG} as smallest – or identify <u>their</u> smallest from two+ pairings, two totals

A1ft from two+ pairings and totals

[7]

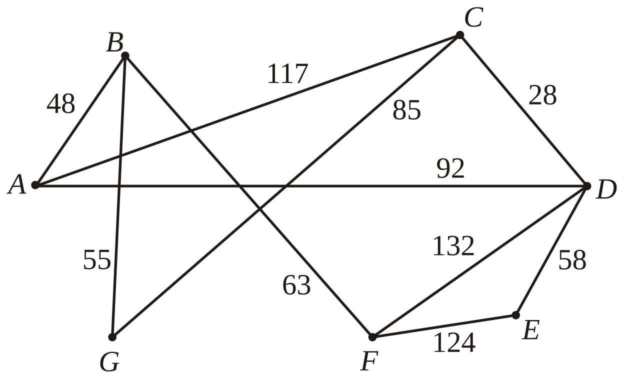
10. (a) AB, BG, BF | GC, CD, DE {1 2 5 6 7 4 3} M1 A1 A1 3

weight 337 m B1



B1ft 2

(b)



M1 A1 A1 3

(c) AB + CF = 48 + 160 = 208 M1 A1

AC + BF = 117 + 63 = 180 A1

AF + B C = 111 + 140 = 251 A1 4

e.g. A $\overline{B\ \underline{F}}\ \underline{B}$ G $\underline{C\ \overline{A}}\ \overline{C}$ D E F D A A1

length 802 + 180 = 982 m M1 A1ft 3

[15]

11. (a) $AC + DF = 8 + 9 = 17 \leftarrow$ M1 A1 A1 3

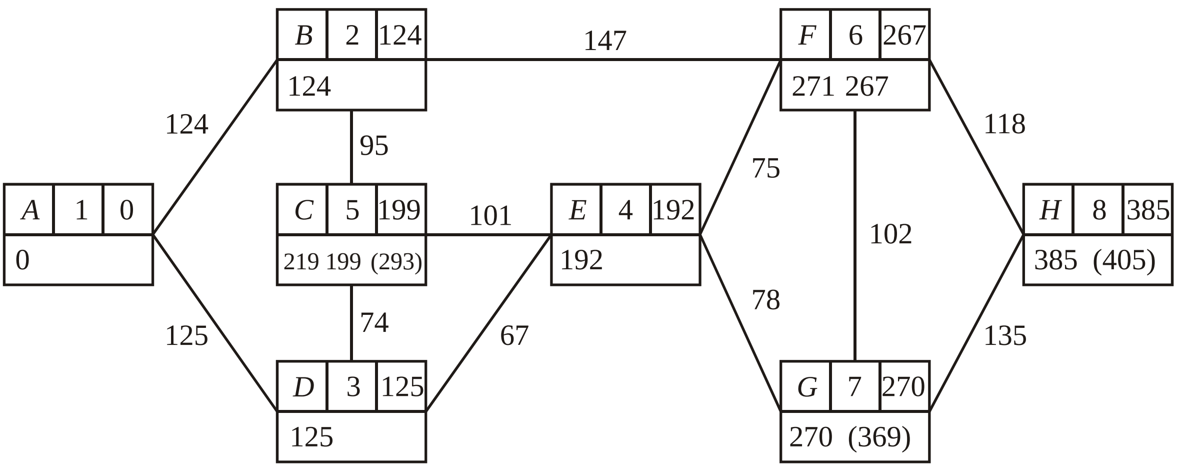
$AD + CF = 15 + 16 = 31$

$AF + CD = 13 + 7 = 20$

	length = 77 + 17 = 94 km	M1 A1ft	2
	M1 3 pairs of 4 odd vertices (different) A C D F		
	A1 2 pairs + "total" correct		
	A1 all 3 pairs + totals correct 17 31 20		
	M1 77 + <u>their</u> shortest or plausible list		
	A1ft cao <u>+km</u>		
(b)	Shortest arc is *CD* (7) so use *A* and *F* as end points	B2,1,0	2
	B2 CD identified as the smallest or arc to be repeated <u>and</u> A + F stated as end points		
	B1 <u>either</u> CD identified as the smallest or arc to be repeated <u>or</u> A + F stated as end points "bad" gets B1 <u>or</u> picks smallest out of least 4 √routes		

[7]

12. (i)



M1 A1 A1ft A1ft

	<u>shortest distance is 385 m</u>	A1	5
(ii)	Odd vertices B, C, D, G	m1	
	BC + DG = 95 + 145 = 240 (*)	A1	
	BD + CG = 169 + 179 = 348	A1	
	BG + CD = 249 + 74 = 323	A1	4
	Repeat BC, DE and EG		
	eg. A B C B F H G F E G E C D E D A	B1	
	length 1241 + 240 = <u>1481 m</u>	B1	2

[11]

13. (a)	$B_1G + B_2E = 26 + 30 = 56$	M1A1	
	$B_1B_2 + EG = 65 + 18 = 83$	A1	
	$B_1E + B_2G = 41 + 42 = 83$	A1	4

		Repeat B,D , DG , B_2A AE			
		Route e.g. F A B_2 A C E A E F D B_1 D H G D G F	B1		
		length = 129 + 56 = 185 km	M1A1ft	3	
	(b)	now only E and G are odd – repeat EF, FG only	B1		
		length = 129 + 18	M1A1	3	
		= 47 km			
					[10]
14.	(a)	Idea of travelling along each arc at least once and seeking to do so in a minimum total. Practical meaning of arcs/numbers.	B1	1	
	(b)	$AB + DF = 32 + 9 = 41$	M1 A1		
		$AD + BF = 26 + 15 = 41$			
		$AF + BD = 18 + 24 = 42$	A1		
		Repeat either $A(E + E)B$ and DF or AD and BF	A1 ft	4	
	(c)	Not unique, e.g. gives other solution	A1 ft		
	(d)	258 + 41 = 299	B1	2	
	(e)	DF is the shortest so start/finish at A/B	M1 A1	2	
					[9]
15.	(a)	BD + FG = 1.3 + 0.9 = 2.2 *	M1		
		BF + DG = 1.5 + (1.3 + 0.7) = 3.5	A1		
		BG + DF = 0.7 + (0.9 + 0.8) = 2.4	A1	3	
		Repeat BD and FG			
		Route e.g. GABCDBFEDBGFG	B1		
		Length = 8.9 + 2.2 = 11.1 km	M1 A1	3	
	(b)	Only now need to repeat BF of length 1.5 < 2.2	M1 A1 ft		
		Length = 8.9 +1.5 = 10.4 km saving 0.7 (km)	A1 ft	3	
					[9]
16.	(a)	All arcs must be traversed twice. (So no arc needs repeating more than twice.) All valencies therefore even.	B1	1	
	(b)	e.g. CECAEFEAFABFBACDBDGFGDC	M1 A1		
		length = 2 × 6			
		= 12 km	A1	3	
					[4]

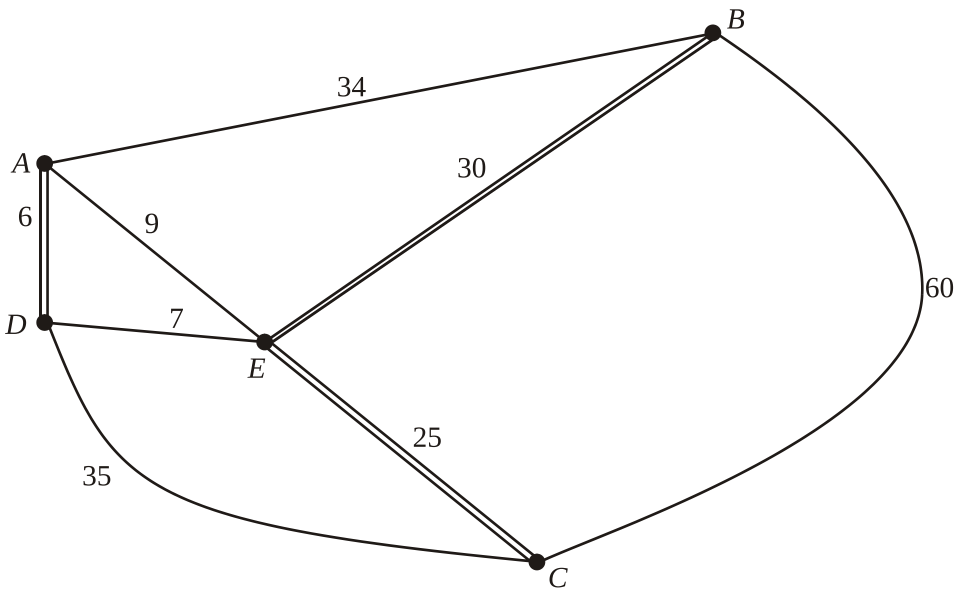
17.	(a)	Each arc contributes 2 to the sum of degrees, hence this sum must be even. Therefore there must be an even (or zero) number of vertices of odd degree.	B2, 1, 0	2
	(b)	If $x > 9$, $10\frac{1}{2}x - 26 = 100, \Rightarrow x = 12$	B1, M1 A1	4
		(If $x < 9$, $11\frac{1}{2}x - 35 = 100 \Rightarrow x = 11\frac{17}{23}$ inconsistent)		

[6]

18.	(a)	*B* and *E* are the only odd vertices, repeating a route between them will make them even	B1	1
	(b)	$BA + AE = 17 + x$ $BD + DE = 2x + 9$ $BC + CE = 21$	M1 A1	2
	(c)	$2x + 9 < x + 17$ and $2x + 9 < 21$ $x < 8$ and $x < 6$	M1 A1	
		$\therefore\ 0 < x < 6$ for both to be true in context	A1	3
	(d)	If $x = 7$, repeated route is $BC + CE$	B1	
		Total time is $(3(7) + 47) + 21 = 89$	M1 A1	3

[9]

19. (a)



Odd vertices are *A* (3), *B* (3), *C* (3), *D* (3). B1

Possible pairings	Shortest distances			
(*A* & *B*) (*C* & *D*)	34	+	32	=66
	(*AB*)		(*CED*)	
(*A* & *C*) (*B* & *D*)	34	+	37	=71
	(*AEC*)		(*BED*)	

(A & D) (B & C)	6	+	55	=61	(*)
	(AD) (BEC)				

So we repeat edges AD, BE and EC. The length of the minimum route is

(6 + 9 + 7 + 34 + 30 + 60 + 25 + 35) + 61;

206 + 61 = 267 m A1

A route of this length is $A\ B\ C\ E\ B\ E\ C\ D\ A\ E\ D\ A$ B1 6

(b) If her walk starts at B and finishes at any other vertex then that vertex must be odd. B1

(i) If the other vertex is B then the distance is that found above.

(ii) If the other vertex is A then she will have to repeat edges CED, length 32.

(iii) If the other vertex is C then she will have to repeat edge AD, length 6.

(iv) If the other vertex is D then she will have to repeat edges AEC, length 34. M1 A3 (–1 eeoo)

Therefore to cover the minimum distance she will have to start at B and finish at C. The length of her route will be 206 + 6 = 212. B1

A possible route is $B\ A\ D\ C\ E\ D\ A\ E\ B\ C$ B1 7

[13]

1. No Report available for this question.

2. Around 70% of the candidates gained at least 7 marks.

In part (a) most candidates were able to state three pairings of the four correct odd vertices, and did so with pleasing few arithmetical errors, far fewer candidates realised that the path BC comprised of arcs BA and AC.

In part (b) most candidates calculated the length of the route correctly but many did not list a route.

Many candidates were able to gain at least one mark in (c) but only the more able secured both marks. Candidates needed to refer to BE specifically and present a numerical argument.

3. Almost all candidates found three pairings of the correct odd nodes, but few found all three correct totals. The answers 85, 83 and 112 were very common. Some weaker candidates listed all six couples, without pairing them, gaining no marks. After identifying the shortest arcs from their parings most candidates were able to calculate the length of the shortest route, but some wasted time listing the shortest route. Examiners were pleased that more candidates then previously were able to make progress in part (b). A significant number failed to indicate, when choosing DE, that it was the shortest route between two odd nodes. Some incorrectly chose EC at 39 as the shortest, since it was in their current route. Some candidates lost a mark by not working out 660 even if they had got the other two marks in this part.

4. There were many good attempts seen to part (a), which can be a challenging topic for candidates. The most common errors were: failing to have one end point; missing arrows – especially important on the dummies; omitting an activity, often J. Some candidates used activity on node. In part (b) candidates struggled to give good explanations for the two dummies, so full marks were very rarely awarded. Many did not give enough detail of the activities involved in the first dummy and did not make it clear that each activity has to be uniquely expressible in terms of its end events.

5. Part (a) was very well done in general, with only a few slips. The most common was CD + EG = 44, and a few omitted the totals for each pairing. Some candidates used one or more even vertex, but most found the three pairings efficiently and concisely, with only a very few failing to pair up the six separate paths. A number of candidates did not read the question carefully and wasted time finding a route. Part (b) was less well done. Many candidates only considered arcs CD and EG (from part (a)), others chose C and G so that they could eliminate the longest path, others chose G and D because they had the 'highest valencies', others chose C and E saying that the path between them was the shortest (which is correct, but therefore CE should be the path chosen to be repeated).

6. (a) Most candidates found this question an excellent source of marks. The majority were able to find the three pairings of the four odd nodes and only the weakest listed their arcs without pairing them. Most were able to determine a suitable route and find its length correctly. Part (b) caused difficulty for some candidates, particularly in part (i). Poor use of technical terms caused problems for candidates trying to make their meaning clear, others stated that each arc would be traversed 'at least twice' and some indicated that some arcs would have to be traversed four times. Most candidates gained the last two marks for 22km, with only a few omitting the units, but some incorrectly doubled their route length from part (a).

7. The three pairings were successfully found by most candidates, the majority of whom went on to find the correct lengths too, with a small but significant number getting 40 instead of 39. A lot of time was wasted on drawing the network and giving the actual route, and this occasionally resulted in failure to give the total length. Few candidates got both marks in part (b), most just looked at their repeated paring, or identified BH as the largest distance, or AB as the smallest edge on the graph. Poor notation was often seen here, such as reference to 'the vertice DF'.

8. Whilst some very good, concise answers to (a) were seen, the majority of candidates had great difficulty, the use of technical terms was poor with many confusing vertices and edges. Many made reference to the handshaking lemma but then did not explain its relevance or made contradictory statements. Part (b) was often well done, although some candidates did not consider all three pairings of the odd vertices. Many candidates did not spot that the shortest route between C and D was via A. Some candidates wasted time seeking a route, when only the length was required.

9. Parts (a) and (b) proved very successful for most candidates. A substantial minority failed to identify the correct four odd vertices, either by miscounting or some, having listed the all the valencies, selected an even vertex. Similarly some did not select their least pairing, but selecting the pairing using the least number of edges. Candidates were required to list the arc to

10. This proved an accessible question, but careless slips resulted in few gaining full credit. Not all candidates used Prim's algorithm correctly and of those that did, some did not clearly state the order in which the edge were selected – as directed in the question. A disappointing number of candidates used the nearest neighbour algorithm instead of Prim's algorithm. Many omitted to state the weight of the tree. The vast majority were able to draw the network in part (b). The majority did three pairings but many did not find all the shortest routes between the pairs. Some discarded any route involving more than one edge and some did not state the totals of their pairings. A few did not state a route and some found a semi-Eulerian solution.

11. The first two parts of this question were well answered with most candidates identifying the three pairings and their weights. Many forgot the units when stating the length of the minimum route and some wasted time by listing their route. Only the better candidates were able to answer part (c) correctly – some tried to exclude the longest arc in the pairings rather than seeking to include the shortest and some only considered the two arcs used in their initial shortest route. Some correctly identified CD as the arc they should repeat and some stated that they should start and finish at A and F, but only the best gave both parts for a complete answer.

12. This was mostly well done, but Dijkstra's algorithm had to be very carefully applied to gain full marks. Common errors were to award a final label to vertex C before vertex E and an incorrect order of working values. Some candidates did not write down the shortest distance, but wrote down the shortest route instead. Some candidates did not realise what was being asked in part (ii) and explained how they achieved their shortest route from their labelled diagram. The majority, who did attempt the route inspection, were usually fairly successful. The commonest error was to state that the pairing BG + CD = 348. Most were able to state a correct route and its length.

13. Many good answers were seen to part (a), but some candidates did not make their method clear and just listed a route. Most were able to list the three pairings of the 4 odd vertices however. Part (b) was generally well-answered.

14. Many candidates found part (a) tricky – with a sizable minority describing the travelling salesman problem, others did not find a suitable context, or state the need for a minimum route. This was usually well answered although some candidates lost marks by not giving all three pairings of odd nodes. Parts (c) and (d) were usually well answered. Only the better candidates were able to answer part (e) correctly – some tried to exclude the longest arc in the pairings rather than seeking to include the shortest, of those who correctly identified DF as the arc they should repeat, some did not then go on to answer the question and say that they should therefore start and finish at A and B.

15. This was well answered by most candidates, although part (b) caused difficulties for some. Most candidates applied the route inspection algorithm correctly with only a few failing to explore all three pairings of odd nodes; almost all the candidates were able to find a correct route and its length. In part (b) those candidates who correctly identified BF as the only path that needed to be repeated were usually successful, although once again a number did not state a conclusion.

16 Most candidates were able to answer part (a) successfully, stating that all arcs needed to be traversed twice, although some confused arcs and vertices. In part (b) however very few candidates acted on their answer to part (a) and most solved the standard route inspection problem, often making errors in pairings. Many candidates did not state a route.

17. A number of candidates found this a tough question especially (by virtue of it total mark) occurring so early in the paper, and a number of candidates gave evidence of spending t long on this question. Complete clear answers to part (a) were rarely seen, although most candidates were able to score some credit. A large number tried to argue that three odd nodes were impossible because it would then not be possible to use the route inspection algorithm. A significant number were able to state that a network must have an even number of odd nodes but few were able to explain the reason behind this by equating the sum of the degrees and twice the number of edges. In part (b) all but the very strongest candidates assumed that the least route form B to C was x, but then most continued correctly, although some omitted to repeat the route with $9\frac{1}{2}x - 26 = 100$ being a very popular incorrect answer. A surprising number of candidates had difficulty solving their linear equation, often making errors in collecting the number term. The more able candidates were able to compare the different routes between B and C and verify that the shortest consistent route was indeed $x = 12$.

18. Most candidates were able to complete parts (a) and (b) correctly. However very few were able to set up and solve the correct inequalities in part (c), and those that did often did not state the full solution. Many candidates did not identify the correct set of repeated arcs in part (d) with both the routes *BAE* and *BDE* being much more popular than the correct repeated route.

19. No Report available for this question.