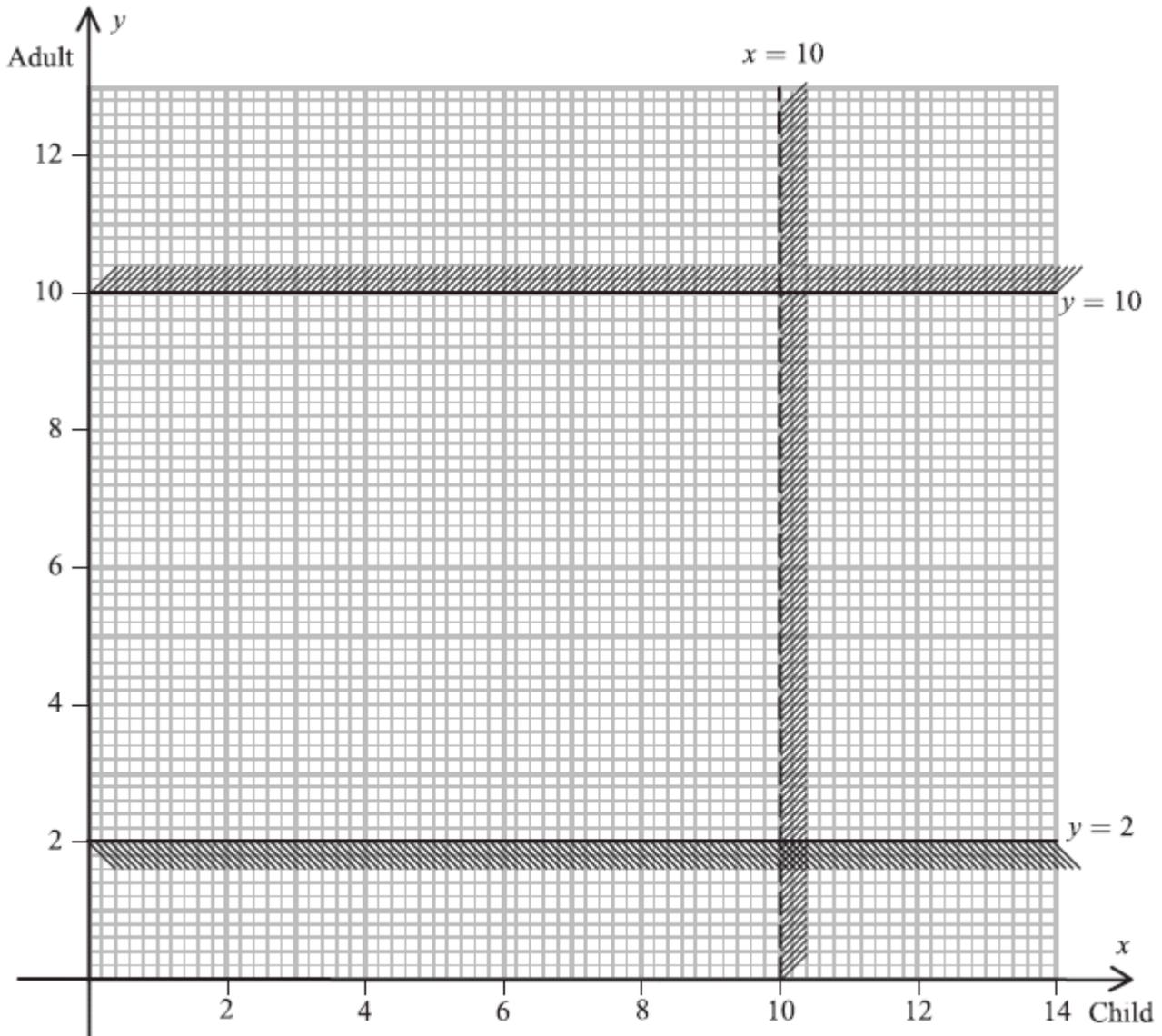


1.



The captain of the *Malde Mare* takes passengers on trips across the lake in her boat.

The number of children is represented by  $x$  and the number of adults by  $y$ .

Two of the constraints limiting the number of people she can take on each trip are

$$x < 10$$

and

$$2 \leq y \leq 10$$

These are shown on the graph in the figure above, where the rejected regions are shaded out.

(a) Explain why the line  $x = 10$  is shown as a dotted line. (1)

(b) Use the constraints to write down statements that describe the number of children and the number of adults that can be taken on each trip. (3)

For each trip she charges £2 per child and £3 per adult. She must take at least £24 per trip to cover costs.

The number of children must not exceed twice the number of adults.

(c) Use this information to write down two inequalities. (2)

(d) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R. (4)

(e) Use your graph to determine how many children and adults would be on the trip if the captain takes:

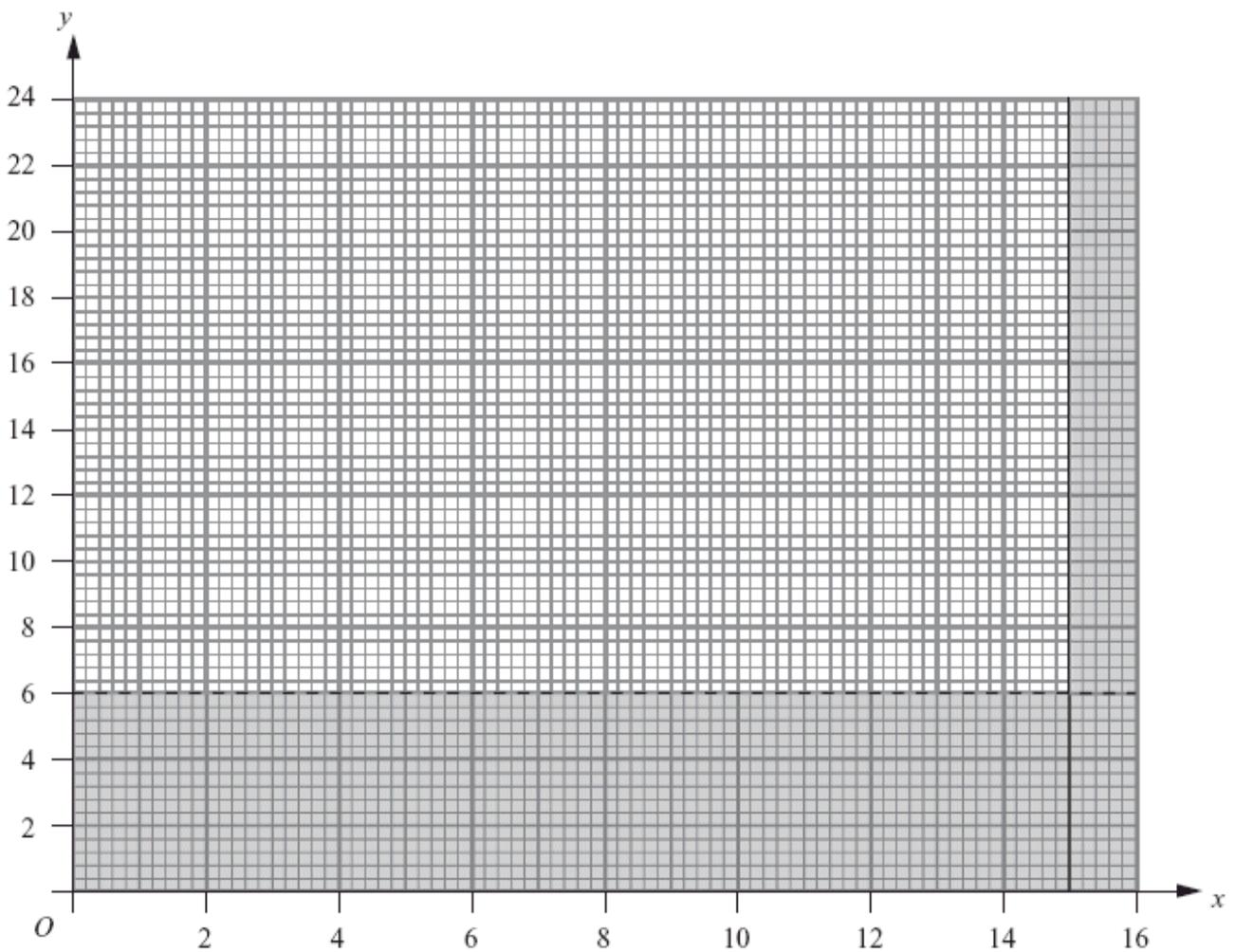
(i) the minimum number of passengers,

(ii) the maximum number of passengers.

(4)

**(Total 14 marks)**

2.



Keith organises two types of children's activity, 'Sports Mad' and 'Circus Fun'. He needs to determine the number of times each type of activity is to be offered.

Let  $x$  be the number of times he offers the 'Sports Mad' activity. Let  $y$  be the number of times he offers the 'Circus Fun' activity.

Two constraints are

$$x \leq 15$$

$$\text{and } y > 6$$

These constraints are shown on the graph below, where the rejected regions are shaded out.

(a) Explain why  $y = 6$  is shown as a dotted line.

(1)

Two further constraints are

$$3x \leq 2y$$

and  $5x + 4y \leq 80$

- (b) Add two lines and shading to the diagram above book to represent these inequalities. Hence determine the feasible region and label it R.

(3)

Each 'Sports Mad' activity costs £500.

Each 'Circus Fun' activity costs £800.

Keith wishes to minimise the total cost.

- (c) Write down the objective function, C, in terms of  $x$  and  $y$ .

(2)

- (d) Use your graph to determine the number of times each type of activity should be offered and the total cost. You must show sufficient working to make your method clear.

(5)

**(Total 11 marks)**

3. You are in charge of buying new cupboards for a school laboratory. The cupboards are available in two different sizes, standard and large. The maximum budget available is £1800. Standard cupboards cost £150 and large cupboards cost £300. Let  $x$  be the number of standard cupboards and  $y$  be the number of large cupboards.

- (a) Write down an inequality, in terms of  $x$  and  $y$ , to model this constraint.

(2)

The cupboards will be fitted along a wall 9 m long. Standard cupboards are 90 cm long and large cupboards are 120 cm long.

- (b) Show that this constraint can be modelled by

$$3x + 4y \leq 30.$$

You must make your reasoning clear.

(2)

Given also that  $y \geq 2$ ,

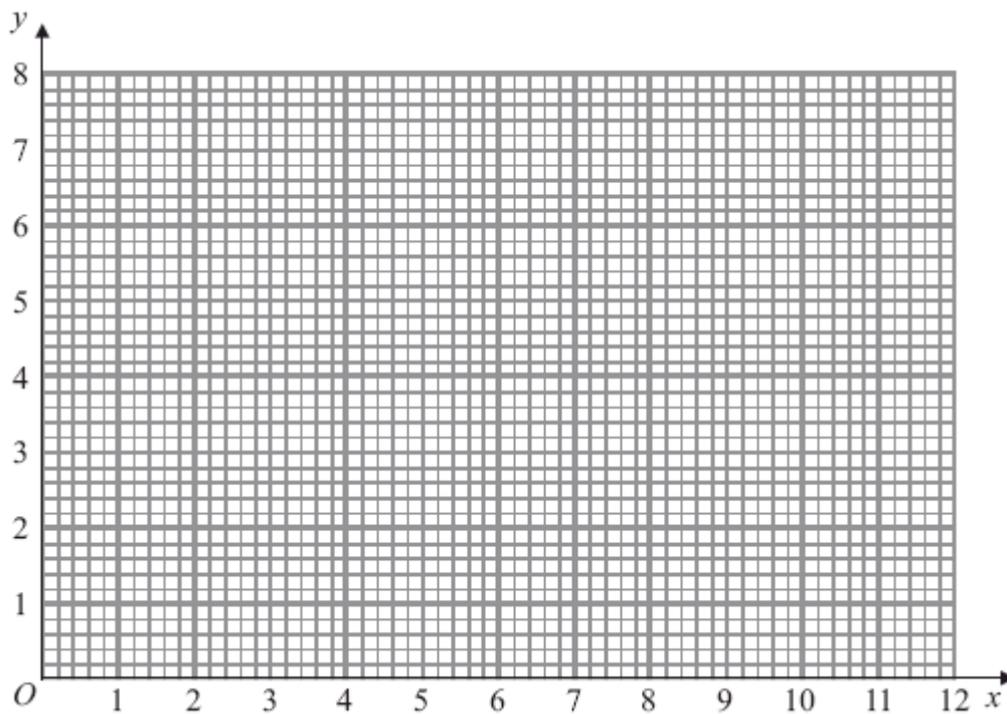
- (c) explain what this constraint means in the context of the question. (1)

The capacity of a large cupboard is 40% greater than the capacity of a standard cupboard. You wish to maximise the total capacity.

- (d) Show that your objective can be expressed as

$$\text{maximise } 5x + 7y \quad (2)$$

- (e) Represent your inequalities graphically, on the axes below, indicating clearly the feasible region, R.



(6)

- (f) Find the number of standard cupboards and large cupboards that need to be purchased. Make your method clear.

(4)

**(Total 17 marks)**

4. Rose makes hanging baskets which she sells at her local market. She makes two types, large and small. Rose makes  $x$  large baskets and  $y$  small baskets.

Each large basket costs £7 to make and each small basket costs £5 to make. Rose has £350 she can spend on making the baskets.

- (a) Write down an inequality, in terms of  $x$  and  $y$ , to model this constraint.

(2)

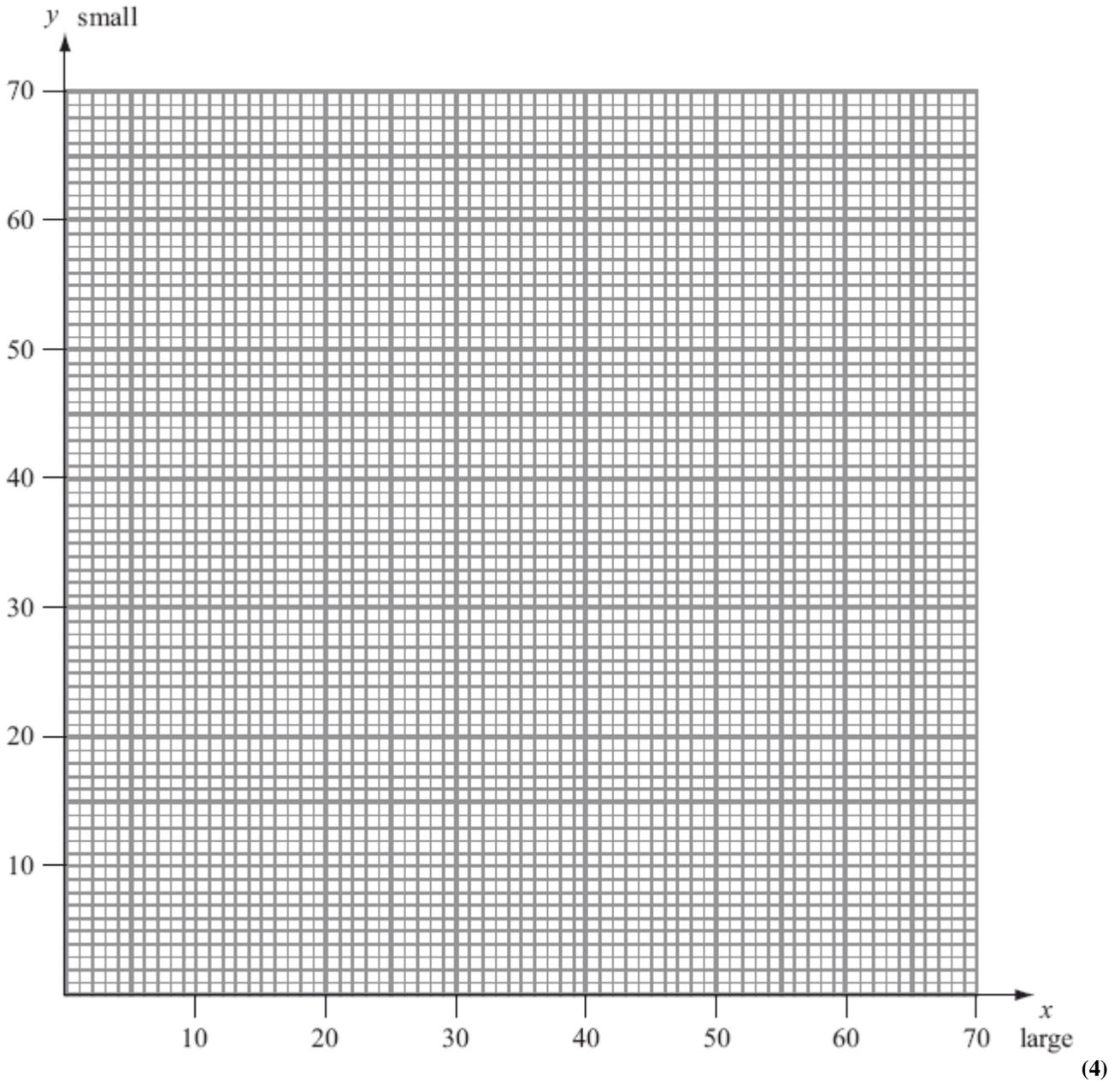
Two further constraints are

$$y \leq 20 \text{ and}$$
$$y \leq 4x$$

- (b) Use these two constraints to write down statements that describe the numbers of large and small baskets that Rose can make.

(2)

- (c) On the grid below, show these three constraints and  $x \geq 0, y \geq 0$ . Hence label the feasible region, R.



Rose makes a profit of £2 on each large basket and £3 on each small basket. Rose wishes to maximise her profit, £P.

- (d) Write down the objective function. (1)

- (e) Use your graph to determine the optimal numbers of large and small baskets Rose should make, and state the optimal profit.

(5)

(Total 14 marks)

5. A linear programming problem is modelled by the following constraints

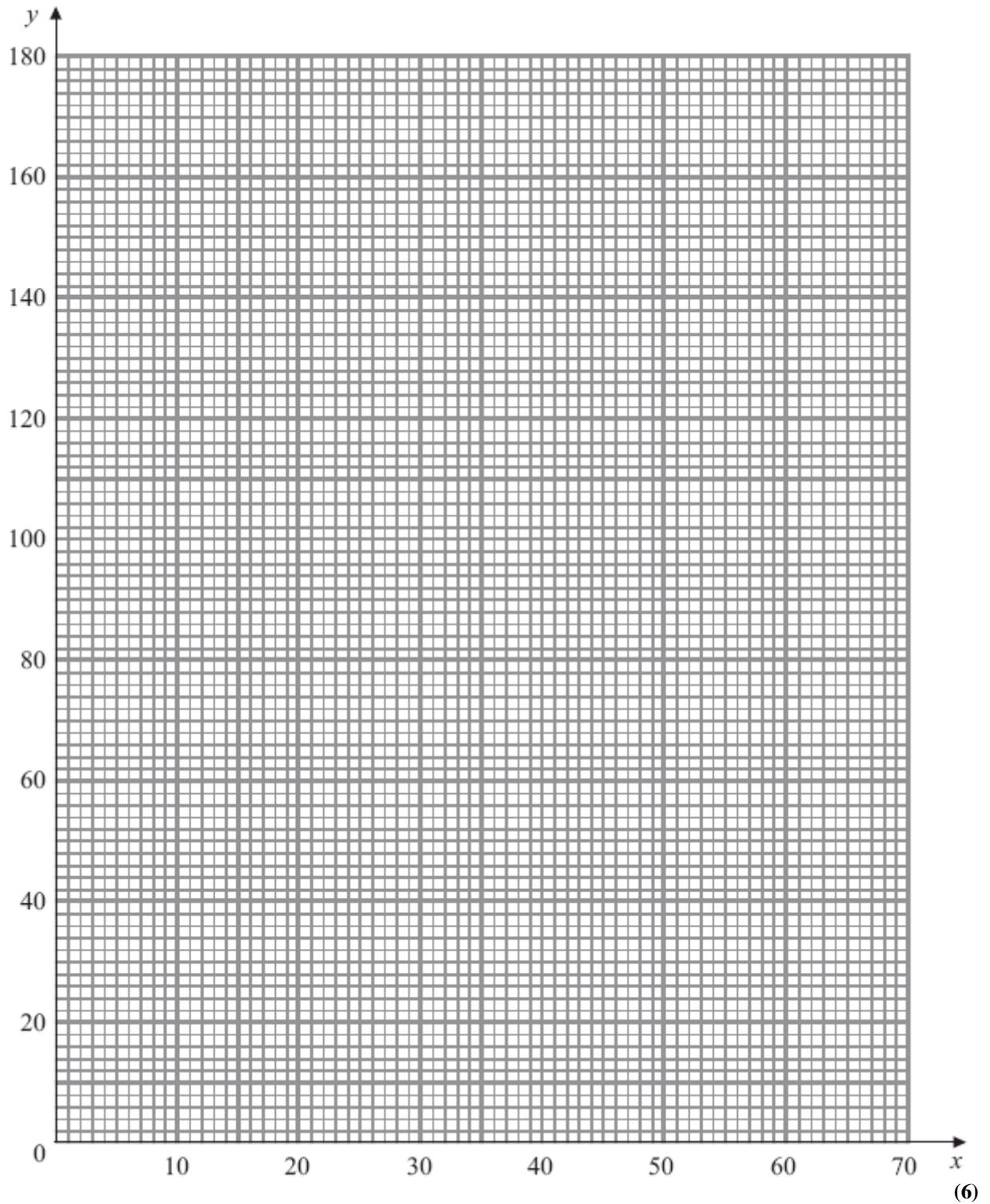
$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

- (a) Use the grid below to represent these inequalities graphically. Hence determine the feasible region and label it R.



The objective function,  $F$ , is given by

$$F = 3x + y$$

- (b) Making your method clear, determine
- (i) the minimum value of the function  $F$  and the coordinates of the optimal point,
  - (ii) the maximum value of the function  $F$  and the coordinates of the optimal point.

(6)

(Total 12 marks)

6. Phil sells boxed lunches to travellers at railway stations. Customers can select either the vegetarian box or the non-vegetarian box.

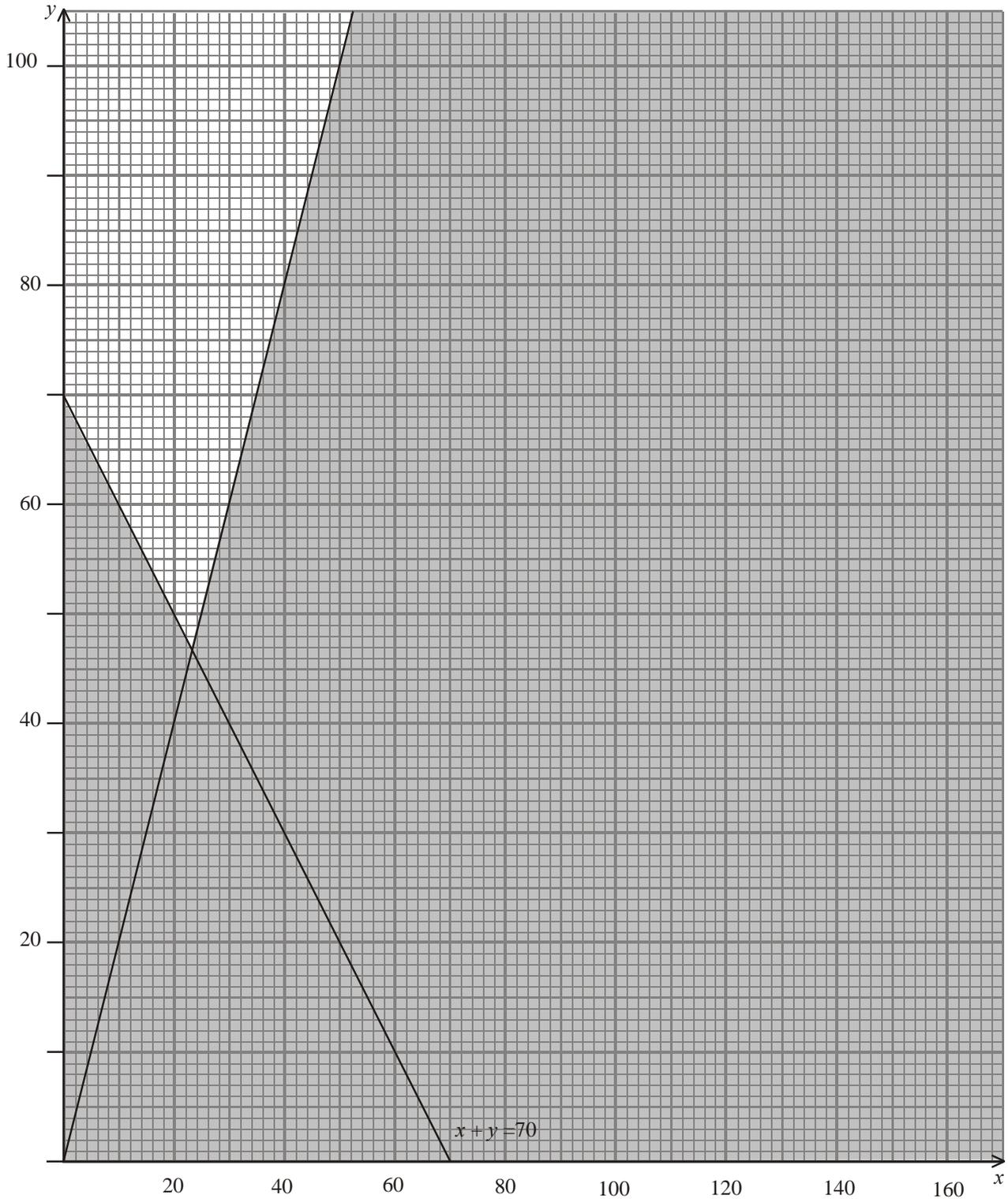
Phil decides to use graphical linear programming to help him optimise the numbers of each type of box he should produce each day.

Each day Phil produces  $x$  vegetarian boxes and  $y$  non-vegetarian boxes.

One of the constraints limiting the number of boxes is

$$x + y \geq 70.$$

This, together with  $x \geq 0$ ,  $y \geq 0$  and a fourth constraint, has been represented in the diagram below. The rejected region has been shaded.



- (a) Write down the inequality represented by the fourth constraint. (2)

Two further constraints are:

$$x + 2y \leq 160$$

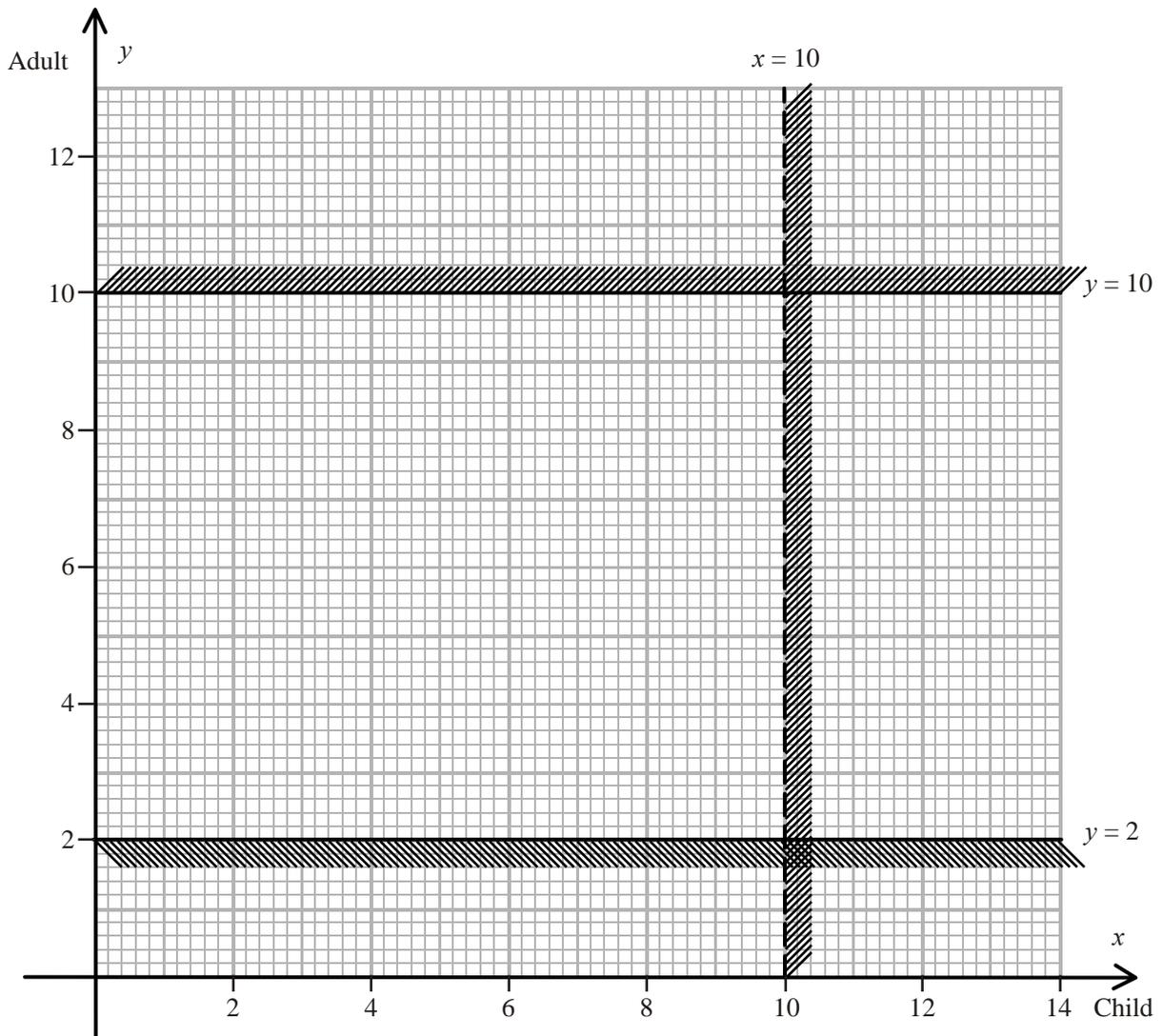
$$\text{and } y > 60.$$

- (b) Add two lines and shading to the diagram above to represent these inequalities. (4)
- (c) Hence determine and label the feasible region, R. (1)
- (d) Use your graph to determine the minimum total number of boxes he needs to prepare each day. Make your method clear. (3)

Phil makes a profit of £1.20 on each vegetarian box and £1.40 on each non-vegetarian box. He wishes to maximise his profit.

- (e) Write down the objective function. (1)
- (f) Use your graph to obtain the optimal number of vegetarian and non-vegetarian boxes he should produce each day. You must make your method clear. (4)
- (g) Find Phil's maximum daily profit. (1)
- (Total 16 marks)**

7.



The captain of the *Malde Mare* takes passengers on trips across the lake in her boat.

The number of children is represented by  $x$  and the number of adults by  $y$ .

Two of the constraints limiting the number of people she can take on each trip are

$$x < 10$$

and  $2 \leq y \leq 10$

These are shown on the graph above, where the rejected regions are shaded out.

(a) Explain why the line  $x = 10$  is shown as a dotted line.

(1)

- (b) Use the constraints to write down statements that describe the number of children and the number of adults that can be taken on each trip.

(3)

For each trip she charges £2 per child and £3 per adult. She must take at least £24 per trip to cover costs.

The number of children must not exceed twice the number of adults.

- (c) Use this information to write down two inequalities.

(2)

- (d) Add two lines and shading to the diagram above to represent these inequalities. Hence determine the feasible region and label it R.

(4)

- (e) Use your graph to determine how many children and adults would be on the trip if the captain takes:

- (i) the minimum number of passengers,  
(ii) the maximum number of passengers.

(4)

**(Total 14 marks)**

8. A company produces two types of party bag, Infant and Junior. Both types of bag contain a balloon, a toy and a whistle. In addition the Infant bag contains 3 sweets and 3 stickers and the Junior bag contains 10 sweets and 2 stickers.

The sweets and stickers are produced in the company's factory. The factory can produce up to 3000 sweets per hour and 1200 stickers per hour. The company buys a large supply of balloons, toys and whistles.

Market research indicates that at least twice as many Infant bags as Junior bags should be produced.

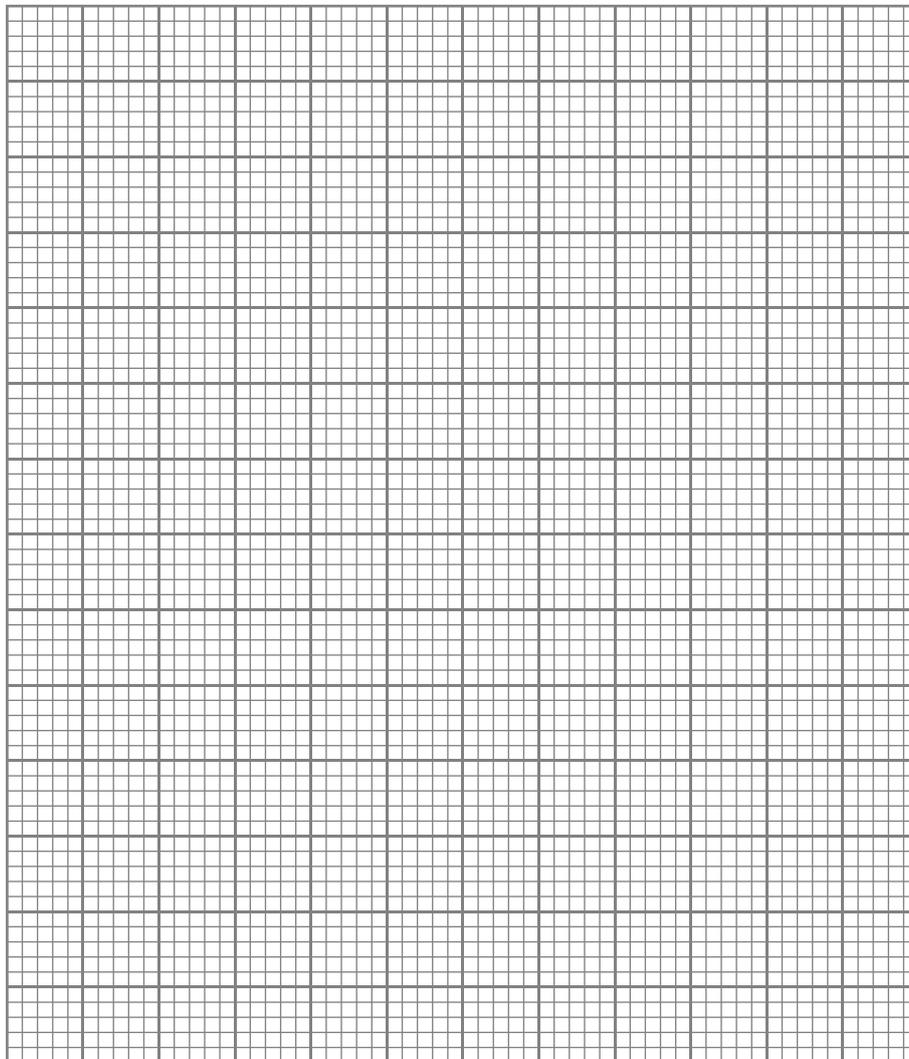
Both types of party bag are sold at a profit of 15p per bag. All the bags are sold. The company wishes to maximise its profit.

Let  $x$  be the number of Infant bags produced and  $y$  be the number of Junior bags produced per hour.

- (a) Formulate the above situation as a linear programming problem.

(5)

- (b) Represent your inequalities graphically, indicating clearly the feasible region.



(6)

- (c) Find the number of Infant bags and Junior bags that should be produced each hour and the maximum hourly profit. Make your method clear.

(3)

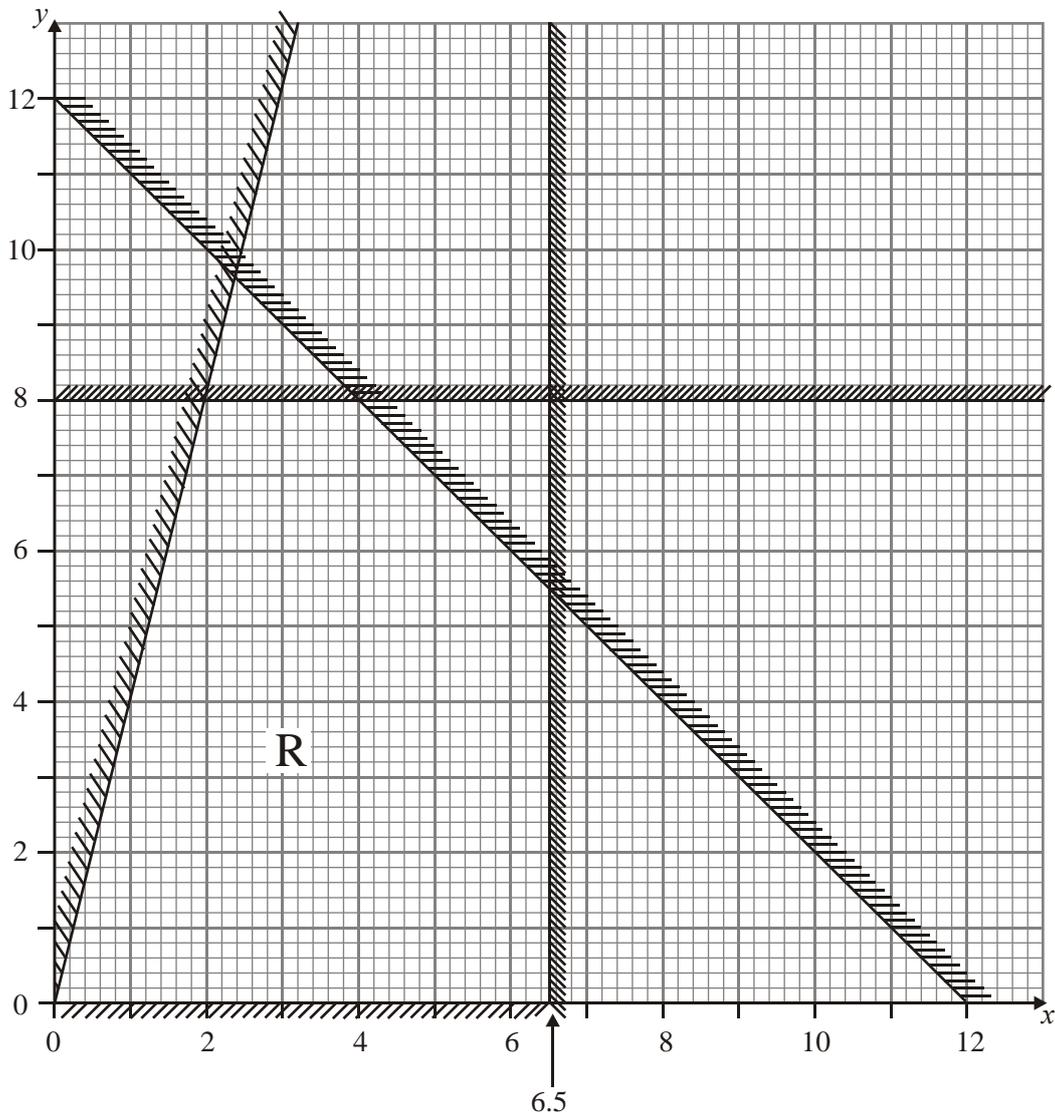
In order to increase the profit further, the company decides to buy additional equipment. It can buy equipment to increase the production of **either** sweets **or** stickers, but **not both**.

- (d) Using your graph, explain which equipment should be bought, giving your reasoning. (2)

The manager of the company does not understand why the balloons, toys and whistles have not been considered in the above calculations.

- (e) Explain briefly why they do not need to be considered. (2)
- (Total 18 marks)

9.



The company EXYCEL makes two types of battery, X and Y. Machinery, workforce and predicted sales determine the number of batteries EXYCEL make. The company decides to use a graphical method to find its optimal daily production of X and Y.

The constraints are modelled in the diagram above where

$x$  = the number (in thousands) of type X batteries produced each day,

$y$  = the number (in thousands) of type Y batteries produced each day.

The profit on each type X battery is 40p and on each type Y battery is 20p. The company wishes to maximise its daily profit.

- (a) Write this as a linear programming problem, in terms of  $x$  and  $y$ , stating the objective function and all the constraints. (6)
- (b) Find the optimal number of batteries to be made each day. Show your method clearly. (3)
- (c) Find the daily profit, in £, made by EXYCEL. (2)
- (Total 11 marks)**

10. The Young Enterprise Company “Decide”, is going to produce badges to sell to decision maths students. It will produce two types of badges.

Badge 1 reads “I made the decision to do maths” and  
Badge 2 reads “Maths is the right decision”.

“Decide” must produce at least 200 badges and has enough material for 500 badges.

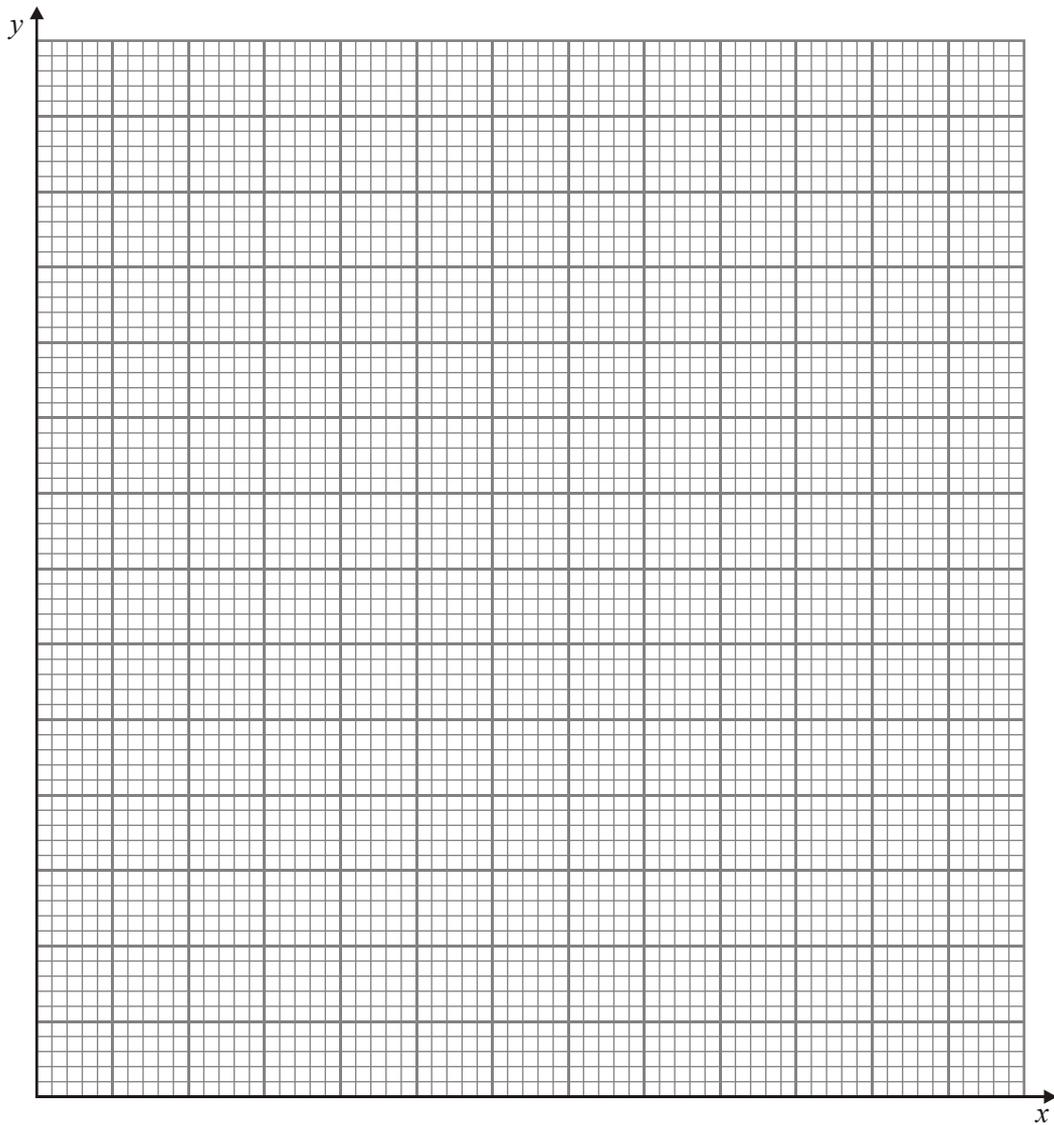
Market research suggests that the number produced of Badge 1 should be between 20% and 40% of the total number of badges made.

The company makes a profit of 30p on each Badge 1 sold and 40p on each Badge 2. It will sell all that it produced, and wishes to maximise its profit.

Let  $x$  be the number produced of Badge 1 and  $y$  be the number of Badge 2.

- (a) Formulate this situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers. (6)

- (b) On the grid provided below, construct and clearly label the feasible region.



(5)

- (c) Using your graph, advise the company on the number of each badge it should produce. State the maximum profit “Decide” will make.

(3)

**(Total 14 marks)**

11. Becky's bird food company makes two types of bird food. One type is for bird feeders and the other for bird tables. Let  $x$  represent the quantity of food made for bird feeders and  $y$  represent the quantity of food made for bird tables. Due to restrictions in the production process, and known demand, the following constraints apply.

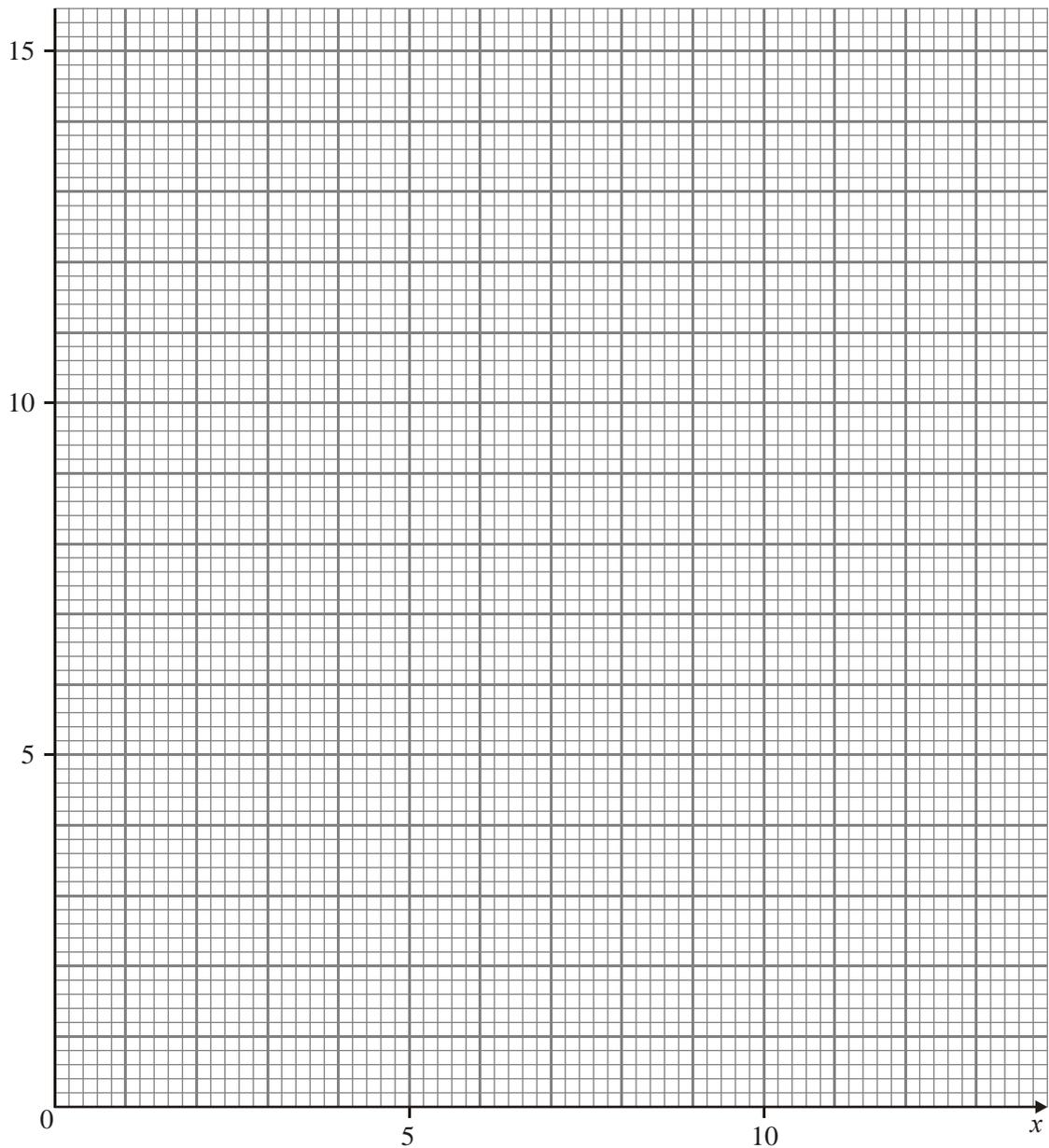
$$x + y \leq 12,$$

$$y < 2x,$$

$$2y \geq 7,$$

$$y + 3x \geq 15.$$

- (a) On the axes provided, show these constraints and label the feasible region  $R$ .



(5)

The objective is to minimise  $C = 2x + 5y$ .

- (b) Solve this problem, making your method clear. Give, as fractions, the value of  $C$  and the amount of each type of food that should be produced.

(4)

Another objective (for the same constraints given above) is to maximise  $P = 3x + 2y$ , where the variables must take integer values.

- (c) Solve this problem, making your method clear. State the value of  $P$  and the amount of each type of food that should be produced.

(4)

(Total 13 marks)

12. A company produces two types of self-assembly wooden bedroom suites, the ‘Oxford’ and the ‘York’. After the pieces of wood have been cut and finished, all the materials have to be packaged. The table below shows the time, in hours, needed to complete each stage of the process and the profit made, in pounds, on each type of suite.

	Oxford	York
Cutting	4	6
Finishing	3.5	4
Packaging	2	4
Profit (£)	300	500

The times available each week for cutting, finishing and packaging are 66, 56 and 40 hours respectively.

The company wishes to maximise its profit.

Let  $x$  be the number of Oxford, and  $y$  be the number of York suites made each week.

- (a) Write down the objective function.

(1)

- (b) In addition to

$$2x + 3y \leq 33,$$

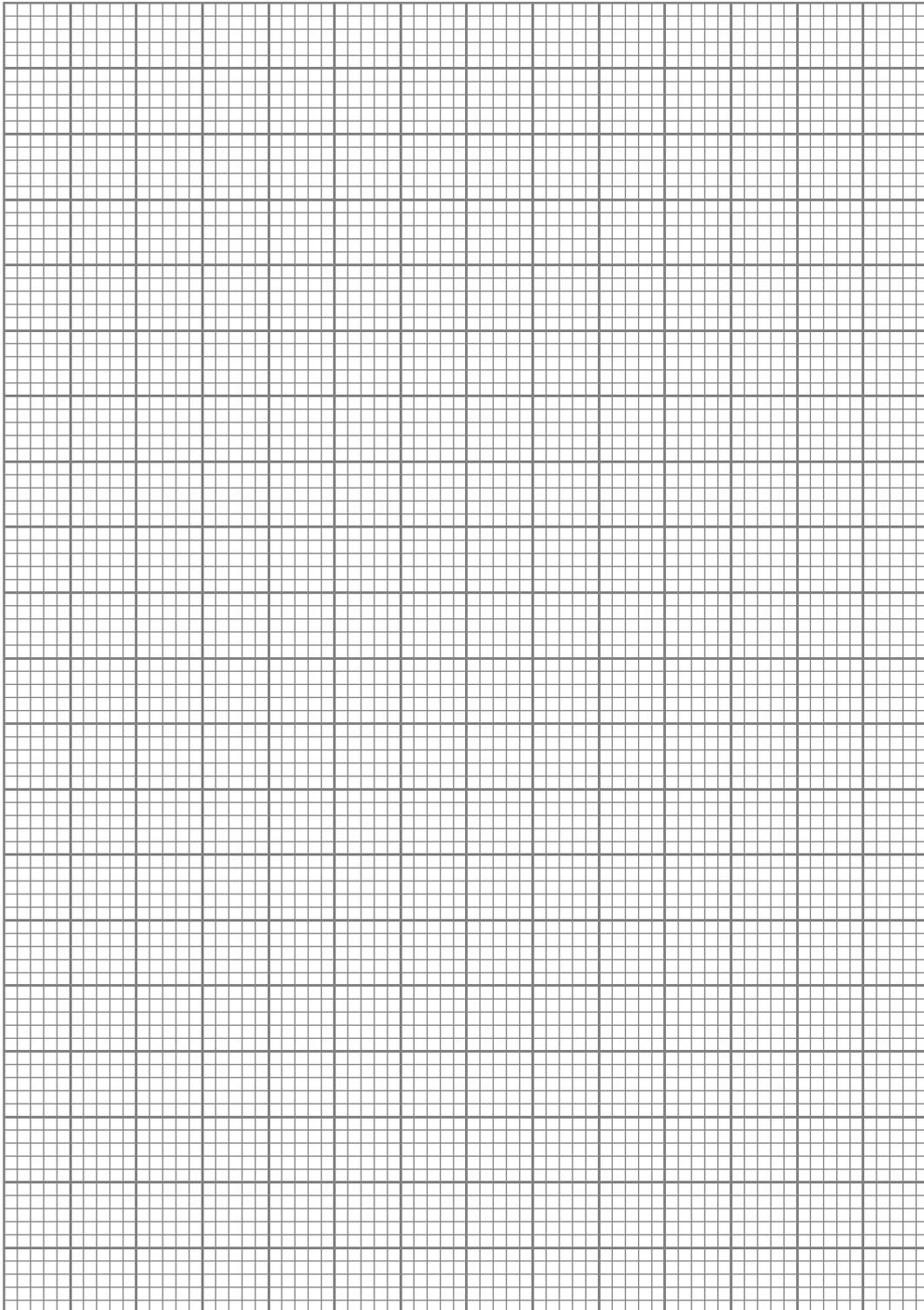
$$x \geq 0,$$

$$y \geq 0,$$

find two further inequalities to model the company’s situation.

(2)

- (c) On the grid below, illustrate all the inequalities, indicating clearly the feasible region.



(4)

- (d) Explain how you would locate the optimal point.

(2)

- (e) Determine the number of Oxford and York suites that should be made each week and the maximum profit gained.

(3)

It is noticed that when the optimal solution is adopted, the time needed for one of the three stages of the process is less than that available.

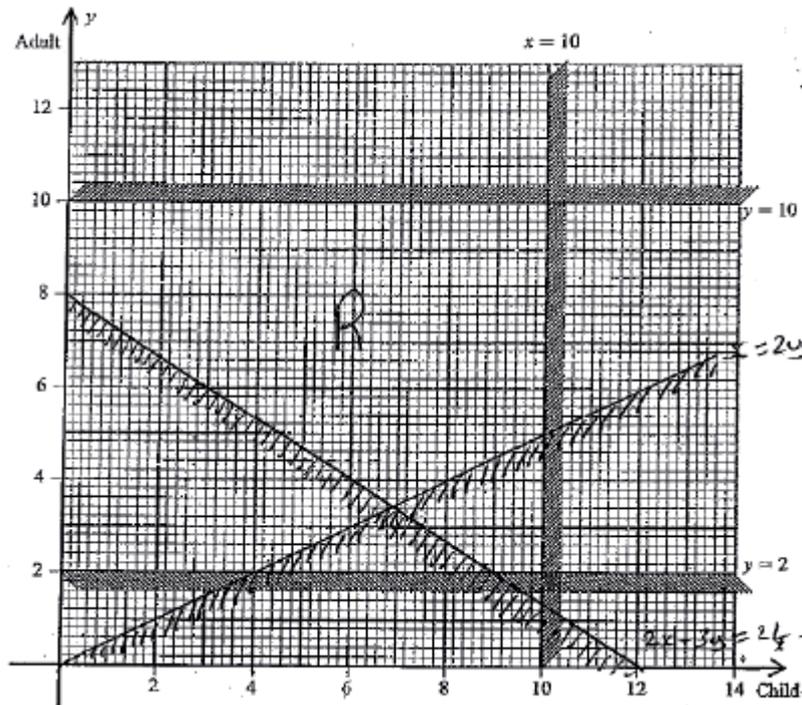
- (f) Identify this stage and state by how many hours the time may be reduced.

(3)

**(Total 15 marks)**

1. (a) To show a strict inequality B1 1
- (b) There must be fewer than 10 children B1  
 There must be between 2 and 10 adults inclusive B2, 1, 0 3
- (c)  $2x + 3y \geq 24$  B1  
 $x \leq 2y$  B1 2

(d)



- B1ft ( $2x + 3y = 24$ )  
 B1ft ( $x = 2y$ )  
 B1ft (shading)  
 B1 4

(e)	Minimum	0 Children	8 Adults	-8 Passengers	M1A1	
	Maximum	9 Children	10 Adults	-19 Passengers	B1 B1	4

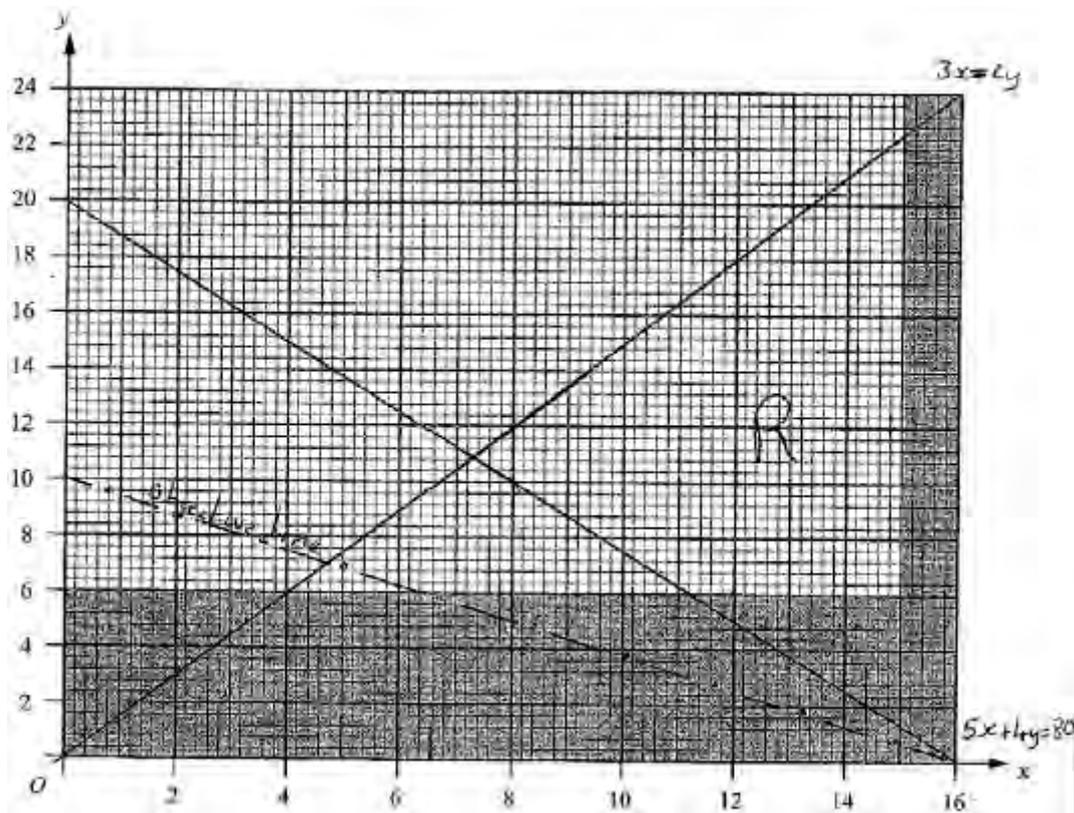
[14]

2. (a) To indicate the strict inequality B1 1

**Note**

1B1: CAO

(b)  $3x = 2y$  and  $5x + 4y = 80$  added to the diagram. B1, B1  
 R correctly labelled. B1 3



**Note**

1B1:  $3x = 2y$  passing through 1 small square of (0, 0) and (12, 18), but must reach  $x = 15$

2B1:  $5x + 4y = 80$  passing through 1 small square of (0, 20) and (16, 0) (extended if necessary) but must reach  $y = 6$

3B1: R CAO (condoning slight line inaccuracy as above.)

(c) [Minimise C =]  $500x + 800y$  B1, B1 2

**Note**

1B1: Accept expression and swapped coefficients. Accept  $5x + 8y$  for 1 mark

2B1: CAO (expression still ok here)

- (d) Point testing or Profit line M1 A1  
 Seeking integer solutions M1  
 (11, 7) at a cost of £ 11 100. B1, B1 5

**Note**

1M1: Profit line [gradient accept reciprocal, minimum length line passes through (0, 2.5) (4, 0)] **OR** testing 2 points in their FR near two different vertices.

1A1: Correct profit line **OR** 2 points correctly tested in correct FR (my points)

e.g

$(7\frac{3}{11}, 10\frac{10}{11}) = 12\,363\frac{7}{11}$	or	$(7, 11) = 12\,300$
		$(8, 10) = 12\,000$
		$(8, 11) = 12\,800$
$(11\frac{1}{5}, 6) = 10\,400$	or	$(11, 6) = 10\,300$
$(15, 6) = 12\,300$	or	$(15, 7) = 13\,100$
$(15, 22\frac{1}{2}) = 25\,500$	or	$(15, 22) = 25\,100$
$(11, 7) = 11\,100$		

2M1: Seeking integer solution in correct FR (so therefore no  $y = 6$  points)

1B1: (11, 7) CAO

2B1: £11 100 CAO

[11]

3. (a)  $x + 2y \leq 12$  ( $150x + 300y \leq 1800$ ) M1A1 2

**Note**

1M1 – correct terms, accept = here, accept swapped coefficients.

1A1 – cao does not need to be simplified.

- (b)  $0.9x + 1.2y \leq 9$  M1  
 $\rightarrow 3x + 4y \leq 30$  (\*) A1 cso 2

**Note**

1M1 – correct terms, must deal with cm/m correctly, accept = here.

1A1 – cso **answer given.**

- (c) (You need to buy) at least 2 large cupboards. B1 1

**Note**

1B1 – cao ‘at least’ and ‘2’ and ‘large’.

- (d) Capacity C and 140% C

So total is  $Cx + \frac{140}{100}Cy$

M1

Simplify to  $7y + 5x$  (\*)

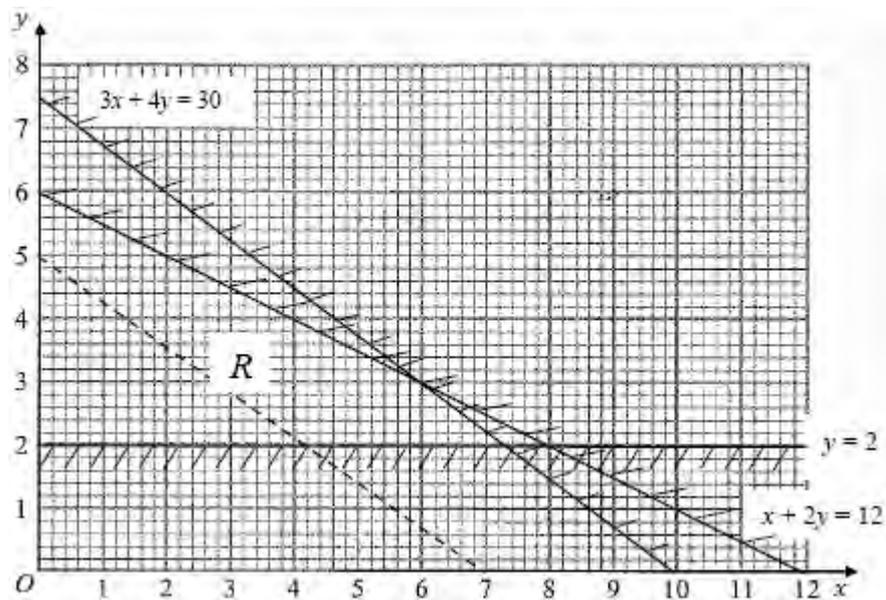
A1cso 2

**Note**

1M1 – ‘1.4’ or ‘5 × 40%’ maybe ‘5+2’ seen, they **must** be seen to engage with 140% in some way.

1A1 – cso **answer given.**

- (e)



Graph:

- |  |          |
|--|----------|
| $y \geq 2$                                   | B1       |
| $0.9x + 1.2y \leq 12$ ( $3x + 4y \leq 30$ )  | B1       |
| $x + 2y \leq 12$ ( $150x + 300y \leq 1800$ ) | B1       |
| Lines labelled & drawn with a ruler          | B1       |
| Shading, Region identified                   | B1, B1 6 |

**Note**

**Lines should be within 1 small square of correct point at axes.**

1B1 – correctly drawing  $y = 2$ .

2B1 – correctly drawing  $3x + 4y = 30$   
 $[0.9x + 1.2y = 12]$

3B1 – correctly drawing  $x + 2y = 12$   
 $[150x + 300y = 1800]$ , **ft only** if swapped coefficients in (a) (6,0) (2,8).

**These next 3 marks are only available for candidates who have drawn at least 2 lines, including at least one ‘diagonal’ line with negative gradient.**

4B1 – Ruler used. At least 2 lines labelled including one ‘diagonal’ line.

5B1 – Shading, or R correct, b.o.d. on their lines.

6B1 – all lines and R correct.

(f) Consider points and value of  $5x + 7y$ : M1A1

Or draw a clear profit line

(7,2) → 49 or  $(7\frac{1}{3}, 2)$   $50\frac{2}{3}$ , or (7.3, 2)  
 → 50.5

(6,3) → 51

(0,6) → 42 A1

(0,2) → 14

Best option is to buy 6 standard cupboards  
 and 3 large cupboards. A1 4

**Note**

1M1 At least 2 points tested **or** objective line drawn with correct m or 1/m, minimum intercepts 3.5 and 2.5.

1A1 – 2 points correctly tested **or** objective line correct.

2A1 – 3 points correctly tested **or** objective line correct and distinct/labelled.

3A1 – 6 standard and 3 large, accept (6,3) if very clearly selected in some way.

[17]

4. (a)  $7x + 5y \leq 350$  M1 A1 2

**Note**

1M1: Coefficients correct (condone swapped  $x$  and  $y$  coefficients) need 350 and any inequality

1A1: cso.

- (b)  $y \leq 20$  e.g. make at most 20 small baskets B1

$Y \leq 4x$  e.g. the number of small ( $y$ ) baskets is at most 4 times the number of large baskets ( $x$ ).

B1 2

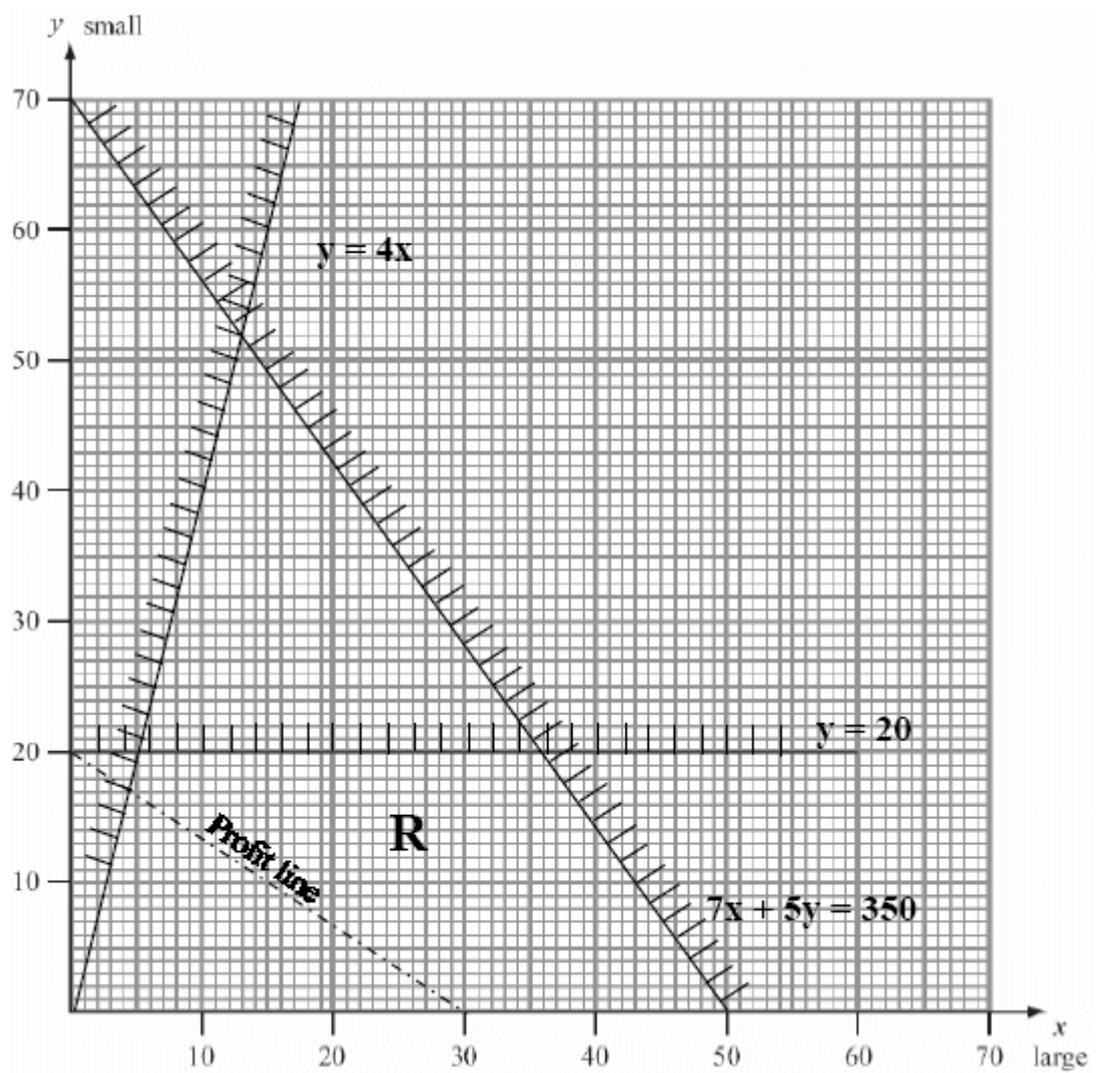
{E.g if  $y = 40$ ,  $x = 10, 11, 12$  etc. or  
if  $x = 10$ ,  $y = 40, 39, 38$ }

**Note**

1B1: cao

2B1: cao, test their statement, need both  
= and < aspects.

(c)



Draw three lines correctly

B3 2 1 0

Label R

B1 4

**Note**

- 1B1: One line drawn correctly
- 2B1: Two lines drawn correctly
- 3B1: Three lines drawn correctly.  
Check (10, 40) (0, 0) and axes
- 4B1: R correct, but allow if one line is slightly out (1 small square).

(d)  $(P=) 2x + 3y$

B1 1

**Note**

- 1B1: cao accept an expression.

- (e) Profit line or point testing. M1 A1  
 $x = 35.7$   $y = 20$  precise point found. B1  
 Need integers so optimal point in R is  
 (35, 20); Profit (£)130 B1;B1 5

**Note**

1M1: Attempt at profit line or attempt to test at least two vertices in their feasible region.

1A1: Correct profit line or correct testing of at least three vertices.

**Point testing:** (0,0) P= 0; (5,20) P = 70; (50,0) P = 100

$$\left(35\frac{5}{7}, 20\right) = \left(\frac{250}{7}, 20\right) P = 131\frac{3}{7} = \frac{920}{7}$$

also (35, 20) P = 130. Accept (36,20)  
 P = 132 for M but not A.

**Objective line:** Accept gradient of 1/m for M mark or line close to correct gradient.

1B1: cao – accept x co-ordinates which round to 35.7

2B1: cao

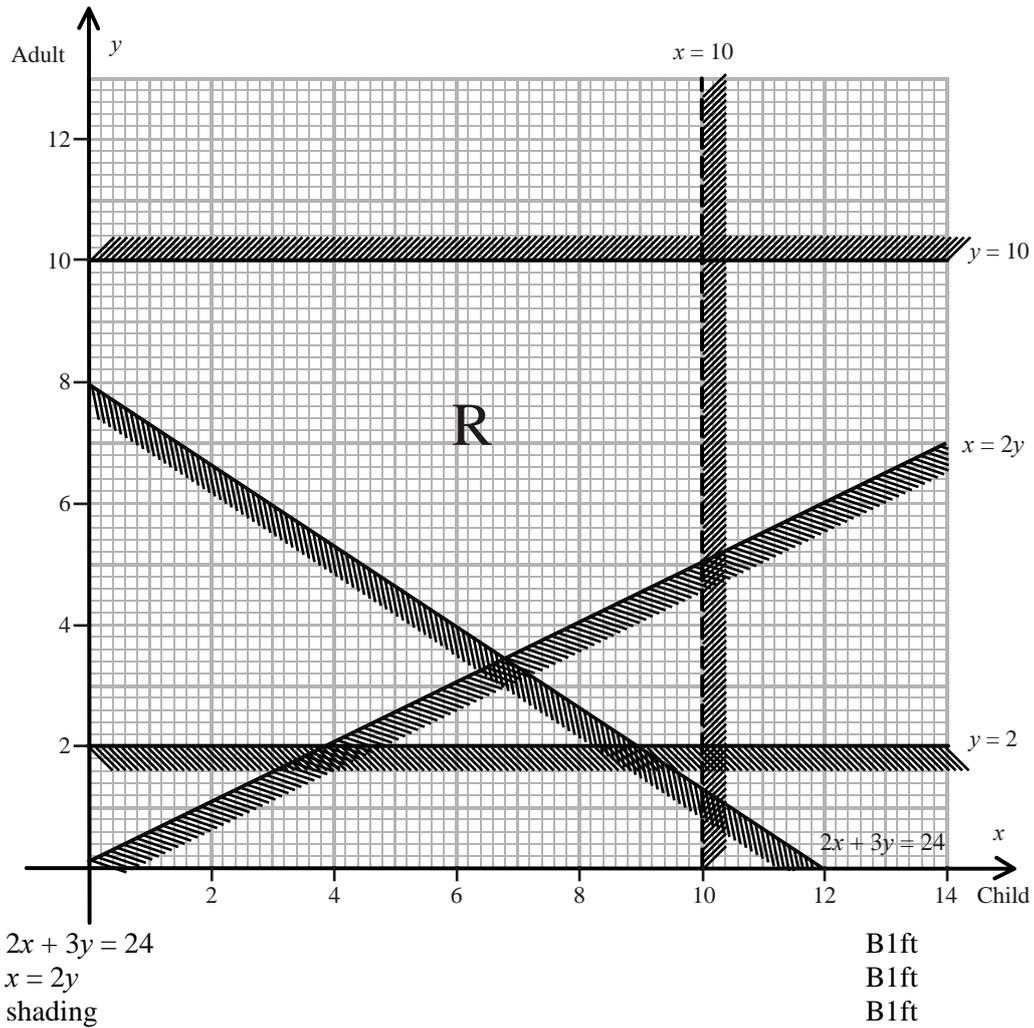
3B1: cao

[14]



6.	(a)	$y \geq 2x$	B2,1,0	2	
	(b)	$x + 2y = 160$ correctly drawn $y = 60$ correctly drawn and distinctive (strict inequality) shading correct	B4,3,2,1,0	4	
	(c)	R correct	B1ft	1	
	(d)	Profit line added or Point testing seen correctly done. 70 boxes identified	M1 A1 B1	3	
	(e)	$(P =) 1.2x + 1.4y$	B1	1	
	(f)	Profit line added or Point testing seen correctly done (32, 64) identified	M1 A1ftA1 B1	4	
	(g)	£128.00	B1	1	
					<b>[16]</b>
7.	(a)	To show a strict inequality	B1	1	
	(b)	There must be fewer than 10 children There must be between 2 and 10 adults, inclusive.	B1 B2,1,0	3	
	(c)	$2x + 3y \geq 24$ $x \leq 2y$	B1 B1	2	

(d)



(e) minimum 0 children 8 adults – 8 passenger  
 maximum 9 children 10 adults – 19 passengers

M1A1  
 B1B1    4

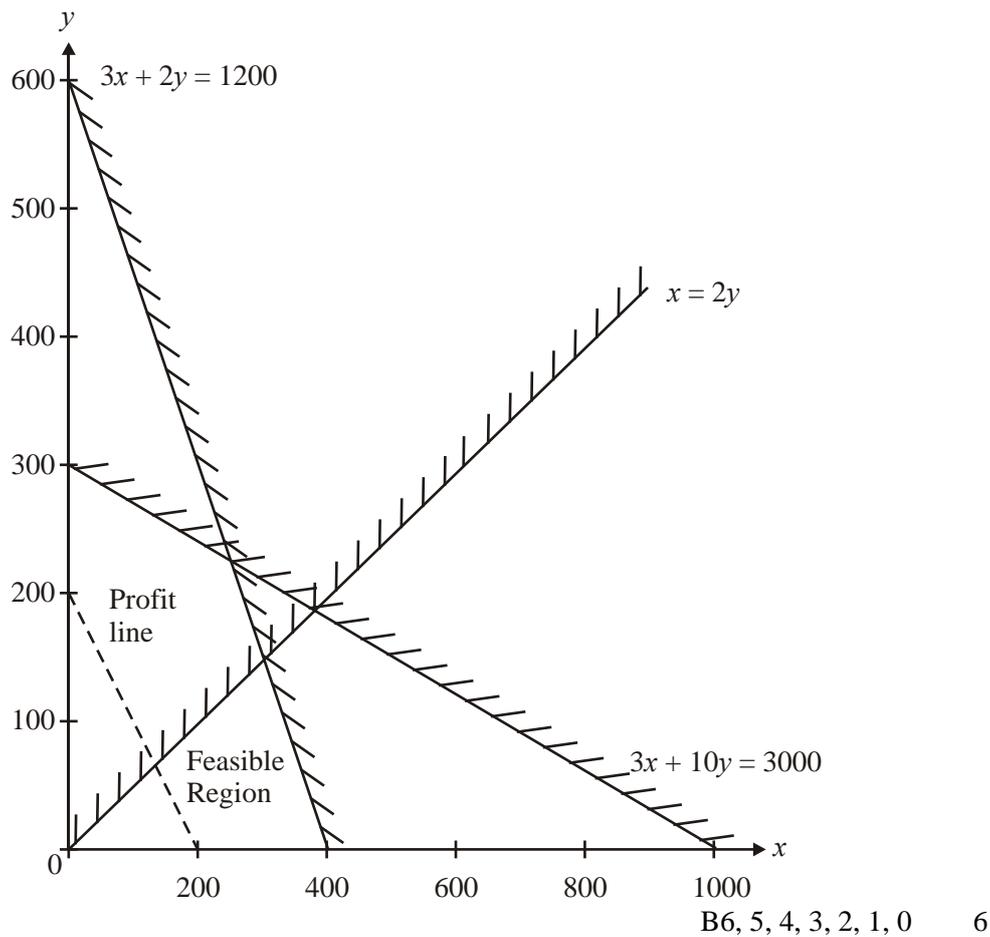
[14]

8. (a) Maximise, (P =)  $15x + 15y$

Subject to  $3x + 10y \leq 3000$   
 $3x + 2y \leq 1200$   
 $x \geq 2y$   
 $x, y \geq 0$

B1, B1  
 B3, 2, 1, 0    5  
 B3, 2, 1, 0

(b)



(c) Profit line or vertex testing, (300, 150). Profit = £67.50    M1 A1ft A1ft    3

(d) Production of stickers should be increased since this would be more the intersection point further from the origin.    B2, 1ft, 0    2

(e) e.g. The constraint line, would be for outside the feasible region – so they would not effect it.    B2, 1, 0    2

[18]

9. (a) Maximum (P=)  $0.4x + 0.2y$     B1  
*accept*  $40x + 20y$   
 Subject to  $x \leq 6.5$   
 $y \leq 8$   
 $x + y \leq 12$     B5,4,3,2,1,0    6  
 $y \leq 4 - x$   
 $y \geq 0$

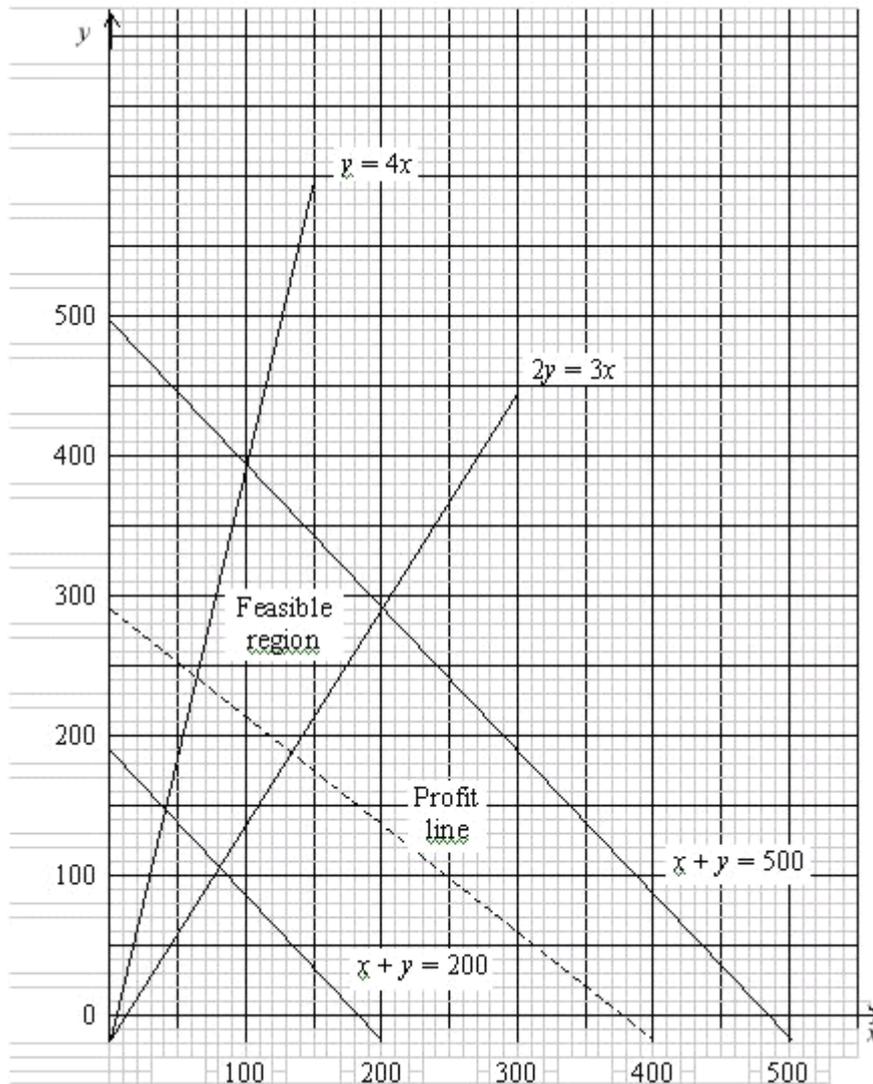
(b) Point testing or Profit line  
 $(6.5, 5.5) \Rightarrow 65\omega$  type  $x$  and  $55\omega$  type  $y$  M1A1A1 3

(c)  $P = 0.4(65\omega) + 0.22(55\text{co})$   
 $= \text{£}3.7 \text{ co}$  M1  
 A1 2

[11]

10. (a) Maximise  $P = 30x + 40y$  (or  $P = 0.3x + 0.4y$ ) B1  
 subject to  $\underline{x + y \geq 200}$  B1  
 $\underline{x + y \leq 500}$  B1  
 $x \geq \frac{20}{100}(x + y) \Rightarrow 4x \geq y$  M1 A1  
 $x \leq \frac{40}{100}(x + y) \Rightarrow 3x \geq 2y$  A1 6

(b)



B5,4,3,2,1,0 5

$(x + y = 200, x + y = 500)$

B1 ft

$(y = 4x)$

B1 ft

$(2y = 3x)$

B1 ft

(labels)

B1 ft

FR

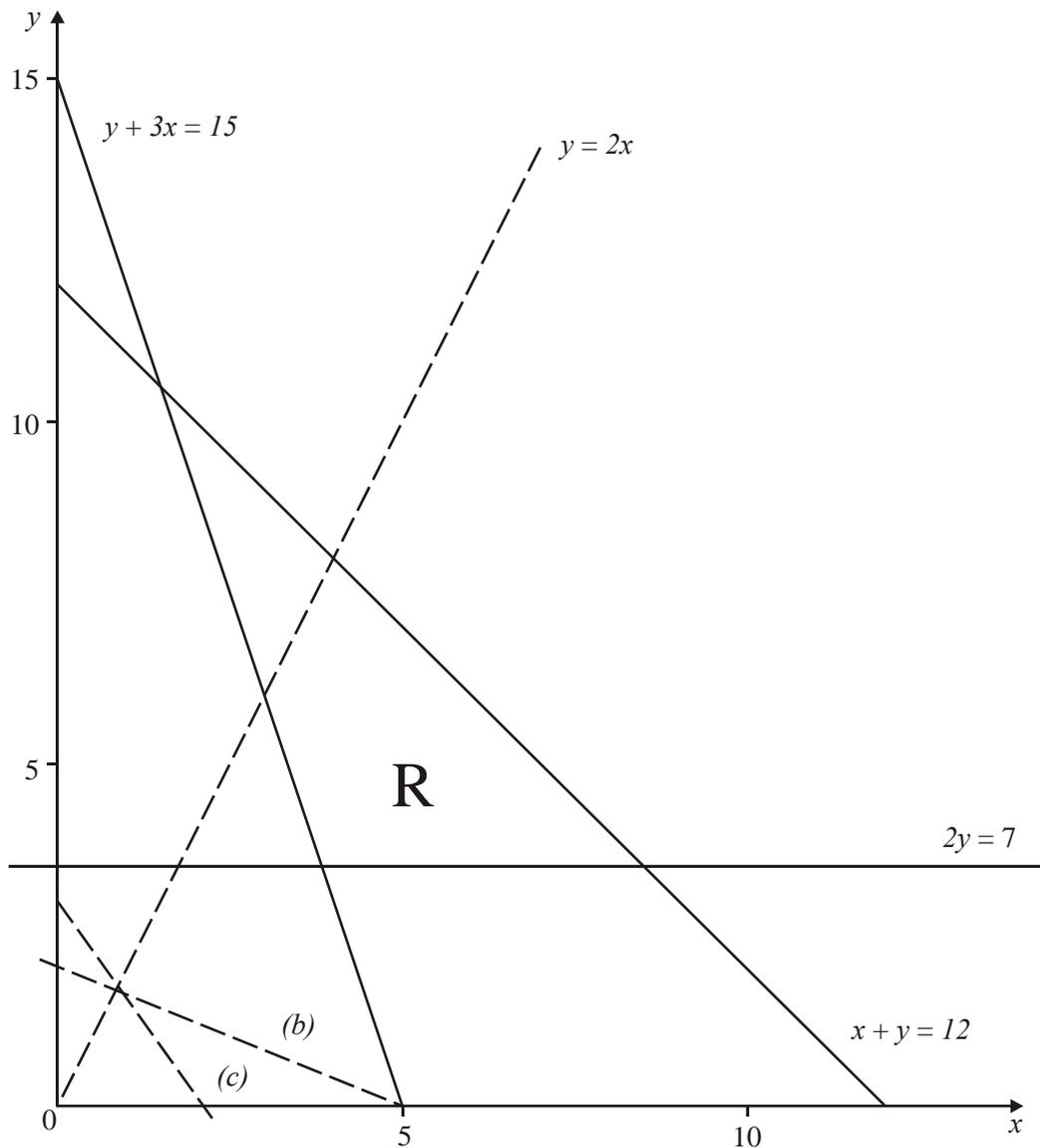
B1

(NB: Graph looks OK onscreen at 75% magnification but may print out misaligned)

- (c) Point testing or profit line  
 Intersection of  $y = 4x$  and  $x + y = 500$   
 (100, 400) Profit = £190 (units must be clear)
- |    |   |
|----|---|
| M1 |   |
| A1 |   |
| A1 | 3 |

[14]

11. (a)



B5, 4, 3, 2, 1, 0

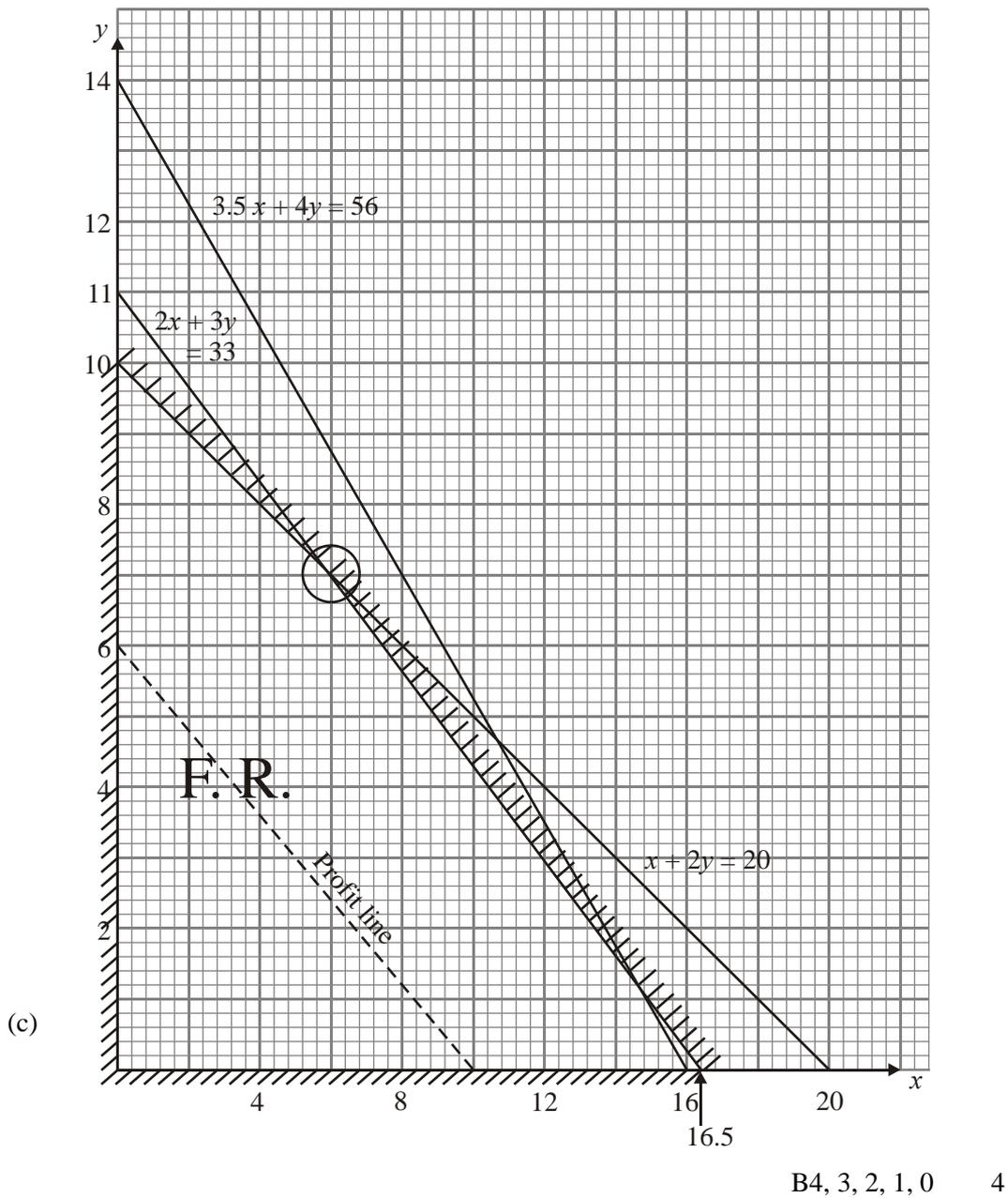
5

- (b) Either point testing or profit line M1  
 $A (3\frac{5}{6}, 3\frac{1}{2}) \rightarrow 25\frac{1}{6}, B (8\frac{1}{2}, 3\frac{1}{2}) \rightarrow 34\frac{1}{2},$   
*Accept C (4,8) → 48 and D (3,6) → 36*  
 Profit line gradient  $-\frac{2}{5}$  A1  
 Identifies  $A (3\frac{5}{6}, 3\frac{1}{2})$  cost  $25\frac{1}{6}$  A1, A1 4

- (c) Either point testing or profit line M1  
 $A (3\frac{5}{6}, 3\frac{1}{2}) \rightarrow$  not integer so try (4, 4) → 20 Profit line  
 $B (8\frac{1}{2}, 3\frac{1}{2}) \rightarrow$  not integer so try (8,4) → 32  
 → try (7,5) → 31 gradient  $-\frac{3}{2}$   
*Accept C (4,8) → 28 and D (3,6) → 21* A1  
 Identifies (8,4) profit 32. A1 A1 4

**[13]**

12. (a)  $(P =) 300x + 500y$  B1  
 (b) Finishing  $3.5x + 4y \leq 56 \Rightarrow 7x + 8y \leq 112$  (or equivalent) B1  
 Packing  $2x + 4y \leq 40 \Rightarrow x + 2y \leq 20$  (or equivalent) B1 3



- (d) e.g.:
- Point testing:* test corner points in feasible region
  - find profit at each and select point yielding maximum
  - Profit line:* draw profit lines
  - select point on profit line furthest from the origin      B2,1,0      2
- (e) Using a correct, complete method      M1
- make 6 Oxford and 7 York      Profit = £5300      A1 ft A1 ft      3
- (f) The line  $3.5x + 4y = 49$  passes through (6, 7)
- so reduce finishing by 7 hours      M1 A1 ft A1      3

[15]

1. No Report available for this question.

2. This question gave rise to a good spread of marks. Most candidates completed part (a) correctly although some very lengthy responses were seen.  $5x + 4y = 80$  was drawn correctly more often than  $3x = 2y$  in part (b), with many candidates drawing the latter with a negative gradient. Pleasingly most candidates used a ruler to draw their lines, a great improvement on previous years. The feasible region was often incorrectly identified and labels were often absent.

Most were able to complete part (c) correctly.

Those who used the objective line method in (d) usually gained more marks than those who used the point testing method. Some of those using the latter method seemed confused by the y-axis scale and only considered vertices with even values of y, many tested points by reading from the graph rather than solving simultaneous equations.

A large number of solutions had  $y = 6$  despite answering part (a) correctly. Some found the maximum solution. Many did not make their method clear.

3. Most candidates were able to score at least 12 out of 17 marks on this question. Parts (a), (b) and (c) were usually correct, with only a very few making slips with the inequality in (a) or muddling 'small' with 'large' in part (c). The units in part (b) caused difficulty for some candidates, but most changed all lengths into cm and proceeded correctly. Many candidates struggled with part (d). When the answer is printed on the paper candidates must ensure that their reasoning is both clear and convincing, disappointingly, many candidates were not able to derive the given result and in particular many 'derivations' attempted to start with  $1.4y = x$ . There were many fully correct graphs, helped by widespread use of rulers, a big improvement from past papers. Three correct lines almost always invariably led to the correct region. As always, some lost a mark because they did not label their lines and/or R. In (f) both the vertex testing and profit line methods were often successful. As always it is vital that the method is clearly seen, some lost all 4 marks in (f) because they merely described the use of a profit line but failed to draw it. Others drew a very short profit line – candidates should use sufficiently large values for the axes intercepts to ensure an accurate gradient. Those using the vertex method should be reminded that all vertices should be tested, a number of candidates only tested one or two vertices.

4. There were some very good, and very poor, solutions seen to this question. Almost all candidates were able to write down the correct inequality in part (a) with only a very few getting the wrong coefficients or replacing the inequality with an equals sign. Part (b) proved challenging for many candidates. Candidates struggled in particular to interpret  $y \leq 4x$ . The usual error was to confuse 'small' with 'large' but many failed to refer to, or reversed, the inequality. The most able described the inequality in terms of percentages; where this was seen it was almost always correct. Most candidates drew  $5x + 7y = 350$  and  $y = 20$  correctly. Most candidates used a ruler and most plotted the axes interceptions accurately. Unsurprisingly  $y = 4x$  caused the most difficulty, often replaced by  $x = 4y$ . Most candidates used shading sensibly although some shaded so scruffily that they obscured their line. Most candidates labelled R correctly; most candidates did not label their lines. Most candidates were able to write down the correct objective function. Part (e) was often poorly done with many candidates failing to make their method clear; if using the objective line method candidates MUST draw an objective line, and of a sensible length, so that its accuracy can be checked; if using point testing then the points and their values must be stated. As always those who use the objective line method are more successful than those who use point testing. When point testing, all vertices in the feasible region must be tested. Many candidates assumed that the point (36, 20) was a vertex; it was pleasing to see a small number of scripts where this was tested and found to be outside the feasible region. Others found the precise point but then did not seek integer solutions to complete their answer.

5. While a number of candidates scored the full 12 marks on this question, many could only achieve a handful of marks.

The first two lines were often drawn correctly however  $y = 4x$  was frequently incorrect. The most common error was to draw  $y = (1/4)x$ , not drawing the line long enough, or not noticing the differing scales on the axes. The shading on two of the lines was often correct but only the better candidates were able to get the shading correct on all three lines, so many gave the FR as the central triangle. Many candidates did not label their lines. There continues to be evidence that candidates are not going in to the examination properly equipped with (30cm) rulers.

In part (b) candidates who gained full marks did so most frequently and easily by using the objective line method. If errors were seen in this method, it was often due to a reciprocal gradient, or an inability to read the scale on the graph accurately. A significant number of candidates continue to state they are using this method without showing any evidence of this, these gain no credit. If the objective line method is being used, the examiners need to see an accurately drawn objective line (of decent length). Those who chose to use the point testing method frequently lost marks through the inaccuracy of their extreme points, or by not testing all of their extreme points. There was a tendency to try to read the coordinates from the LP-graphs, and unsurprisingly the point  $(15 \frac{5}{9}, 62 \frac{2}{9})$  was therefore rarely seen. Candidates should be reminded that if they choose to use the vertex method they must show their complete working, and will be expected to use simultaneous equations to find the exact coordinates of any vertices. A few candidates stated that they were using this method but then tested points other than the vertices of their FR, another frequently seen error was to find the coordinates of the vertices but then not use them to find any F values.

6. This question proved challenging for many. Some candidates used very lengthy methods to determine the equation of the straight line in part (a), and many got the 2 on the wrong side. Many were able to determine the correct inequality but others reversed it. The lines in part (b) were sometimes very disappointing, many failed to label their lines and/or make  $y = 60$  distinctive, but most disappointing of all was the lack of ruler use and inaccurate plotting of axis intercepts. Part (c) was often well done with examiners following through on candidate's lines were possible. Many candidates identified an optimal point in part (d) but some failed to state that 70 was the total number of boxes, the method used was not always apparent to the examiners and candidates need to be reminded that they should always make their method clear on this 'methods' paper.

Most of the candidates who did display their method used point-testing. Almost all the candidates were able to find an objective function in part (e). Once again many candidates did not make their method clear in part (f). If point-testing candidates should list the points they are testing and show the value of the objective at each point. If using a profit line they should clearly draw a profit line - of good length and label it. Point testing proved a more popular method in this question, but few tested all five vertex points. Poor notation was often seen with a number of candidates writing  $32x + 64y$  instead of  $x = 32, y = 64$ . Those who located the optimal point in part (e) were usually successful in calculating the profit in part (f).

7. Parts (a) and (b) were often well-answered, the commonest error was imprecise use of language, especially use of the phrase 'between 2 and 10', without the word 'inclusive'. Although there were the usual sign reversals and some strict inequalities seen, part (c) was often well-answered with the large majority of candidates successfully finding the first inequality. The second inequality proved more difficult, with the 2 often on the wrong side. Most candidates were able to draw  $2x + 3y = 24$ , but there was confusion (both ways) between  $x = 2y$  and  $y = 2x$ . Some candidates made shading errors and even the best rarely followed the instructions to label the feasible region. Candidates MUST show their working when seeking maximum and minimum points. There was often no evidence of any method being used. If candidates use the point testing method, they should state the points they are testing and the result of the test, if they are using the 'profit line' method, they must draw in a clearly labelled profit line – and this must be long enough for examiners to confirm the accuracy of the gradient. Some candidates tested their point using the cost equation rather than simply totalling the number of passengers. Many candidates simply stated the maximum and minimum totals rather than the number of adults and

8. Most candidates were able to make some progress with part (a), most correctly stated the objective function but often the objective was omitted. The non-negativity constraints were often omitted and many had difficulty in finding the  $x \geq 2y$  inequality. The examiners were all disappointed by the standard of the graph work seen in part (b). Lines were often imprecisely drawn or omitted,  $x = 2y$  (if found in part (a)) was often incorrectly drawn. Labels, scales and/or shading were often omitted and the feasible region was not always indicated. Not all candidates used sharp pencils and rulers. If candidates are going to use the profit line method in part (c) they must draw in, and label, a profit line which should long enough to enable examiners to check the gradient. If candidates are going to use the point testing method they must state and test **every** vertex point in the feasible region, not just the most likely point. Many candidates did not state the profit, and of those that did, some did not state units. Those who drew a correct graph generally answered part (d) well. Part (e) was often well-answered but there were many irrelevant comments seen.
9. Many omitted the instruction to maximise the objective. Most candidates were able to write down at least 3 constraints correctly, although few obtained them all.  $y=0$ , was often omitted and there were often errors in writing down  $x + y = 12$  and  $y = 4x$ . The solution of those candidates using point testing in part (b) was often spoilt by using incorrect coordinates. The y coordinate of the point with x coordinate 6.5 was frequently misstated. Those using the objective line method must draw the line clearly on the diagram, and in such a way that the gradient can be seen to be correct. The clearest way in this case is to draw the line from e. g. (2, 0) to (0, 4). A number of candidates did not pick up that the values of x and y represented 1000's of batteries and this together with problems with pounds and pence caused much confusion in part (c), although many completely correct answers were seen.
10. This question was often poorly done. Poor algebra was often seen in part (a). Most candidates were able to state the objective function but did not state that this was to be maximised. The  $x + y$  inequalities were better handled. Only the better candidates were able to correctly handle the 20% and 40% inequalities. A number tried to combine the four inequalities into two constraints. In part (b) the lines  $x + y = 200$  and  $x + y = 500$  were plotted correctly but the other lines very poorly plotted. Many candidates drew vertical or horizontal lines. Candidates should use rulers and sharp pencils to draw lines. Labels were frequently omitted both of the lines and the feasible region. Most who used the 'profit line' method got the correct answer although a few drew lines with the reciprocal gradient. Some candidates using the 'vertex method' considered that they only need check two points and not all four.

11. In part (a) the lines  $x + y = 12$  and  $y + 3x = 15$  were well drawn by most candidates. The lines  $2y = 7$  and  $y = 2x$  and the identification of the feasible region were less well done. Many did not make any distinction between way they drew the line  $y = 2x$  and the other lines, not picking up on the strict inequality. Others did not use rulers to draw their lines and others did not label their lines. In parts (b) and (c) candidates could use either the point testing method or the profit line method: generally those using the latter were more successful.

The presentation of the solution was sometimes very difficult to follow. Those using the profit line method must draw, and label, a profit line so that its gradient can be checked and the method seen. Those using point testing must make the points they are testing and the value of the function at these points clear. The fractions caused problems for many candidates. Part (c) was less well done with many candidates ignoring the integer constraint.

12. Parts (a) and (b) were well done by the vast majority of candidates. Most candidates were able to draw the lines correctly in part (c) but many did not label their lines or the axis. Some very poor choices of scale were often seen, so that sometimes the whole feasible region was not shown, or was too small to be useful. Most candidates were able to score some credit in part (d) but few were able to give complete, clear explanations. A surprisingly large number of candidates failed to show any evidence of working in part (e), no point testing or drawing of a profit line giving no marks. Part (f) was often omitted, but well done by those who did attempt it, with candidates either using their graphs or testing in all three inequalities.