1.

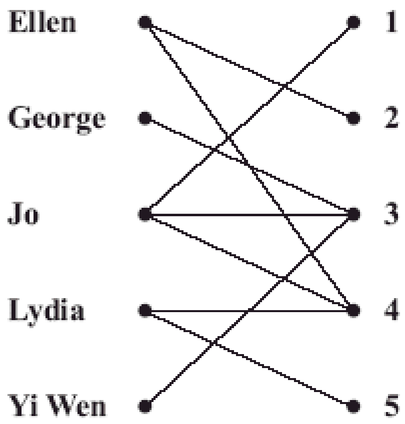


Figure 1

Figure 2

Figure 1 shows the possible allocations of five people, Ellen, George, Jo, Lydia and Yi Wen to five tasks, 1, 2, 3, 4 and 5.

Figure 2 shows an initial matching.

(a) Find an alternating path linking George with 5. List the resulting improved matching this gives.

(3)

(b) Explain why it is not possible to find a complete matching.

(1)

George now has task 2 added to his possible allocation.

(c) Using the improved matching found in part (a) as the new initial matching, find an alternating path linking Yi Wen with task 1 to find a complete matching. List the complete matching.

(3)
(Total 7 marks)

2.

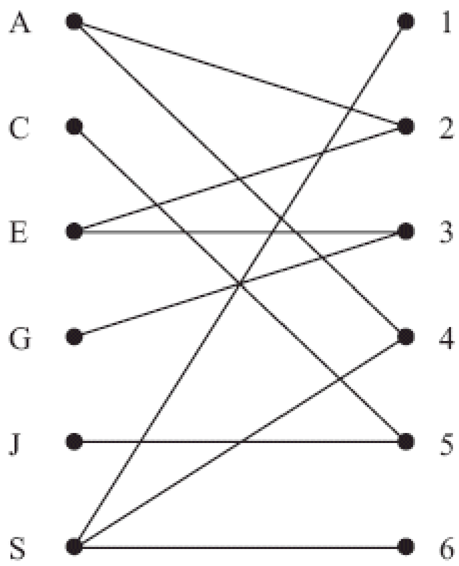


Figure 1

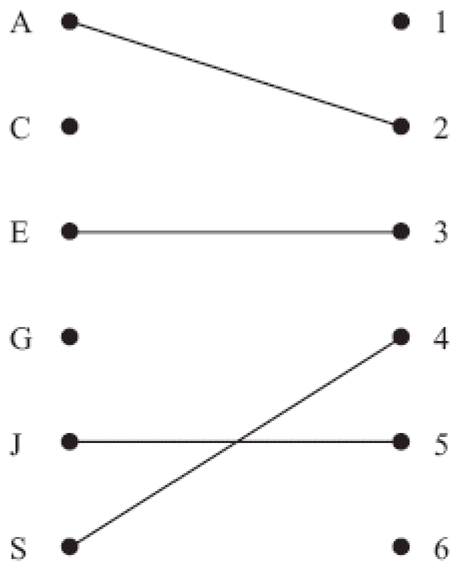


Figure 2

Figure 1 shows the possible allocations of six people, Amelia, Charlie, Ellie, Gemma, Jimmy and Saskia, to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 2 shows an initial matching.

(a) Use the maximum matching algorithm once to find an improved matching. You must state the alternating path used and your improved matching.

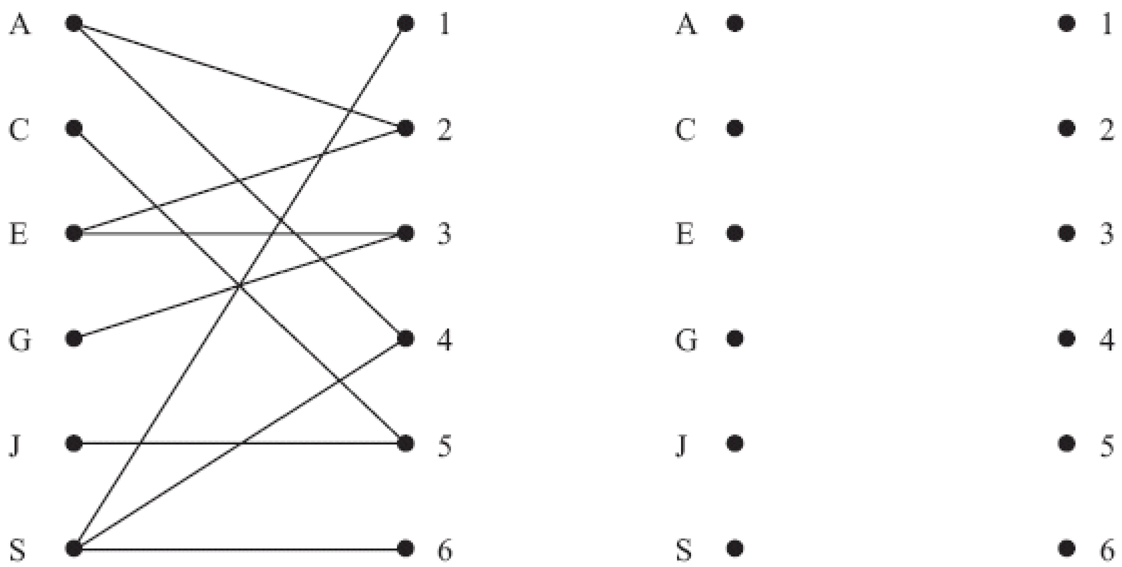
(3)

(b) Explain why a complete matching is not possible.

(2)

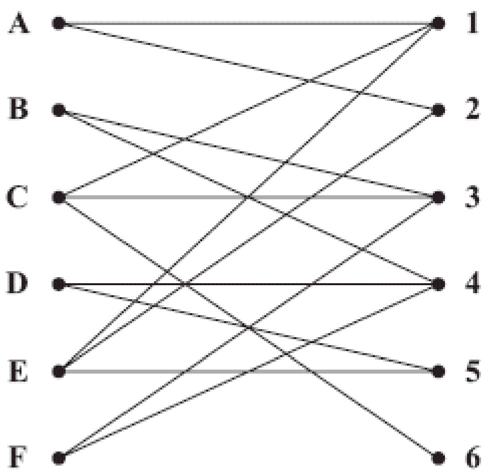
After training, Jimmy can be assigned to tasks 4 or 5 and Ellie to tasks 2, 3, 5 or 6.

(c) **Starting with your current maximal matching**, use the maximum matching algorithm to obtain a complete matching. You must state the alternating path used and your final matching.



(3)
(Total 8 marks)

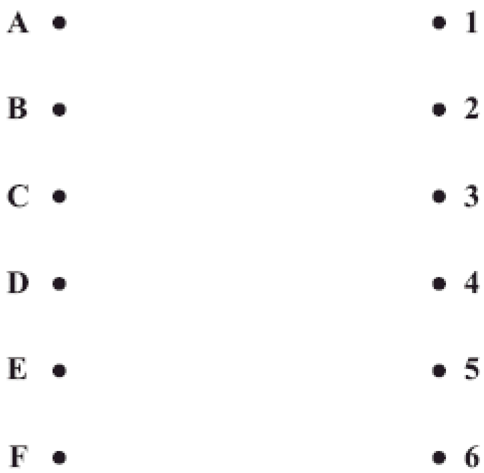
3.



The diagram above shows the possible allocation of six people, Alice (A), Brian (B), Christine (C), David (D), Elizabeth (E) and Freddy (F), to six tasks, 1, 2, 3, 4, 5 and 6.

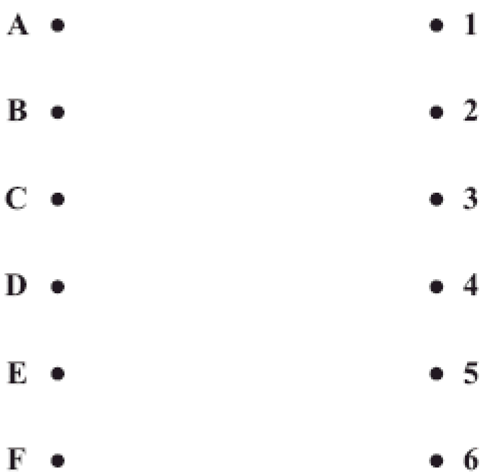
An initial matching is Alice to task 1, Christine to task 3, David to task 4 and Elizabeth to task 5.

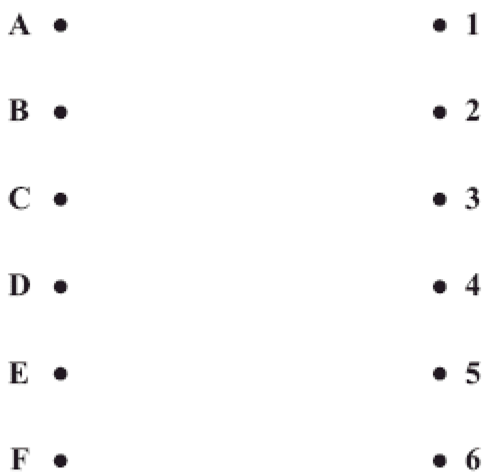
(a) Show this initial matching on the diagram below.



(1)

(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating paths that you use, and give your final matching.





(5)
(Total 6 marks)

4.

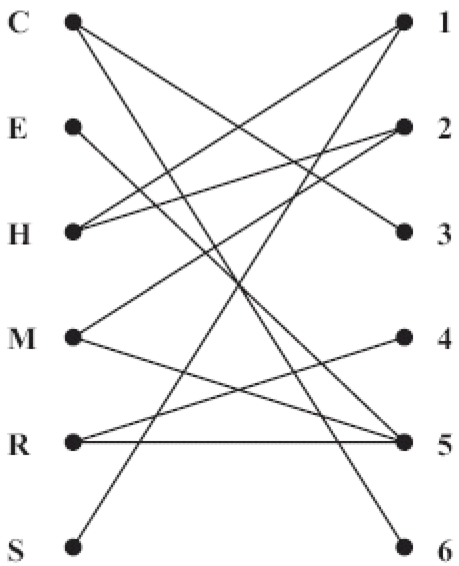


Figure 1

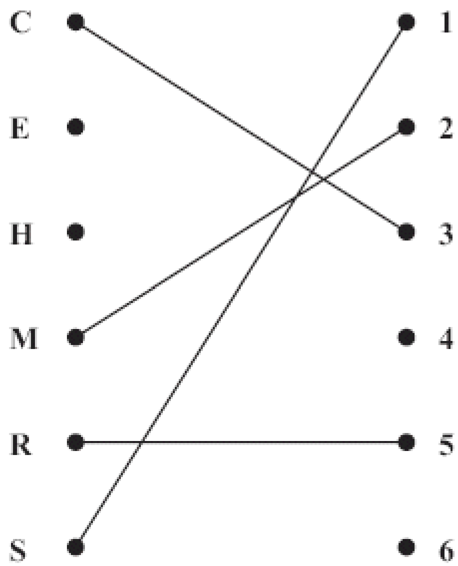


Figure 2

Figure 1 shows the possible allocations of six workers, Charlotte (C), Eleanor (E), Harry (H), Matt (M), Rachel (R) and Simon (S) to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 2 shows an initial matching.

(a) List an alternating path, starting at H and ending at 4. Use your path to find an improved matching. List your improved matching.

(3)

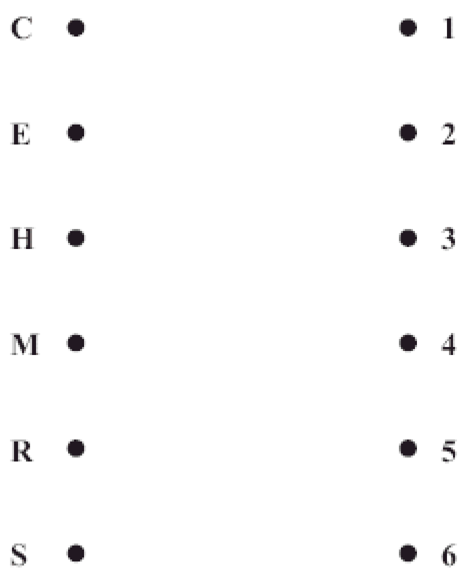
(b) Explain why it is not possible to find a complete matching.

(1)

Simon (S) now has task 3 added to his possible allocation.

(c) Taking the improved matching found in (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating path you use and your complete matching.

(3)
(Total 7 marks)



5.

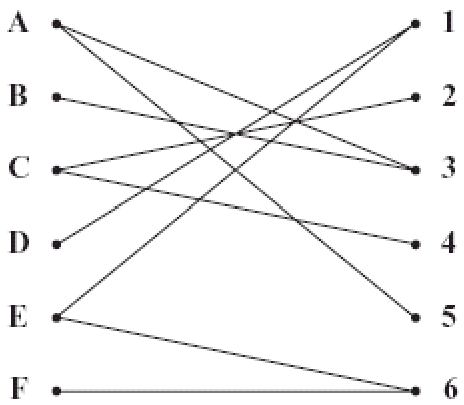


Figure 1

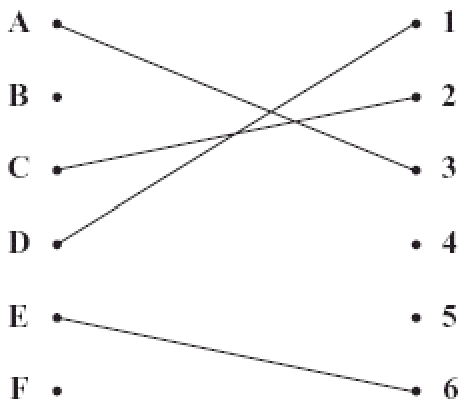


Figure 2

Figure 1 shows the possible allocations of six people, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 2 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You must list the alternating path used, and your improved matching.

(3)

(b) Explain why it is not possible to find a complete matching.

(2)

D now has task 2 added to their possible allocation.

(c) Using the improved matching found in part (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. You must list the alternating path used and your complete matching.

(3)
(Total 8 marks)

6.

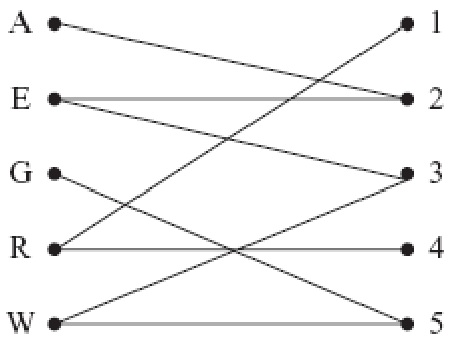


Figure 1

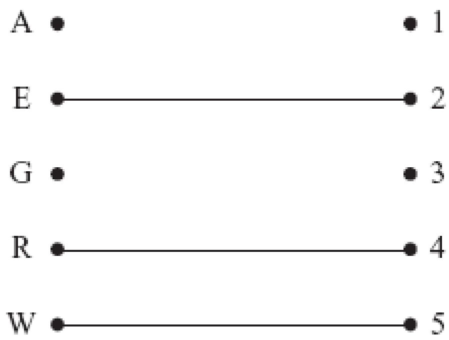


Figure 2

Five tour guides, Alice, Emily, George, Rose and Weidi, need to be assigned to five coach trips, 1, 2, 3, 4 and 5. A bipartite graph showing their preferences is given in Figure 1 and an initial matching is given in Figure 2.

(a) Use the maximum matching algorithm, starting with vertex G, to increase the number of matchings. State the alternating path you used.

(2)

(b) List the improved matching you found in (a).

(1)

(c) Explain why a complete matching is not possible.

(2)

Weidi agrees to be assigned to coach trip 3, 4 or 5.

(d) **Starting with your current maximal matching**, use the maximum matching algorithm to obtain a complete matching.

(3)

(Total 8 marks)

7. (a) Define the terms

(i) alternating path,

(2)

(ii) matching.

(2)

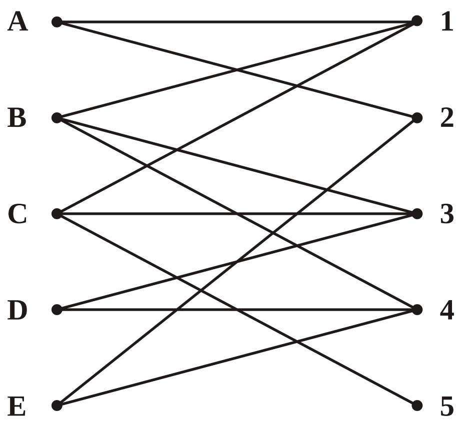


Figure 1

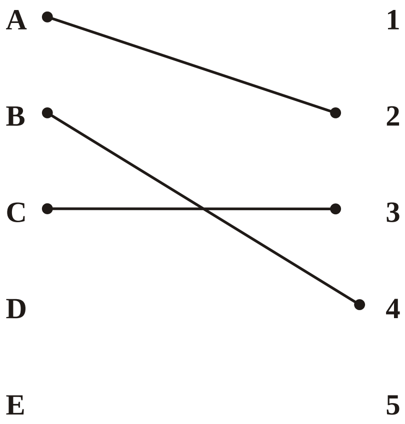


Figure 2

At a school fair, five teachers, *A*, *B*, *C*, *D* and *E*, are to supervise five stalls, 1, 2, 3, 4 and 5.

A bipartite graph showing their possible allocations is given in Figure 1. An initial matching is given in Figure 2.

(b) Use the maximum matching algorithm twice to obtain a complete matching.
List clearly the alternating paths you use.

(5)
(Total 9 marks)

8.

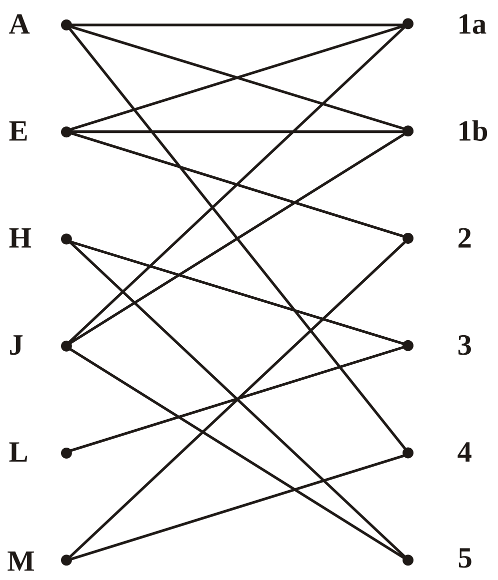


Figure 1

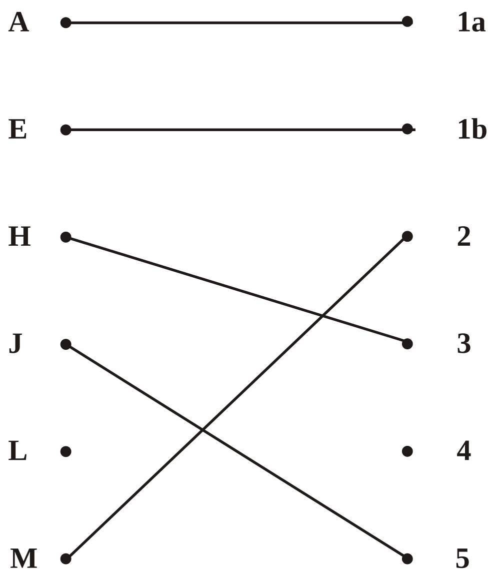


Figure 2

Six workers, Annie, Emma, Hannah, Jerry, Louis and Morand, are to be assigned to five tasks, 1,2,3,4 and 5.

For safety reasons, task 1 must be done by two people working together.A bipartite graph showing the possible allocations of the workers is given in Figure 1 and an initial matching is given in Figure 2.

The maximum matching algorithm will be used to obtain a complete matching.

(a) Although there are five tasks, six vertices have been created on the right hand side of each bipartite graph. Explain why this is necessary when applying this algorithm.
(2)

(b) Find an alternating path and the complete matching it gives.
(3)

Hannah is now unable to do task 5 due to health reasons.

(c) Explain why a complete matching is no longer possible.
(2)
(Total 7 marks)

9.

Figure 1

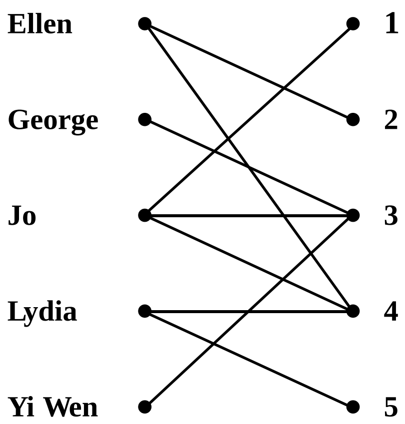


Figure 2

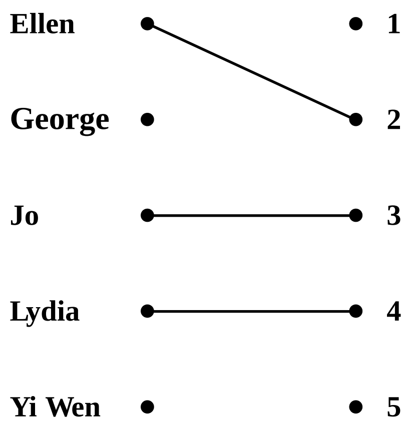


Figure 1 shows the possible allocations of five people, Ellen, George, Jo, Lydia and Yi Wen to five tasks, 1, 2, 3, 4 and 5.

Figure 2 shows an initial matching.

(a) Find an alternating path linking George with 5. List the resulting improved matching this gives.

(3)

(b) Explain why it is not possible to find a complete matching.

(1)

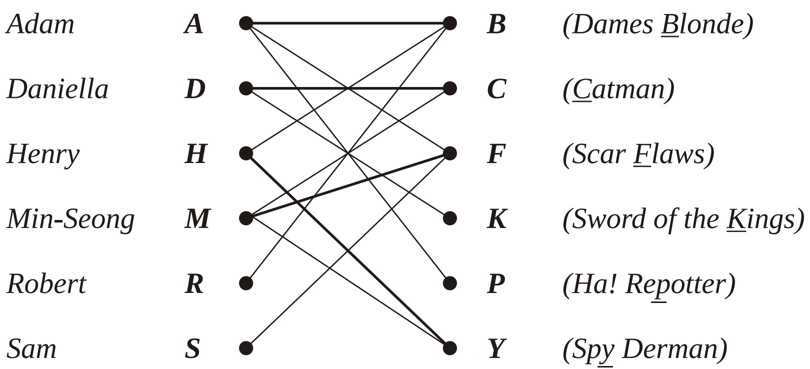
George now has task 2 added to his possible allocation.

(c) Using the improved matching found in part (a) as the new initial matching, find an alternating path linking Yi Wen with task 1 to find a complete matching. List the complete matching.

(3)
(Total 7 marks)

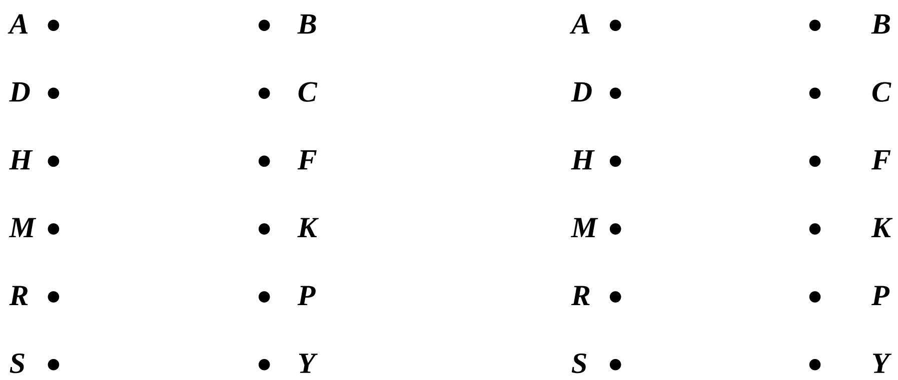
10. (a) Define the term ‘alternating path’.

(2)



The bipartite graph in the figure above shows the films that six customers wish to hire this Saturday evening. The shop has only one copy of each film. The bold lines indicate an initial matching.

(b) Starting from this initial matching use the maximum matching algorithm twice to obtain a complete matching. You should clearly state the alternating paths that you use.



(5)
(Total 7 marks)

11.

Figure 1

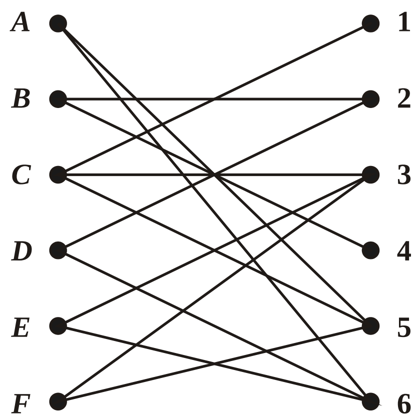
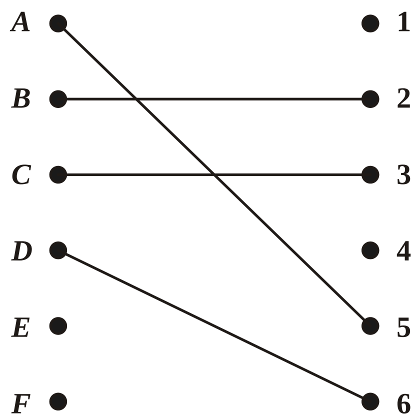


Figure 2



A taxi firm has six taxis *A*, *B*, *C*, *D*, *E* and *F*, available for six journeys, 1, 2, 3, 4, 5 and 6, which are booked for 9 am tomorrow.

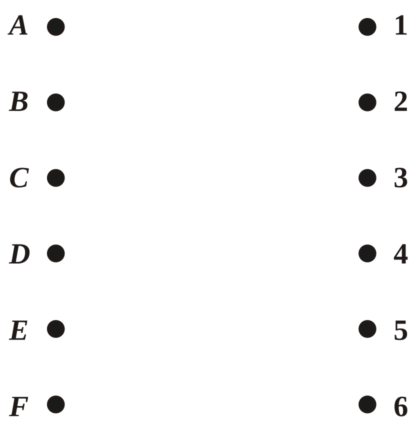
The bipartite graph shown in Figure 1 shows the possible matchings.

Initially *A*, *B*, *C* and *D* are matched to 5, 2, 3 and 6 respectively, as indicated in Figure 2.

(a) Explain why it is necessary to perform the maximum matching algorithm twice in order to try to obtain a complete matching.

(1)

(b) Use the maximum matching algorithm twice to obtain a complete matching. List clearly the alternating paths you use.



(6)
(Total 7 marks)

12.

Figure 1

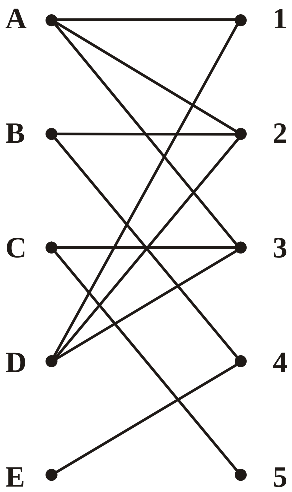
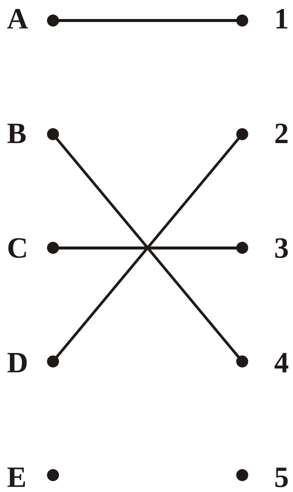


Figure 2



A film critic, Verity, must see five films A, B, C, D and E over two days.

The films are being shown at five special critics' preview times:

1 (Monday 4 pm),
2 (Monday 7 pm),
3 (Tuesday 1 pm),
4 (Tuesday 4 pm),
5 (Tuesday 7 pm).

The bipartite graph in Figure 1 shows the times at which each film is showing.

Initially Verity intends to see

Film A on Monday at 4 pm,
Film B on Tuesday at 4 pm,
Film C on Tuesday at 1 pm,
Film D on Monday at 7 pm.

This initial matching is shown in Figure 2.

Using the maximum matching algorithm and the given initial matching,

(a) find two distinct alternating paths and complete the matchings they give.

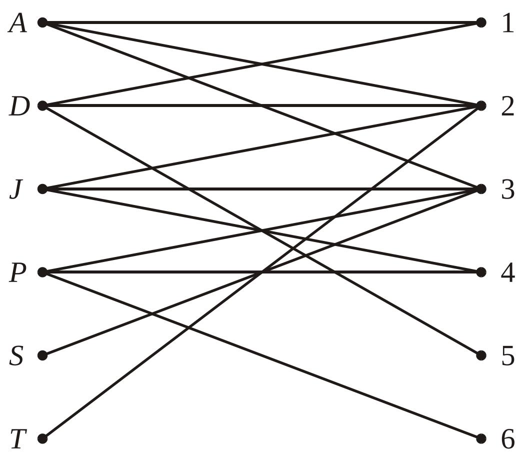
(6)

Verity's son is very keen to see film D, but he can only go with his mother to the showing on Monday at 7 pm.

(b) Explain why it will not be possible for Verity to take her son to this showing and still see all five films herself.

(2)
(Total 8 marks)

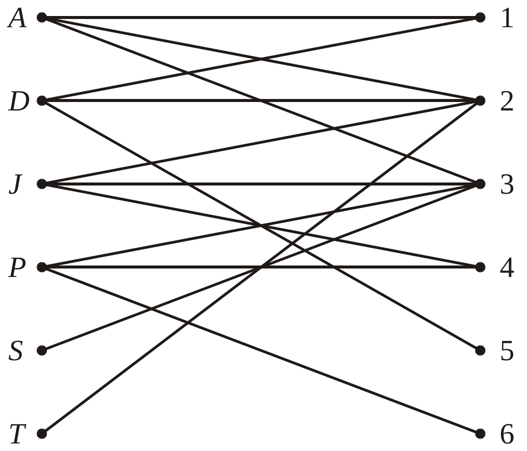
13.

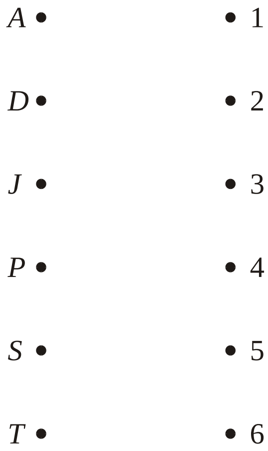


The bipartite graph in the diagram shows a mapping between six people, Andy (A), David (D), Joan (J), Preety (P), Sally (S) and Trevor (T), and six tasks, 1, 2, 3, 4, 5 and 6.

The initial matching is A to 2, D to 1, J to 3 and P to 4.

(a) Indicate this initial matching in a distinctive way on the bipartite graph below.





(1)

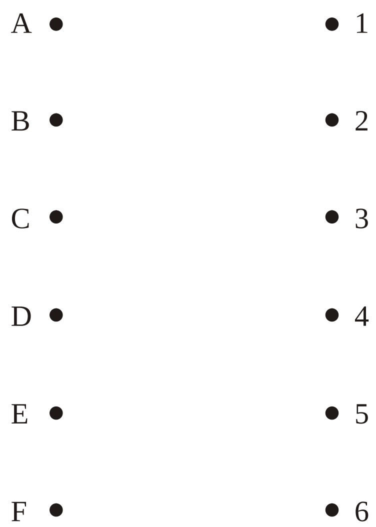
(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating paths you use.

(5)
(Total 13 marks)

14. Six newspaper reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E) and Fred (F), are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign (4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. For example, Chris can be assigned to any one of stories 1, 2 or 4.

	1	2	3	4	5	6
A					✓	
B	✓			✓		
C	✓	✓		✓		
D					✓	
E			✓		✓	✓
F				✓		

(a) Show these possible allocations on the bipartite graph on the diagram below.



(1)

A possible matching is

A to 5, C to 1, E to 6, F to 4

(b) Show this information, in a distinctive way, on the diagram.

(1)

(c) Use an appropriate algorithm to find a maximal matching. You should list any alternating paths you have used.

(4)

(d) Explain why it is not possible to find a complete matching.

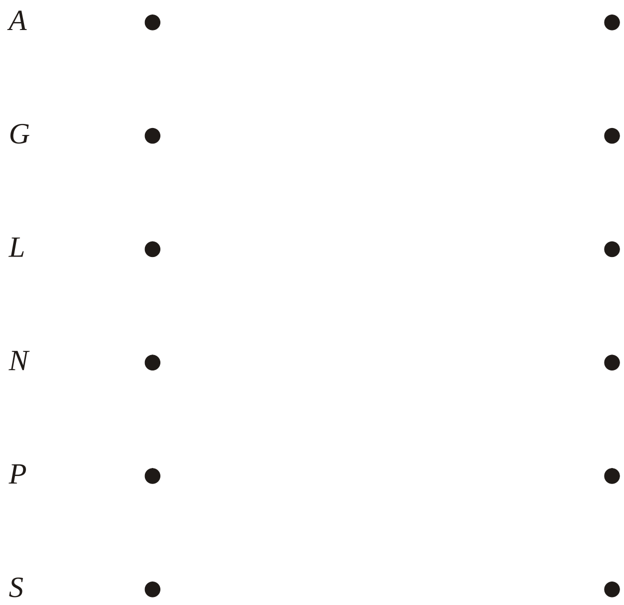
(2)
(Total 8 marks)

15. The organiser of a sponsored walk wishes to allocate each of six volunteers, Alan, Geoff, Laura, Nicola, Philip and Sam to one of the checkpoints along the route. Two volunteers are needed at checkpoint 1 (the start) and one volunteer at each of checkpoint 2, 3, 4 and 5 (the finish). Each volunteer will be assigned to just one checkpoint. The table shows the checkpoints each volunteer is prepared to supervise.

Name	Checkpoints
Alan	1 or 3
Geoff	1 or 5
Laura	2, 1 or 4
Nicola	5
Philip	2 or 5
Sam	2

Initially Alan, Geoff, Laura and Nicola are assigned to the first checkpoint in their individual list.

(a) Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.



(2)

(b) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. Clearly list any alternating paths you use.

(3)

(c) Explain why it is not possible to find a complete matching.

(2)
(Total 7 marks)

16. Define the terms

(a) bipartite graph,

(2)

(b) alternating path,

(2)

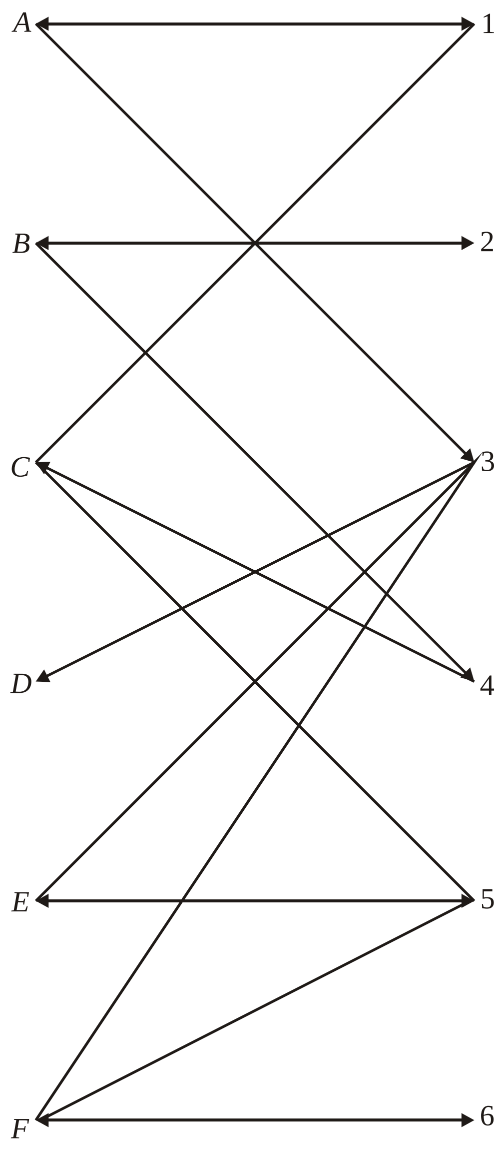
(c) matching,

(1)

(d) complete matching.

(1)
(Total 6 marks)

17.

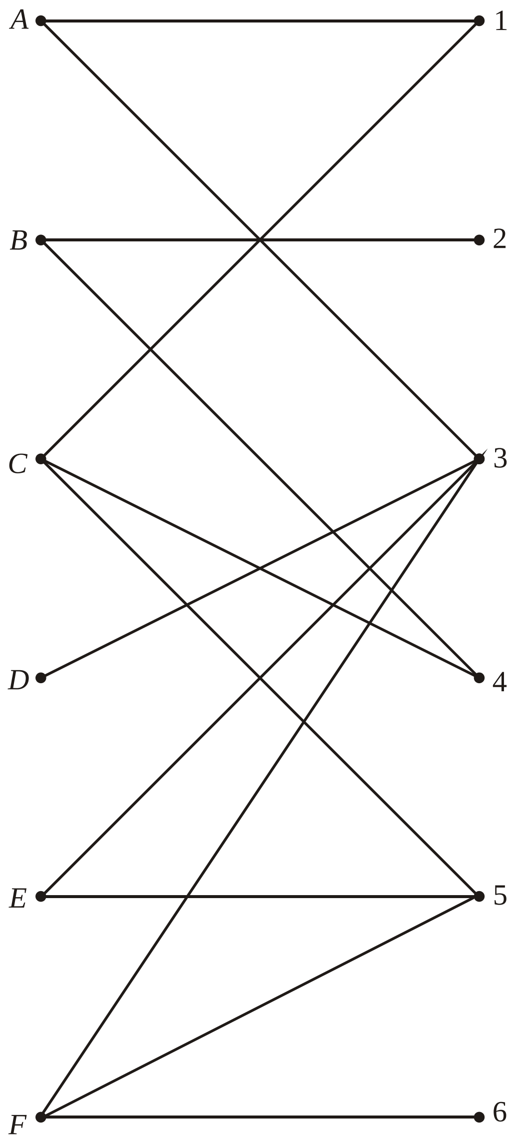


The bipartite graph in the diagram above shows the possible allocations of people *A*, *B*, *C*, *D*, *E* and *F* to tasks 1, 2, 3, 4, 5 and 6.

An initial matching is obtained by matching the following pairs

A to 3, *B* to 4, *C* to 1, *F* to 5.

(a) Show this matching in a distinctive way on the diagram below.



(1)

(b) Use an appropriate algorithm to find a maximal matching. You should state any alternating paths you have used.

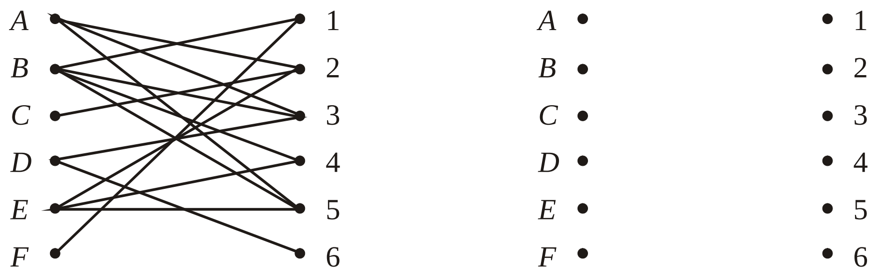
(5)
(Total 6 marks)

18. Six workers *A*, *B*, *C*, *D*, *E* and *F* are to be matched to six tasks 1, 2, 3, 4, 5 and 6.

The table below shows the tasks that each worker is able to do.

Worker	Tasks
A	2, 3, 5
B	1, 3, 4, 5
C	2
D	3, 6
E	2, 4, 5
F	1

A bipartite graph showing this information is drawn below.



Initially, *A*, *B*, *D* and *E* are allocated to tasks 2, 1, 3 and 5 respectively.

Starting from the given initial matching, use the matching improvement algorithm to find a complete matching, showing your alternating paths clearly.

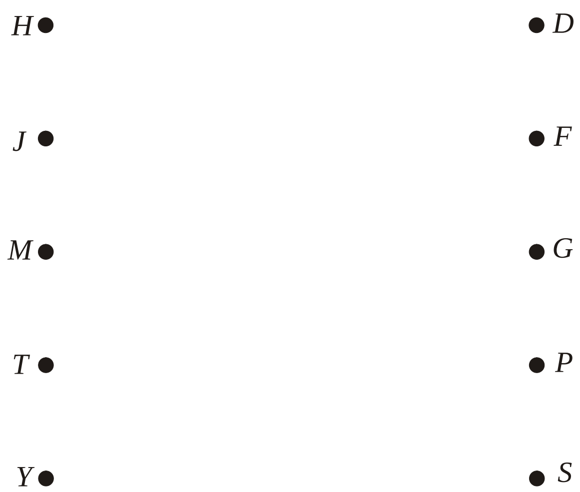
(Total 5 marks)

19. At Tesafe supermarket there are 5 trainee staff, Homan (H), Jenna (J), Mary (M), Tim (T) and Yoshie (Y). They each must spend one week in each of 5 departments, Delicatessen (D), Frozen foods (F), Groceries (G), Pet foods (P), Soft drinks (S). Next week every department requires exactly one trainee. The table below shows the departments in which the trainees have yet to spend time.

Trainee	Departments
H	D, F, P
J	G, D, F
M	S, P, G
T	F, S, G
Y	D

Initially H, J, M and T are allocated to the first department in their list.

(a) Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.



(2)

Starting from this matching,

(b) use the maximum matching algorithm to find a complete matching. You must make clear your alternating path and your complete matching.

(4)
(Total 6 marks)

20. *This question should be answered on the page below.*

A college wishes to staff classes in French (F), German (G), Italian (I), Russian (R) and Spanish (S). Five language teachers are available – Mr Ahmed (A), Mrs Brown (B), Ms Corrie (C), Dr Donald (D) and Miss Evans (E). The languages they can teach are shown in the table.

Mr Ahmed (A)	French and German
Mrs Brown (B)	French and Italian
Ms Corrie (C)	Russian
Dr Donald (D)	Russian and Spanish
Miss Evans (E)	French and Spanish

The first three teachers were allowed to choose their preferences. Their choices were:

Mr Ahmed (A) – French (F)
Dr Donald (D) – Russian (R)
Miss Evans (E) – Spanish (S)

(a) Using the dots on the answer sheet, draw a bipartite graph to show the information in the table. Indicate the above choices in a distinctive way.

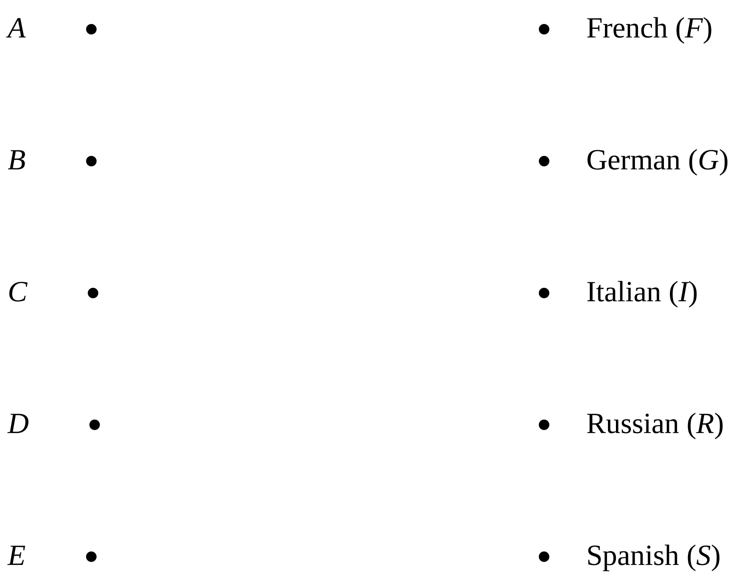
(1)

(b) Using your answer to part (a) as the initial matching, apply the maximum matching algorithm to obtain a complete matching. Alternating paths and the final matching should be stated.

(5)
(Total 6 marks)

Sheet for use in answering this question.

(a)



(b) Alternating paths

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

(c) Complete matching

…………………………………………………………………………………………

…………………………………………………………………………………………

1.	(a)	G – 3 = J – 4 = L – 5	M1	
		Change status: G – 3 = J – 4 = L – 5	A1	
		Improved matching: E = 2 G = 3 J = 4 L = 5	B1	3
	(b)	e.g George and Yi Wen may both only be assigned to 3	B1	1
	(c)	Y – 3 = G – 2 = E – 4 = J – 1	M1	
		Change status: Y = 3 – G = 2 – E = 4 –J = 1	A1	
		Complete Matching E = 4 G = 2 J = 1 L = 5 Y = 3	A1	3

[7]

2.	(a)	e.g. G – 3 = E – 2 = A – 4 = S – 6	M1	
		Change status G = 3 – E = 2 – A = 4 – S = 6	A1	
		Improved matching A = 4 (C unmatched) E = 2 G = 3 J = 5 S = 6	A1	3

Alt

G – 3 = E – 2 = A – 4 = S – 1 c.s. G = 3 – E = 2 – A = 4 – S = 1
A = 4, (C unmatched), E = 2, G = 3, J = 5, S = 1

<u>Note</u>

1M1: Path from G to 6 or 1
1A1: CAO including change status (stated or shown), chosen path clear.
2A1: CAO must ft from stated path, diagram ok

	(b)	e.g. Both C and J can only be matched to 5 Both 1 and 6 can only be done by S	B2, 1, 0	2

<u>Note</u>

1B1: Correct answer, may be imprecise or muddled (bod gets B1) all relevant nodes should be referred to and must be correct, but condone one (genuine) slip.
2B1: Good, clear, correct answer.

(c) C – 5 = J – 4 = A – 2 = E – 6 = S – 1 M1
Change status C = 5 – J = 4 – A = 2 – E = 6 – S = 1 A1

Complete matching
A = 2 C = 5 E = 6 G = 3 J = 4 S = 1 A1 3

Alt

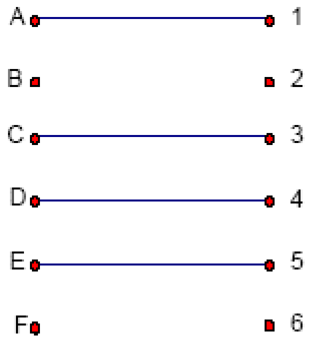
C – 5 = J – 4 = A – 2 = E – 6 c.s. C = 5 – J = 4 – A = 2 – E = 6
A = 2, C = 5, E = 6, G = 3, J = 4, S = 1

<u>Note</u>

1M1: Path from C to 1 or 6 [whichever they didn't use before.]
1A1: CAO including change status (stated or shown), chosen path clear. (Don't penalise change status twice.)
2A1: CAO must ft from stated path, diagram ok

[8]

3. (a)

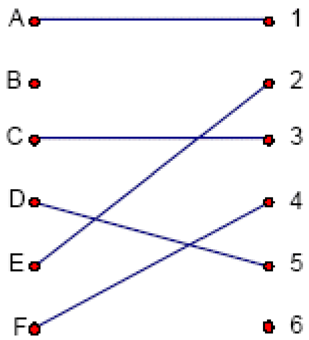


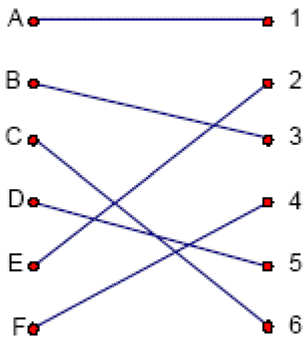
Initial map B1 1

<u>Note</u>

B1 cao preferably just 4 lines, but accept if unambiguous.

(b)





E.g.	Path 1	F-4-D-5-E-2	M1 A1	
	Path 2	B-3-C-6	M1 A1	
	Matching	A : 1, B : 3, C : 6, D : 5, E : 2, F : 4	A1	5

<u>Alternative</u>

	Path 1	Path 2	Matching					
			A	B	C	D	E	F
1	B-3-C-1-A-2	F-3-B-4-D-5-E-1-C-6	2	4	6	5	1	3
2	B-3-C-1-A-2	F-3-B-4-D-5-E-2-A-1-C-6	1	4	6	5	2	3
3	B-3-C-1-A-2	F-4-D-5-E-1-C-6	2	3	6	5	1	4
4	B-3-C-1-A-2	F-4-D-5-E-2-A-1-C-6	1	3	6	5	2	4
5	B-3-C-4-D-5-E-1-A-2	F-3-B-4-C-6	2	4	6	5	1	3
6	B-3-C-4-D-5-E-1-A-2	F-4-C-6	2	3	6	5	1	4
7	B-3-C-6	F-3-B-4-D-5-E-1-A-2	2	4	6	5	1	3
8	B-3-C-6	F-3-B-4-D-5-E-2	1	4	6	5	2	3
9	B-3-C-6	F-4-D-5-E-1-A-2	2	3	6	5	1	4
10	B-3-C-6	F-4-D-5-E-2	1	3	6	5	2	4
11	B-4-D-5-E-2	F-3-C-6	1	4	6	5	2	3
12	B-4-D-5-E-2	F-4-B-3-C-6	1	3	6	5	2	4
13	B-4-D-5-E-1-A-2	F-3-C-6	2	4	6	5	1	3
14	B-4-D-5-E-1-A-2	F-4-B-3-C-6	2	3	6	5	1	4
15	F-3-C-1-A-2	B-3-F-4-D-5-E-1-C-6	2	3	6	5	1	4
16	F-3-C-1-A-2	B-3-F-4-D-5-E-2-A-1-C-6	1	3	6	5	2	4
17	F-3-C-1-A-2	B-4-D-5-E-1-C-6	2	4	6	5	1	3
18	F-3-C-1-A-2	B-4-D-5-E-2-A-1-C-6	1	4	6	5	2	3
19	F-3-C-4-D-5-E-1-A-2	B-3-F-4-C-6	2	3	6	5	1	4
20	F-3-C-4-D-5-E-1-A-2	B-4-C-6	2	4	6	5	1	3
21	F-3-C-6	B-3-F-4-D-5-E-1-A-2	2	3	6	5	1	4
22	F-3-C-6	B-3-F-4-D-5-E-2	1	3	6	5	2	4
23	F-3-C-6	B-4-D-5-E-1-A-2	2	4	6	5	1	3
24	F-3-C-6	B-4-D-5-E-2	1	4	6	5	2	3
25	F-4-D-5-E-2	B-3-C-6	1	3	6	5	2	4
26	F-4-D-5-E-2	B-4-F-3-C-6	1	4	6	5	2	3
27	F-4-D-5-E-1-A-2	B-3-C-6	2	3	6	5	1	4
28	F-4-D-5-E-1-A-2	B-4-F-3-C-6	2	4	6	5	1	3

<u>Note</u>

M1 attempt at a path from B or F to 2 or 6

A1 correct path – including change status

M1 attempt at a second path from F or B to 6 or 2

A1 correct path – including change status (do not penalise change status twice)

A1 correct matching; must follow from 2 correct paths

[6]

4. (a) H – 2 = M – 5 = R – 4 change status to give M1 A1

Note

1M1: Path from H to 4

1A1: correct path and change status

2A1: CAO must follow from correct path.

(b) C = 3 (E unmatched) H = 2 M = 5 R = 4 S = 1 A1 3

Note

1B1: CAO or e.g reference to E 5 M 2 H 1 S

(c) e.g. C is the only person who can do 3 and the only person who can do 6 B1 1

e.g. E – 5 = M – 2 = H – 1 = S – 3 = C – 6 change status to give M1 A1

C = 6 E = 5 H = 1 M = 2 R = 4 S = 3 A1 3

Note

1M1: Path from E to 6

1A1: CAO do not penalise lack of change status a second time.

2A1: CAO must follow from a correct path

[7]

5. (a) Alternating path B – 3 = A – 5 change status B = 3 – A = 5 M1 A1

A = 5 B = 3 C = 2 D = 1 E = 6 F unmatched A1 3

Note

1M1: Path from B to 5.
1A1: Correct path including change status
2A1: CAO my matching, may be drawn but if so 5 lines only and clear.

(b) e.g. C is the only person able to do 2 and the only person able to do 4. B2, 1, 0 2
Or D, E and F between them can only be allocated to 1 and 6.

Note

1B1: Close, a correct relevant, productive statement bod generous
2B1: A Good clear answer generous

	(c)	Alternating path F – 6 = E – 1 = D – 2 = C – 4 change status F = 6 – E = 1 – D = 2 – C = 4	M1 A1	
		A = 5 B = 3 C = 4 D = 2 E = 1 F = 6	A1	3
		<u>Note</u> 1M1: Path from F to 4. No ft. 1A1: Correct path penalise lack of change status once only 2A1: CAO may be drawn but if so 6 lines only and clear		

[8]

6.	(a)	G – 5 = W – 3 change status G = 5 – W = 3	M1A1	2
		1M1: Path from G to 3 1A1: CAO including change status (stated or shown), chosen path clear.		
	(b)	A – no match E = 2 G = 5 R = 4 W = 3	A1	1
		2A1: CAO muts ft from stated path		
	(c)	e.g. R is the only person who can do 1 and the only person who can do 4	B2,1,0	2
		1B1: Correct answer; may be imprecise or muddled (bod gets B1) but all nodes refered to must be correct 2B1: Good, clear, correct answer.		
	(d)	A – 2 = E – 3 = W – 4 = R – 1 change status A = 2 – E = 3 – W = 4 – R = 1	M1A1	
		A = 2 E = 3 G = 5 R = 1 W = 4	A1	3
		1M1: Path from A to 1 1A1: CAO including change status (stated or shown) but don’t penalise twice. Chosen path clear. 1A1: CAO must ft from stated path		

Misread (remove last two A or B marks if earned.)
A – 2 = E – 3 c.s. A = 2 – E = 3 Matching A = 2, E = 3, R = 4 W = 5
Then
G – 5 = W – 4 = R – 1 c.s. G = 5 – W = 4 – R = 1
Matching A = 2, E = 3, G = 5, R = 1, W = 4

[8]

7.	(a)	(i)	A path from an unmatched vertex in one set to an unmatched vertex on the other set......	B1	
			... which alternately uses arcs not in/in the matching.	B1	2
		(ii)	A one-to-one pairing of	B1	
			some elements of one set with the other set	B1	2
	(b)	e.g.	D – 3 = C – 5 change status D = 3 – C = 5	M1A1	
			E – 2 = A – 1 change status E = 2 – A = 1	M1A1	
		A = 1 B = 4 C = 5 D = 3 E = 2		A1	5

[9]

8.	(a)	To obtain a complete matching the number of vertices on each side must be equal.	B2,1,0	2
	(b)	e.g. $L – 3 = H – 5 = J – 1a = A – 4$	M1A1	
		C.S. $L = 3 – H = 5 – J = 1a – A = 4$		
		$A = 4$ $H = 5$ $L = 3$	A1	3
		$E = 1b$ $J = 1a$ $M = 2$		
	(c)	*H* and *L* can now both only do 3. So a complete matching is not possible (other answers possible)	B2,1,0	2

[7]

9.	(a)	G – 3 = J – 4 = L – 5	M1	
		change status: <u>G = 3 – J = 4 – L = 5</u>	A1	
		Improved matching: E = 2 G = 3 J = 4 L = 5	B1	3
	(b)	e.g. George and Yi Wen may both only be assigned b 3	B1	1

	(c)	Y – 3 = G – 2 = E – 4 = J – 1	M1	
		change status: <u>Y = 3 – G = 2 – E = 4 – J = 1</u>	A1	
		Complete matching: E = 4 G = 2 J = 1 L = 5 Y = 3	A1	3

[7]

10.	(a)	A path from an unmatched vertex in X to an unmatched vertex in Y, which alternately uses arcs in/not in the marking (where X and Y are distinct sets of vertices).	B2,1,0	3
		B2 A good, complete answer *B1 Partially correct – unmatched to unmatched <u>or</u> arcs inv / not in the matching <u>one</u> enough "bad" sets B1*		
	(b)	e.g. R – B = A – P c.s. R = B – A = P	M1 A1	2
		S – F = M – C = D – K c.s. S = F – M = C – D = K	M1 A1	
		∴ A = P D = K H = Y M = C R = B S= F	A1	3
		M1 Path from / to R/S – to / from K/P (answer links to second M1 answer below) *A1 c.a.o. including c.s.* *M1 Second path from remaining LH vertex to remaining RH vertex* *A1 c.a.o. including c.s. (penalise c.s. only once*)* *A1 Must ft from two correct paths c.a.o.*		

<u>Notes</u>

(b) (i) R – B = A – P
S – F = M – C = D – K

(ii) R – B = A – F = M – C = D – K
S – F = A – P

(iii) S – F = M – C = D – K
R – B = A – P

(iv) S – F = M – Y = H – B = A – P
R – B = H – Y = M – C = D – K

A = P

D = K

H = Y

M = C

R = B

S = F

[7]

11. (a) There are 2 unmatched vertices on each side – the algorithm only matches one on each side per iteration. B1 1

(b) e.g. E – 3 = C – 1 c.s. E = 3 – C = 1 M1 A1 2

F – 5 = A – 6 = D – 2 = B – 4 c.s. F = 5 – A = 6 – D = 2 =B – 4 M1 A1 2

A = 6 B = 4 C = 1 D = 2 E = 3 F = 5 M1 A1 2

[7]

12. (a) $E - 4 = B - 2 = D - 1 = A - 3 = C - 5$ change states to give M1 A1
matching $A = 3$ $B = 2$ $C = 5$ $D = 1$ $E = 4$ A1 3

$E - 4 = B - 2 = D - 3 = C - 5$ change states to give M1 A1
matching $A = 1$ $B = 2$ $C = 5$ $D = 3$ $E = 4$ A1 3

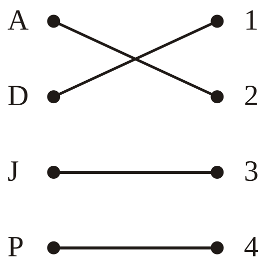
M1 1st path E to S
A1 c.a.o. + c.s.
A1 matching c.a.o. must be clear, must ft
M1 2nd path E to S
A1 c.a.o. + c.s. (don't penalise c.s. twice)
A1 matching c.a.o. must be clear, must ft

(b) e.g. reference to $B + E$ and $4 + 2$ B2, 1, 0 2

B2 full clear explanation B, E, 2 and 4 limited (+0)
o.e. – lots of alternatives
B1 probably 3 out of 4 referred to, may be explanation confused, superflow films or time introduced.
"b.a.d gets B1"

[8]

13 (a)

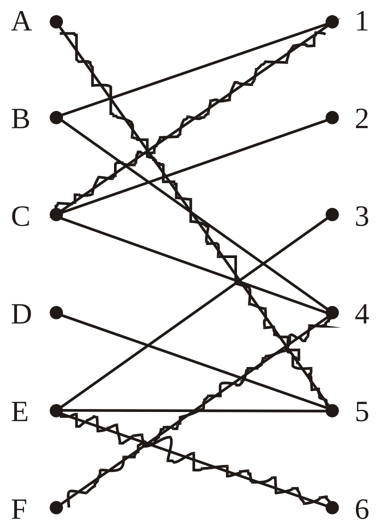


B1 1

(b) e.g. 5 – 3 = J – 4 = P – 6 c.s. S = 3 – J = 4 – P = 6 M1 A1 2
and T – 2 = A – 1 = D – 5 c.s. T = 2 – A = 1 – D = 5 M1 A1
A = 1 D = 5 J = 4 P = 6 S = 3 T = 2 A1 3

[6]

14. (a) (b)



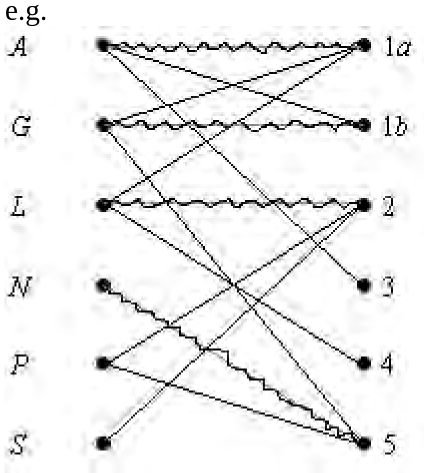
B1B1 2

(c) eg B – 1 = C – 2 c.s. B = 1 – C = 2 M1A1B1(c.s.)
A = 5, B = 1, C = 2, E = 6, F = 4 A1ft 4

(d) e.g. Both A and D are only matched to 5, once one has been assigned the other can not be B2,1,0 2
E is the only person who can do 3, and the only person who can do 6, if they are assigned to one of these the other can not be done

[8]

15. (a) e.g. B1B1 2



(b) e.g.:

(i) $P – 2 = L – 4$ c.s. $P = 2 – L – 4$ M1

(ii) $S – 2 = L – 1a = A – 3$ c.s. $S = 2 – L = 1a – A = 3$ A1

or…
giving $A – 1, G – 1, L – 4, N – 5, P – 2$
$A – 3, G – 1, L – 1, N – 5, S – 2$ A1 3

(c) e.g.:

Sam must do 2 and Nicola must do 5, leaving Philip without a task. B2, 1, 0 2

[7]

16. (a) A graph consisting of <u>two distinct sets of vertices</u> X and Y in B1
which… <u>arcs can only join a vertex in X to a vertex in Y</u>. B1 2

(b) A path <u>from an unmatched vertex in X to an unmatched vertex</u> in Y… B1
...which <u>alternately uses arcs in/not in the matching.</u> B1 2

(c) The (1-1) matching / pairing of <u>some</u> elements of X with elements of Y. B1

(d) A <u>1-1</u> matching between <u>all</u> elements of X onto Y B1 2

[6]

17. (a) Add A to 3, B to 4, C to 1 and F to 5 in a distinctive way B1 1

(b) <u>e.g.</u> D – 3 = A – 1 = C – 4 = B – 2 M1

C.S. D = 3 – A = 1 – C = 4 – B = 2 A1 2

E – 5 = F – 6 M1

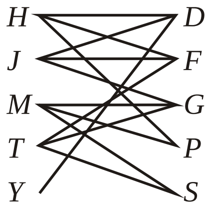
C.S. E = 5 – F = 6 A1

A = 1 B = 2 C = 4 D = 3 E = 5 F = 6 A1 3

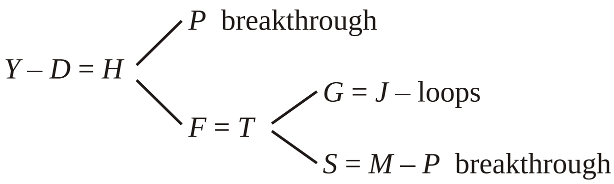
[6]

18. e.g. $C – 2 = A – 5 = E – 4$ c.s. $C = 2 – A = 5 – E = 4$ M1 A1
$F – 1 = B – 3 = D – 6$ c.s. $F = 1 – B = 3 – D = 6$ M1 A1
$\therefore A = 5, B = 3, C = 2, D = 6, E = 4, F = 1$ A1 5

[5]

19. (a) 

B1 B1 2

(b) 

M1 A1

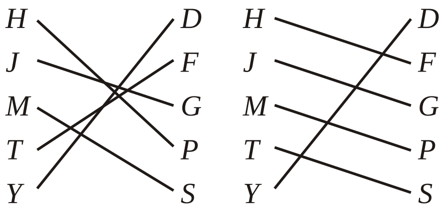
changing status, the possible alternating paths are

(i) $Y = D – H = P$

or (ii) $Y = D – H = F – T = S – M = P$ A1

giving the following matching

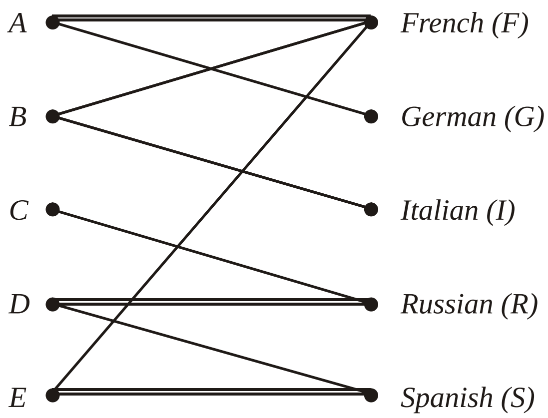
(i)	or	(ii)
$H – P$		$H – F$
$J – G$		$J – G$
$M – S$		$M – P$
$T – F$		$T – S$
$Y –$ D		$Y – D$



A1 4

[6]

20. (a)



B1 1

(b)	Alternating paths:		
	B matched with Italian as both unmatched in initial matching	B1	
	Starting with *C* (unmatched)		
	C—R=D—S=E—F=A—G *(breakthrough)*	M1 A1	3
(c)	Complete matching:		
	Changing status		
	C=R—D=S—E=F—A=G		
	So complete matching is Corrie (Russian), Donald (Spanish), Evans (French), Ahmed (German) and Brown (Italian)	M1 A1	5

1. No Report available for this question.

2. This was a good source of marks for well-prepared candidates with around 80% able to secure at least half marks and around 46% gaining full marks. Most were able to find a path from G to 6 or 1, though some then omitted the change status step and/or did not list the improved matching. Candidates should beware of trying to show too much on the printed diagrams, some try to show their flow augmenting route and the improved solution on the same diagram, often using colour, or highlighter, which may not be used in this paper.

Part (b) was generally well-answered, but candidates should note that for two marks the examiners are looking for a clear, complete answer, some imprecise or muddled answers were seen.

For those that completed part (a) correctly part (c) was often well done, although candidates should remember that an alternating path must go from an unmatched vertex on one side to an unmatched vertex on the other.

3. This proved a good starter, the mean score being 5 out of 6 marks. Almost all candidates were able to complete part (a) correctly but candidates are reminded that colour should not be used: Neither to distinguish arcs in a matching, nor when listing an alternating path. In part (b) most candidates were able to gain at least 4 of the 5 marks, a few losing one mark for not explicitly changing status. A few candidates tried to start an alternating path at an already matched node (often A), and some only found one alternating path. Some candidates showed a tree of possible paths but then did not make it clear which path they had chosen to use.

4. This was a good source of marks for well-prepared candidates. Most are making the alternating paths they used clear. Candidates are reminded that they are instructed to **list** their alternating paths, rather than draw them, which makes them clearer for the examiners to give full credit. The change status step must be made clear, either by writing ‘change status’ or by re-listing the alternating path with the connective symbols switched. Terms such as ‘add’ ‘remove’ ‘switch’ are not accepted as evidence of the change status step. Many candidates did not list the complete improved matching at the end of (a) often omitting S = 1 or C = 3. Candidates are reminded that colour is not visible and so colour should not be used to indicate the change status step or used to ‘pick out’ the improved matching on a bipartite graph showing more than the minimum necessary arcs. In part (b) candidates needed to be specific and name the people and the tasks used in their explanation. The most efficient answer involved C, 3 and 6 only. Alternating paths **must** go from an unmatched node to an unmatched node; in part (c) this meant that the path must go from E to 6, a number of candidates started at S.

5. There were many good responses seen to this question and the general standard of response was better than in previous years. The path from B to 5 was usually given followed by an improved matching. Some candidates tried to find a path, starting at F and often ‘found’ one, ending at D or sometimes 6, both of which are already matched, so this proved a good test of their understanding of ‘breakthrough’. As usual some candidates did not make the ‘change status’ step clear, but the number failing to do so seems less than in the past. Those who had created ‘fragments’ of an alternating path in (a) were often unable to find an alternating path leading to a complete matching in part (c). Most candidates were able to make a good attempt at (b), but with some losing marks for not being specific enough with letters and numbers.

6. This was a good source of marks for well-prepared candidates. Most are remembering to show the ‘change status’ step and are making the alternating paths they used clear. Alternating paths must go from an unmatched node to an unmatched node, a number of candidates added A – 2 = E onto the end of the alternating path found in part (a) for example. Candidates are reminded that they are instructed to list their alternating paths, which makes them clearer for the examiners to give full credit. Many candidates got involved in long explanations in part (c) usually referring to A, E, G, W, 2, 3 and 5 rather than the simpler 1R4 answer. Candidates scoring full marks in (a) and (b) were usually successful in part (d) too.

7. Most candidates were able to gain some credit in part (a) (i) and (ii), but only the best gained full credit. Poor terminology was frequently seen and many confused vertices/nodes with arcs/edges. In part (i) many did not state that the path must start and finish at unmatched vertices and others did not make clear what was ‘alternating’ or wrote that it was ‘alternating left and right’. In (a)(ii) many candidates did not state that the matching should be one-to-one and others explained that a matching consisted of one (and only one) edge. Part (b) was **very** well answered with almost all the candidates stating two clear, acceptable alternating paths, starting and finishing at an unmatched vertex on each side. Candidates should again be reminded about the use of colour in this type of question and should ensure that their responses would clearly identify any differences when photocopied; the use of different types of lines to show changed status of edges is useful with a common approach being - = - = - = - =. Some omitted to show/state the change status step and others omitted the final matching. Only the very weakest attempted to start their alternating paths from an already matched vertex.

8. Many candidates are more practised at finding a matching and an alternating path, with many more candidates able to start and finish at an unmatched vertex on each side. Candidates should again be reminded about the use of colour in this type of question and should ensure that their responses would clearly identify any differences when photocopied; the use of different types of lines to show changed status of edges is useful with a common approach being – = – = – = – =. A number of candidates did not make their ‘change status’ step clear. Only the most able were able to gain full marks in part (a) by identifying that, if a complete matching is to be found, there must be same number of vertices on each side of the bipartite graph. Part (c) was again well answered with many candidates being able to explain this accurately.

9. Some candidate ran out of space here and continued the question elsewhere. Most candidates were able to list the correct alternating path but a substantial number omitted to change status. Many did not list the improved matching in part (a). Part (b) was often very well done, but some candidates were not specific enough. In part (c) many candidates did not take into account their first alternating path when seeking their second.

10. This proved an accessible question, but some poor definitions were seen in part (a), with candidates using technical language inaccurately, however, many were able to score some credit. Part (b) was generally very well done. The great majority of the candidates found at least one correct alternating path, and most both. Some candidates did not indicate the ‘change status’ step and others did not take into account their first alternating path when seeking their second. Candidates are reminded that colour will be indistinguishable as this paper will be marked electronically from next January.

11. This proved a good starter question for the vast majority of the candidates. Part (a) required a clear explanation, and this proved tricky for some candidates, most were able to tackle part (b) successfully however. Some forgot to demonstrate the change of status step, a few tried to create paths starting or finishing at already matched vertices, others did not take into account their earlier alternating path when seeking a second one, some wrote down a multitude of alternating paths and did not indicate which they used, but generally part (b) was very well done.

12. This was a good source of marks for most candidates, with most being able to find at least one correct alternating path between E and 5. Some candidates however split a correct path into 2 loosing both M marks. Many did not offer a second alternating path and a few started or finished their alternating paths at already matched vertices. A number of candidates forgot to make clear the 'change status' step of the algorithm. Most made their two final matchings clear. There were many correct alternative answers to part (b) but most used the argument shown in the mark scheme. The explanation was not always well done with a lot of incomplete or confused explanations.

13. This was well-answered in general, with most candidates scoring nearly full marks. The most common error was in failing to indicate the change of status. Some candidates, who used a 'decision tree' to find more than one alternating path, did not make it clear which one they had selected. Some candidates did not take into account the changes to the matching caused by their first alternating path, when seeking their second.

14. Parts (a) and (b) were very well answered. Most candidates were able to find the alternating path connecting B to 2, but many incorrect paths of one or two arcs were also seen. Most remembered to show the 'change status' step. Some very clear answers to part (d) were seen by the more able candidates, but many candidates did not complete their argument.

15. Few candidates appreciated the need for two vertices to be assigned to checkpoint one and tried to solve the problem with just five vertices on the right hand side. Other than this the bipartite graphs were generally correctly dealt with. Some candidates who used two vertices, 1a and 1b, did not join A and G to both of them and some did not make the initial matching clear. In part (b) most handled their alternating path correctly, although some tried to find an alternating path starting or finishing at a matched vertex. Part (c) was handled correctly by about half the candidates with good clear reasoning given. A number of candidates incorrectly cited the difference between the number of people and the number of checkpoints, even some of those who had created two vertices for checkpoint one.

16. Most candidates were able to score some marks in this question, but few scored full marks. There was a lot of confusion about arcs and vertices in parts (a) and (b) and about complete matchings, complete graphs and maximal matchings in part (d). Many candidates gave examples rather than definitions.

In part (a) most candidates referred to two sets but few clearly explained that the arcs could only join a vertex in X to a vertex inY. In part (b) a number of candidate confused arcs and vertices and few made it clear that the path must both start and finish at an unmatched node, some described an alternative path. In part (d) there was often a lack of clarity about the need for the matching to be 1 – 1.

17. Most candidates were able to complete part (a) correctly. In part (b) most candidates were able to find at least one alternating path, although some tried to start at a matched vertex. Some failed to take their first alternating path into account when finding the second. Most candidates remembered the ‘change status’ step.

18. This was a good starter and most candidates were able to gain full marks. There were a few candidates however who did not make any attempt to list alternating paths and some who tried to solve the problem by reasoning, gaining no marks. There were two alternating paths to find and some candidates stopped after finding just one.

19. The majority of candidates were able to complete this question correctly. Some candidates made errors in the initial matching (with *YD* being the commonest inclusion), and others did not make the ‘change status’ step clear.

20. No Report available for this question.