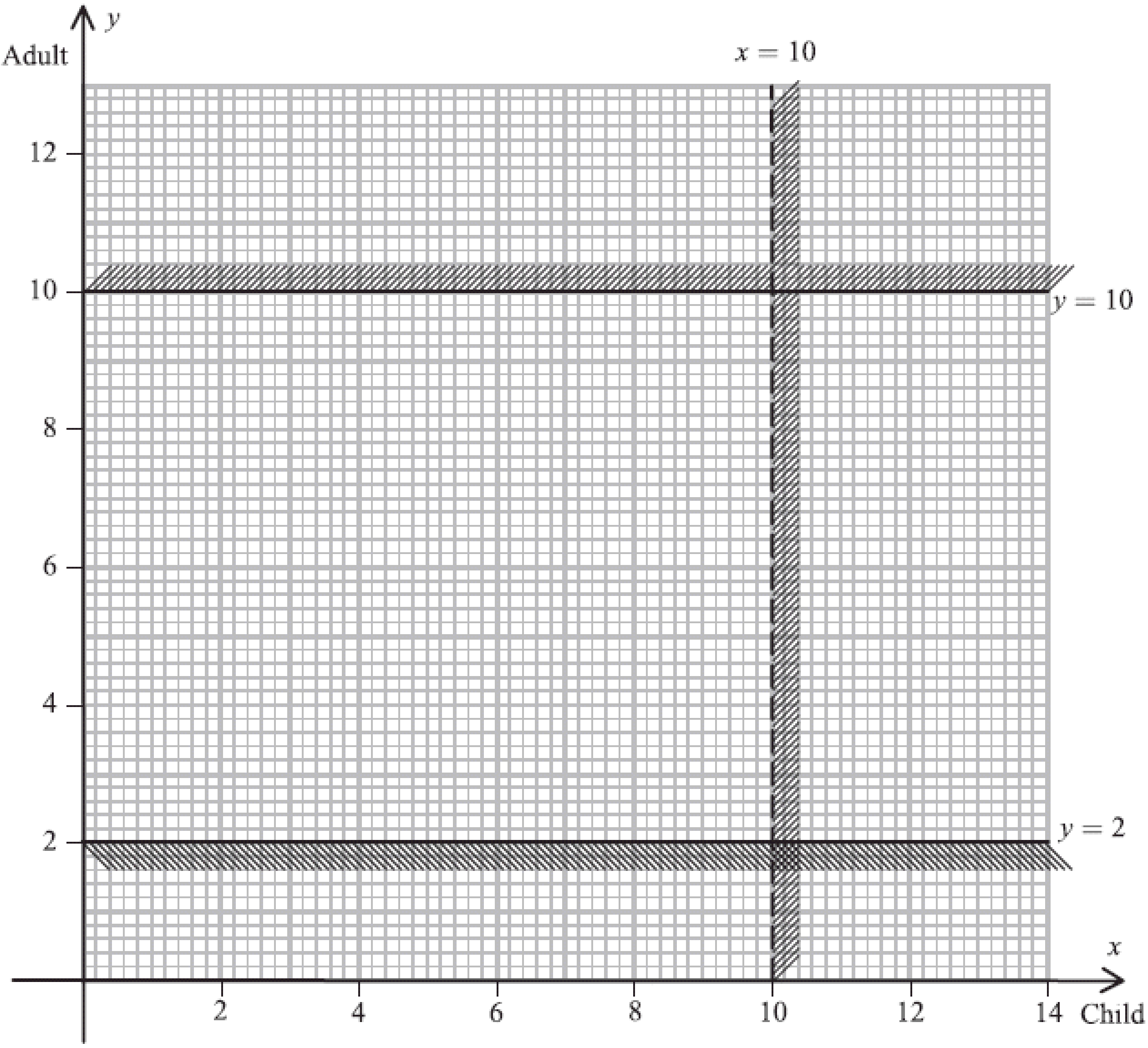
1.



The captain of the *Malde Mare* takes passengers on trips across the lake in her boat.

The number of children is represented by x and the number of adults by y.

Two of the constraints limiting the number of people she can take on each trip are

$$x < 10$$

and

$$2 \leq y \leq 10$$

These are shown on the graph in the figure above, where the rejected regions are shaded out.

(a) Explain why the line $x = 10$ is shown as a dotted line.

(1)

(b) Use the constraints to write down statements that describe the number of children and the number of adults that can be taken on each trip.

(3)

For each trip she charges £2 per child and £3 per adult. She must take at least £24 per trip to cover costs.

The number of children must not exceed twice the number of adults.

(c) Use this information to write down two inequalities.

(2)

(d) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R.

(4)

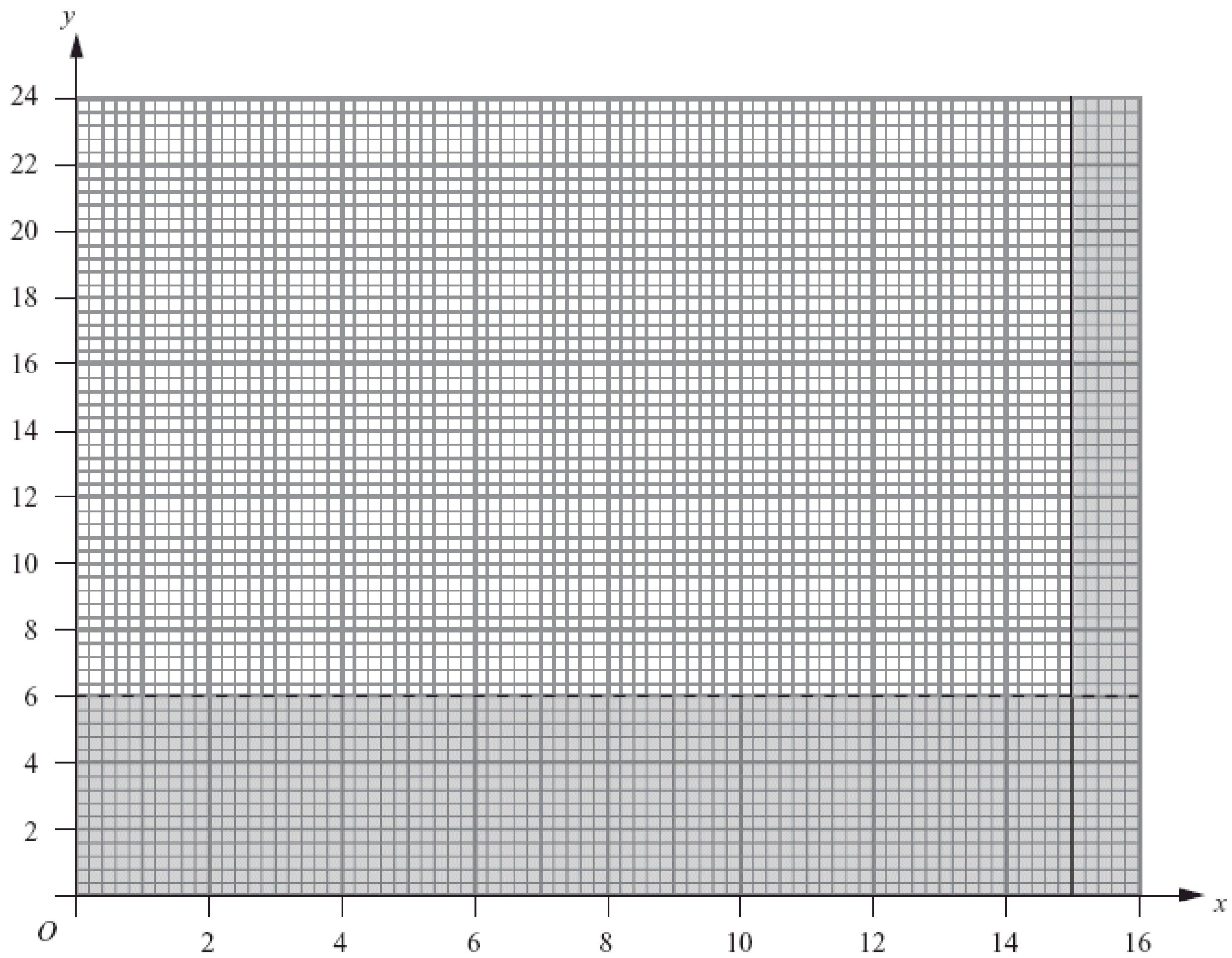
(e) Use your graph to determine how many children and adults would be on the trip if the captain takes:

(i) the minimum number of passengers,

(ii) the maximum number of passengers.

(4)
(Total 14 marks)

2.



Keith organises two types of children's activity, 'Sports Mad' and 'Circus Fun'.
He needs to determine the number of times each type of activity is to be offered.

Let x be the number of times he offers the 'Sports Mad' activity. Let y be the number of times he offers the 'Circus Fun' activity.

Two constraints are

$$x \leq 15$$

and $\quad y > 6$

These constraints are shown on the graph below, where the rejected regions are shaded out.

(a) Explain why $y = 6$ is shown as a dotted line.

(1)

Two further constraints are

$$3x \le 2y$$

and $$5x + 4y \le 80$$

(b) Add two lines and shading to the diagram above book to represent these inequalities. Hence determine the feasible region and label it R.

(3)

Each 'Sports Mad' activity costs £500.
Each 'Circus Fun' activity costs £800.

Keith wishes to minimise the total cost.

(c) Write down the objective function, C, in terms of *x* and *y*.

(2)

(d) Use your graph to determine the number of times each type of activity should be offered and the total cost. You must show sufficient working to make your method clear.

(5)
(Total 11 marks)

3. You are in charge of buying new cupboards for a school laboratory.
The cupboards are available in two different sizes, standard and large.
The maximum budget available is £1800. Standard cupboards cost £150 and large cupboards cost £300.
Let *x* be the number of standard cupboards and *y* be the number of large cupboards.

(a) Write down an inequality, in terms of *x* and *y*, to model this constraint.

(2)

The cupboards will be fitted along a wall 9 m long. Standard cupboards are 90 cm long and large cupboards are 120 cm long.

(b) Show that this constraint can be modelled by

$$3x + 4y \le 30.$$

You must make your reasoning clear.

(2)

Given also that $y \geq 2$,

(c) explain what this constraint means in the context of the question.

(1)

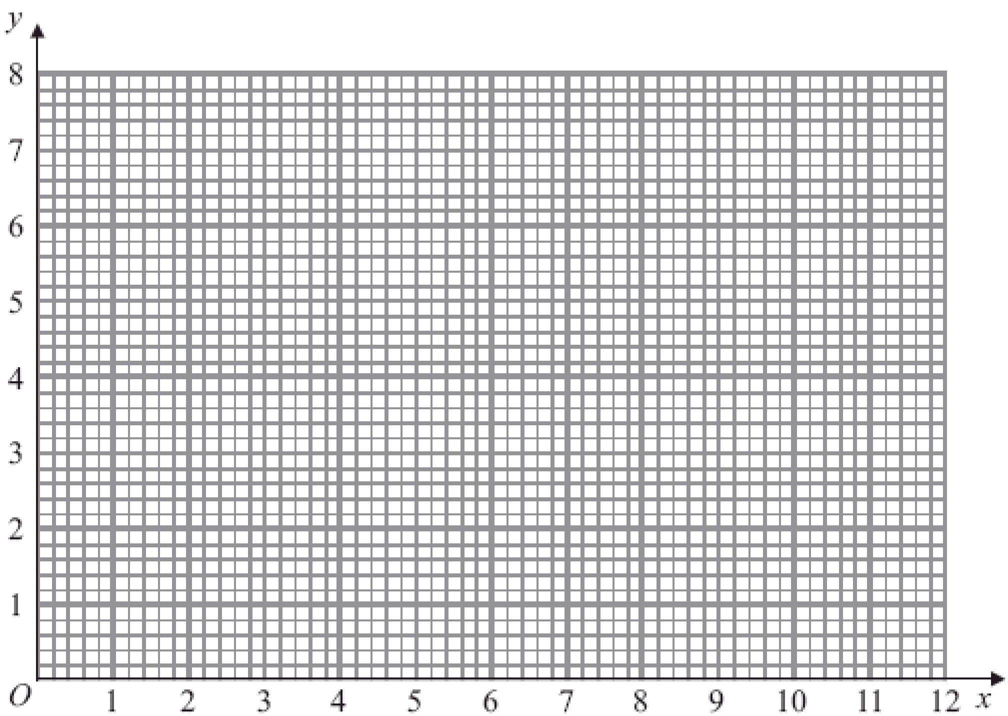
The capacity of a large cupboard is 40% greater than the capacity of a standard cupboard. You wish to maximise the total capacity.

(d) Show that your objective can be expressed as

maximise $5x + 7y$

(2)

(e) Represent your inequalities graphically, on the axes below, indicating clearly the feasible region, R.



(6)

(f) Find the number of standard cupboards and large cupboards that need to be purchased. Make your method clear.

(4)
(Total 17 marks)

4. Rose makes hanging baskets which she sells at her local market. She makes two types, large and small. Rose makes x large baskets and y small baskets.

Each large basket costs £7 to make and each small basket costs £5 to make. Rose has £350 she can spend on making the baskets.

(a) Write down an inequality, in terms of x and y, to model this constraint.

(2)

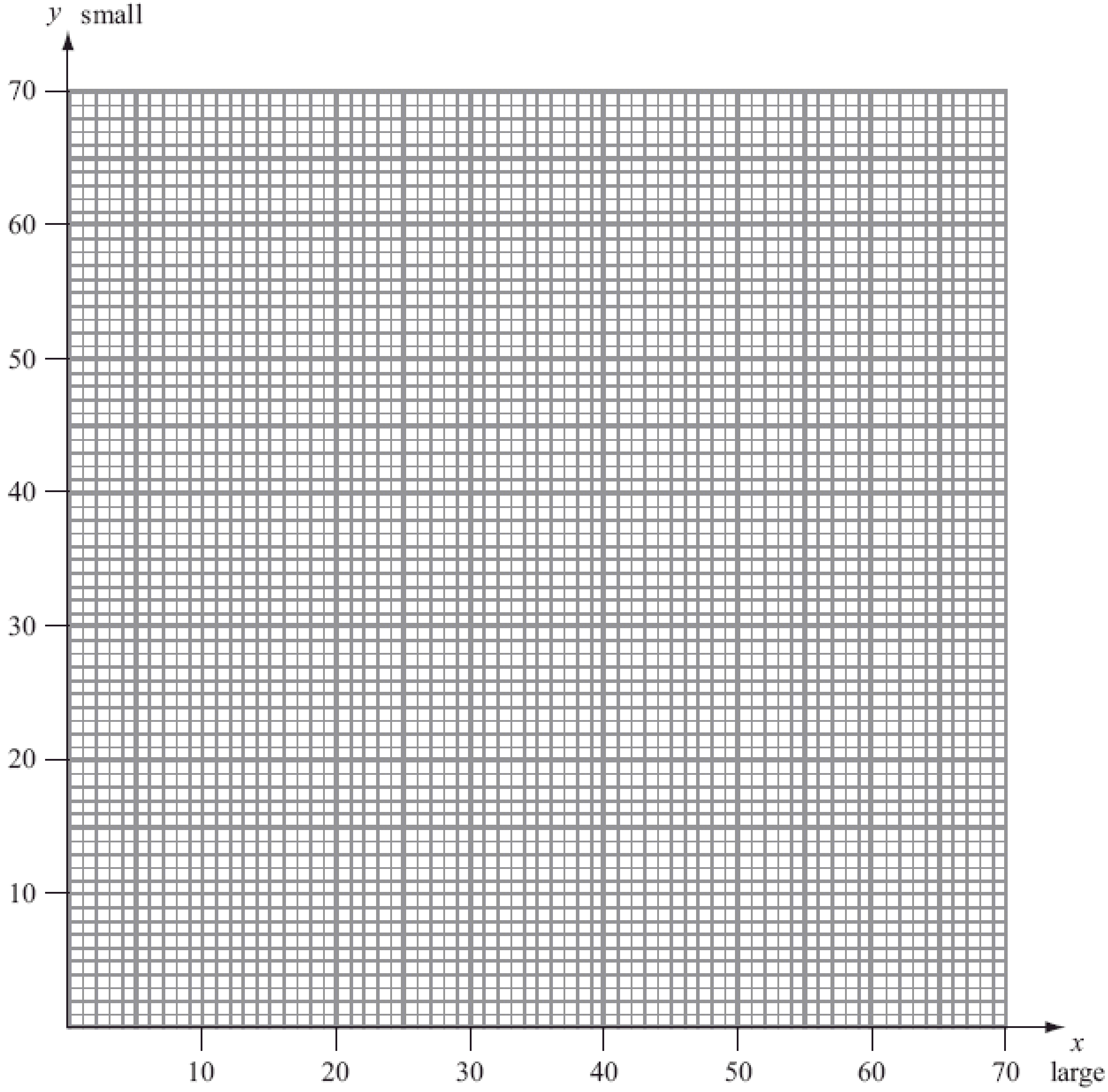
Two further constraints are

$y \leq 20$ and
$y \leq 4x$

(b) Use these two constraints to write down statements that describe the numbers of large and small baskets that Rose can make.

(2)

(c) On the grid below, show these three constraints and $x \geq 0$, $y \geq 0$. Hence label the feasible region, R.



(4)

Rose makes a profit of £2 on each large basket and £3 on each small basket. Rose wishes to maximise her profit, £P.

(d) Write down the objective function.

(1)

(e) Use your graph to determine the optimal numbers of large and small baskets Rose should make, and state the optimal profit.

(5)
(Total 14 marks)

5. Class 8B has decided to sell apples and bananas at morning break this week to raise money for charity. The profit on each apple is 20p, the profit on each banana is 15p. They have done some market research and formed the following constraints.

- They will sell at most 800 items of fruit during the week.
- They will sell at least twice as many apples as bananas.
- They will sell between 50 and 100 bananas.

Assuming they will sell all their fruit, formulate the above information as a linear programming problem, letting *a* represent the number of apples they sell and *b* represent the number of bananas they sell.

Write your constraints as inequalities.

(Total 7 marks)

6. A company produces two types of party bag, Infant and Junior. Both types of bag contain a balloon, a toy and a whistle. In addition the Infant bag contains 3 sweets and 3 stickers and the Junior bag contains 10 sweets and 2 stickers.

The sweets and stickers are produced in the company's factory. The factory can produce up to 3000 sweets per hour and 1200 stickers per hour. The company buys a large supply of balloons, toys and whistles.

Market research indicates that at least twice as many Infant bags as Junior bags should be produced.

Both types of party bag are sold at a profit of 15p per bag. All the bags are sold. The company wishes to maximise its profit.

Let x be the number of Infant bags produced and y be the number of Junior bags produced per hour.

(a) Formulate the above situation as a linear programming problem.

(5)

(b) Represent your inequalities graphically, indicating clearly the feasible region.

(6)

(c) Find the number of Infant bags and Junior bags that should be produced each hour and the maximum hourly profit. Make your method clear.

(3)

In order to increase the profit further, the company decides to buy additional equipment. It can buy equipment to increase the production of **either** sweets **or** stickers, but **not both.**

(d) Using your graph, explain which equipment should be bought, giving your reasoning.

(2)

The manager of the company does not understand why the balloons, toys and whistles have not been considered in the above calculations.

(e) Explain briefly why they do not need to be considered.

(2)
(Total 18 marks)

7. Flatland UK Ltd makes three types of carpet, the Lincoln, the Norfolk and the Suffolk. The carpets all require units of black, green and red wool.

For each roll of carpet,
the Lincoln requires 1 unit of black, 1 of green and 3 of red,
the Norfolk requires 1 unit of black, 2 of green and 2 of red,
and the Suffolk requires 2 units of black, 1 of green and 1 of red.

There are up to 30 units of black, 40 units of green and 50 units of red available each day. Profits of £50, £80 and £60 are made on each roll of Lincoln, Norfolk and Suffolk respectively. Flatland UK Ltd wishes to maximise its profit.

Let the number of rolls of the Lincoln, Norfolk and Suffolk made daily be x, y and z respectively.

(a) Formulate the above situation as a linear programming problem, listing clearly the constraints as inequalities in their simplest form, and stating the objective function.

(4)

This problem is to be solved using the Simplex algorithm. The most negative number in the profit row is taken to indicate the pivot column at each stage.

(b) Stating your row operations, show that after one complete iteration the tableau becomes

Basic variable	x	y	z	r	s	t	Value
r	$\frac{1}{2}$	0	$1\frac{1}{2}$	1	$-\frac{1}{2}$	0	10
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20
t	2	0	0	0	–1	1	10
P	–10	0	–20	0	40	0	1600

(4)

You may not need to use all of the tableaux.

Basic variable	x	y	z	r	s	t	Value	Row operations
r								
s								
t								
P								

-

Basic variable	x	y	z	r	s	t	Value	Row operations

-

Basic variable	x	y	z	r	s	t	Value	Row operations

(c) Explain the practical meaning of the value 10 in the top row.

(2)

(d) (i) Perform one further complete iteration of the Simplex algorithm.

Basic variable	x	y	z	r	s	t	Value	Row operations

-

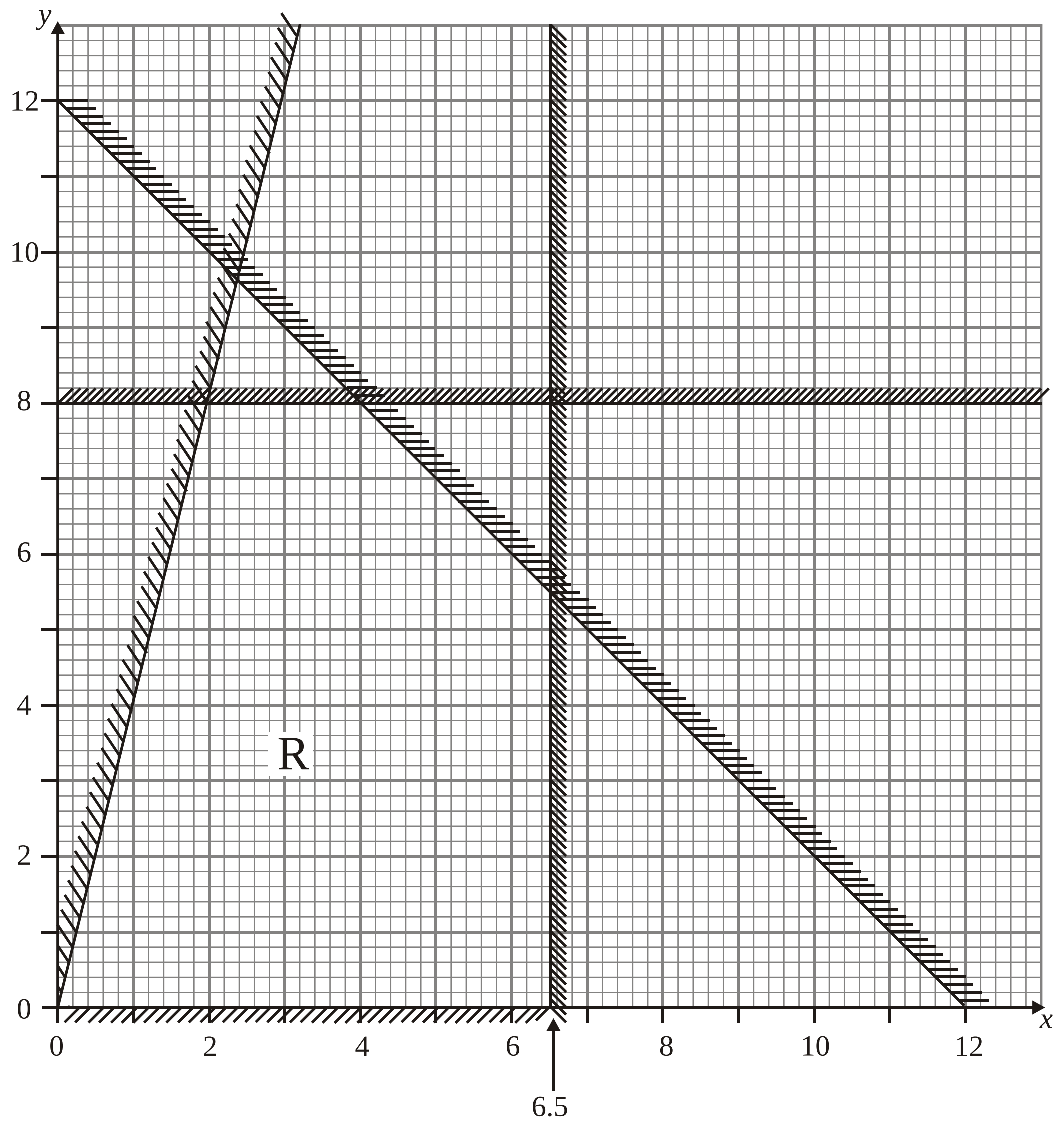
Basic variable	*x*	*y*	*z*	*r*	*s*	*t*	Value	Row operations

(ii) State whether your current answer to part (d)(i) is optimal. Give a reason for your answer.

(iii) Interpret your current tableau, giving the value of each variable.

(8)
(Total 18 marks)

8.



The company EXYCEL makes two types of battery, X and Y. Machinery, workforce and predicted sales determine the number of batteries EXYCEL make. The company decides to use a graphical method to find its optimal daily production of X and Y.

The constraints are modelled in the diagram above where

x = the number (in thousands) of type X batteries produced each day,

y = the number (in thousands) of type Y batteries produced each day.

The profit on each type X battery is 40p and on each type Y battery is 20p. The company wishes to maximise its daily profit.

(a) Write this as a linear programming problem, in terms of x and y, stating the objective function and all the constraints.

(6)

(b) Find the optimal number of batteries to be made each day. Show your method clearly.

(3)

(c) Find the daily profit, in £, made by EXYCEL.

(2)
(Total 11 marks)

9. The Young Enterprise Company "Decide", is going to produce badges to sell to decision maths students. It will produce two types of badges.

Badge 1 reads "I made the decision to do maths" and
Badge 2 reads "Maths is the right decision".

"Decide" must produce at least 200 badges and has enough material for 500 badges.

Market research suggests that the number produced of Badge 1 should be between 20% and 40% of the total number of badges made.

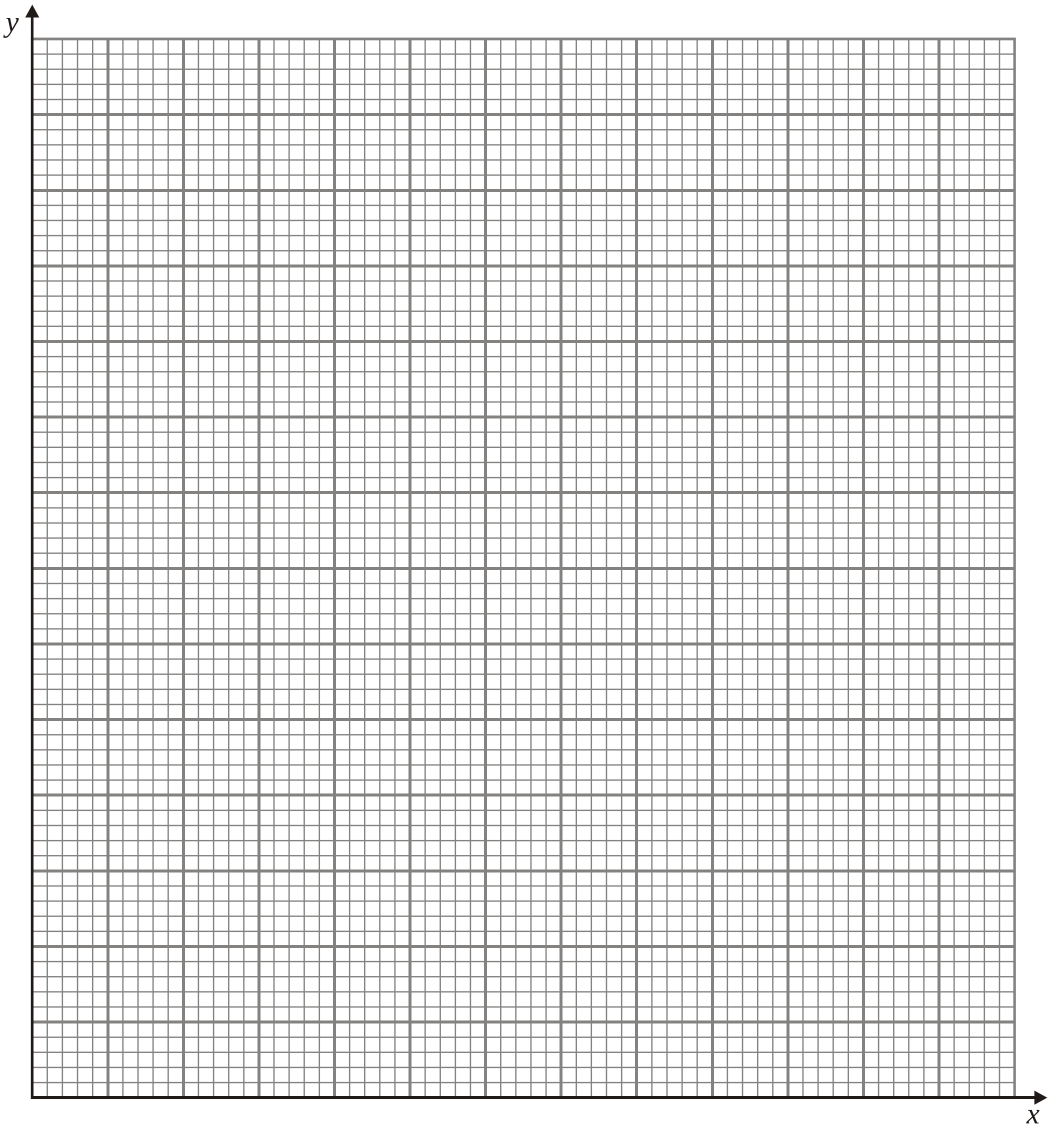
The company makes a profit of 30p on each Badge 1 sold and 40p on each Badge 2. It will sell all that it produced, and wishes to maximise its profit.

Let x be the number produced of Badge 1 and y be the number of Badge 2.

(a) Formulate this situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers.

(6)

(b) On the grid provided below, construct and clearly label the feasible region.



(5)

(c) Using your graph, advise the company on the number of each badge it should produce.
State the maximum profit "Decide" will make.

(3)
(Total 14 marks)

10. A company makes three sizes of lamps, small, medium and large. The company is trying to determine how many of each size to make in a day, in order to maximise its profit. As part of the process the lamps need to be sanded, painted, dried and polished. A single machine carries out these tasks and is available 24 hours per day. A small lamp requires one hour on this machine, a medium lamp 2 hours and a large lamp 4 hours.

Let x = number of small lamps made per day,

y = number of medium lamps made per day,

z = number of large lamps made per day,

where $x \geq 0$, $y \geq 0$ and $z \geq 0$.

(a) Write the information about this machine as a constraint.

(1)

(b) (i) Re-write your constraint from part (*a*) using a slack variable s.

(1)

(ii) Explain what s means in practical terms.

(1)

Another constraint and the objective function give the following Simplex tableau. The profit P is stated in euros.

Basic variable	x	y	z	r	s	Value
r	3	5	6	1	0	50
s	1	2	4	0	1	24
P	–1	–3	–4	0	0	0

You may not need to use all these tableaux

Basic variable	x	y	z	r	s	Value

Basic variable	x	y	z	r	s	Value

-

Basic variable	x	y	z	r	s	Value

Basic variable	x	y	z	r	s	Value

-

Basic variable	x	y	z	r	s	Value

(c) Write down the profit on each small lamp.

(1)

(d) Use the Simplex algorithm to solve this linear programming problem.

(9)

(e) Explain why the solution to part (*d*) is not practical.

(1)

(f) Find a practical solution which gives a profit of 30 euros. Verify that it is feasible.

(2)
(Total 16 marks)

11. A company produces two types of self-assembly wooden bedroom suites, the 'Oxford' and the 'York'. After the pieces of wood have been cut and finished, all the materials have to be packaged. The table below shows the time, in hours, needed to complete each stage of the process and the profit made, in pounds, on each type of suite.

	Oxford	York
Cutting	4	6
Finishing	3.5	4
Packaging	2	4
Profit (£)	300	500

The times available each week for cutting, finishing and packaging are 66, 56 and 40 hours respectively.

The company wishes to maximise its profit.

Let x be the number of Oxford, and y be the number of York suites made each week.

(a) Write down the objective function.

(1)

(b) In addition to

$$2x + 3y \leq 33,$$

$$x \geq 0,$$

$$y \geq 0,$$

find two further inequalities to model the company's situation.

(2)

(c) On the grid below, illustrate all the inequalities, indicating clearly the feasible region.

(4)

(d) Explain how you would locate the optimal point.

(2)

(e) Determine the number of Oxford and York suites that should be made each week and the maximum profit gained.

(3)

It is noticed that when the optimal solution is adopted, the time needed for one of the three stages of the process is less than that available.

(f) Identify this stage and state by how many hours the time may be reduced.

(3)
(Total 15 marks)

12. A manager wishes to purchase seats for a new cinema. He wishes to buy three types of seat; standard, deluxe and majestic. Let the number of standard, deluxe and majestic seats to be bought be x, y and z respectively.
He decides that the total number of deluxe and majestic seats should be at most half of the number of standard seats.
The number of deluxe seats should be at least 10% and at most 20% of the total number of seats.
The number of majestic seats should be at least half of the number of deluxe seats.
The total number of seats should be at least 250.

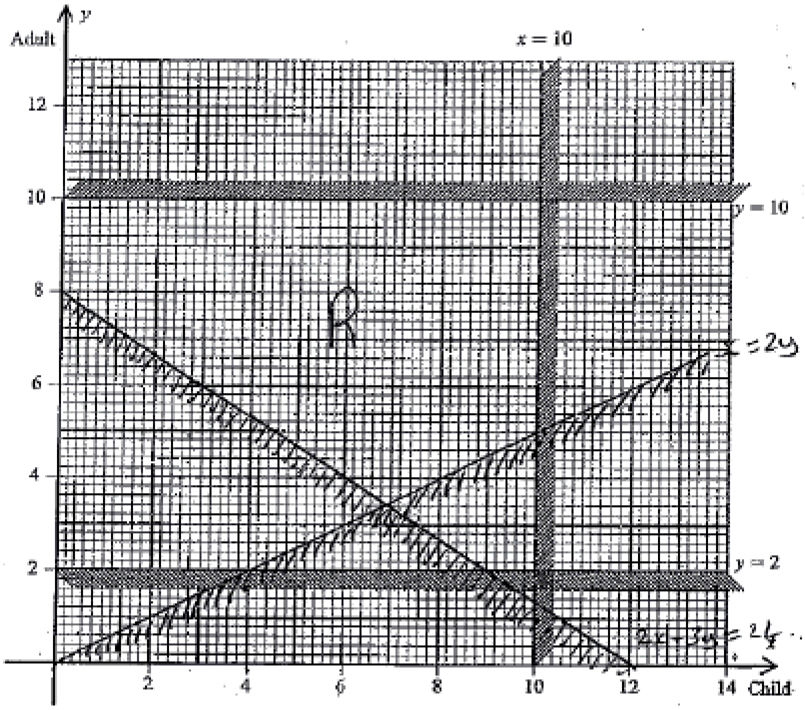
Standard, deluxe and majestic seats each cost £20, £26 and £36, respectively.
The manager wishes to minimise the total cost, £C, of the seats.

Formulate this situation as a linear programming problem, simplifying your inequalities so that all coefficients are integers.

(Total 9 marks)

1.	(a)	To show a strict inequality	B1	1
	(b)	There must be fewer than 10 children	B1	
		There must be between 2 and 10 adults inclusive	B2, 1, 0	3
	(c)	$2x + 3y \geq 24$	B1	
		$x \leq 2y$	B1	2

(d)



B1ft ($2x + 3y = 24$)
B1ft ($x = 2y$)
B1ft (shading)
B1 4

(e)	Minimum	0 Children	8 Adults	–8 Passengers	M1A1	
	Maximum	9 Children	10 Adults	–19 Passengers	B1 B1	4

[14]

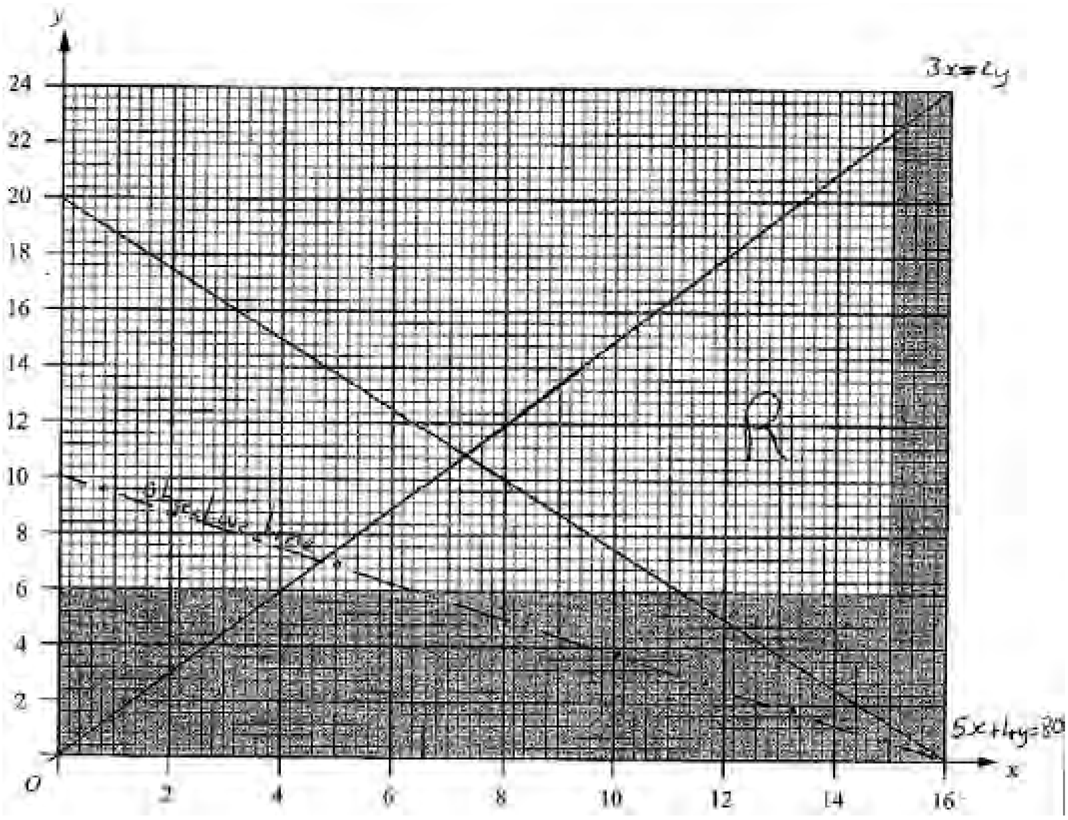
2.	(a)	To indicate the strict inequality	B1	1

Note

1B1: CAO

(b) $3x = 2y$ and $5x + 4y = 80$ added to the diagram. B1, B1

R correctly labelled. B1 3



Note

1B1: $3x = 2y$ passing through 1 small square of (0, 0) and (12, 18), but must reach $x = 15$

2B1: $5x + 4y = 80$ passing through 1 small square of (0, 20) and (16, 0) (extended if necessary) but must reach $y = 6$

3B1: R CAO (condoning slight line inaccuracy as above.)

(c) [Minimise C =] $500x + 800y$ B1, B1 2

Note

1B1: Accept expression and swapped coefficients. Accept 5x + 8y for 1 mark

2B1: CAO (expression still ok here)

(d)	Point testing or Profit line	M1 A1	
	Seeking integer solutions	M1	
	(11, 7) at a cost of £ 11 100.	B1, B1	5

Note

1M1: Profit line [gradient accept reciprocal, minimum length line passes through (0, 2.5) (4, 0)] **OR** testing 2 points in their FR near two different vertices.

1A1: Correct profit line **OR** 2 points correctly tested in correct FR (my points)

e.g

$(7\frac{3}{11}, 10\frac{10}{11}) = 12\ 363\frac{7}{11}$	or	(7, 11) = 12 300
		(8, 10) = 12 000
		(8, 11) = 12 800
$(11\frac{1}{5}, 6)$ 10 400	or	(11, 6) = 10 300
(15, 6) 12 300	or	(15, 7) = 13 100
$(15, 22\frac{1}{2}) = 25\ 500$	or	(15, 22) = 25 100
	(11, 7) = 11100	

2M1: Seeking integer solution in correct FR (so therefore no y = 6 points)
1B1: (11, 7) CAO
2B1: £11 100 CAO

[11]

3.	(a)	$x + 2y \leq 12$ $(150x + 300y \leq 1800)$	M1A1	2

Note

1M1 – correct terms, accept = here, accept swapped coefficients.

1A1 – cao does not need to be simplified.

(b)	$0.9x + 1.2y \leq 9$	M1	
	$\rightarrow 3x + 4y \leq 30$ (*)	A1 cso	2

Note

1M1 – correct terms, must deal with cm/m correctly, accept = here.

1A1 – cso **answer given.**

(c) (You need to buy) at least 2 large cupboards. B1 1

Note

1B1 – cao 'at least' and '2' and 'large'.

(d) Capacity C and 140%C

So total is $Cx + \frac{140}{100}Cy$ M1

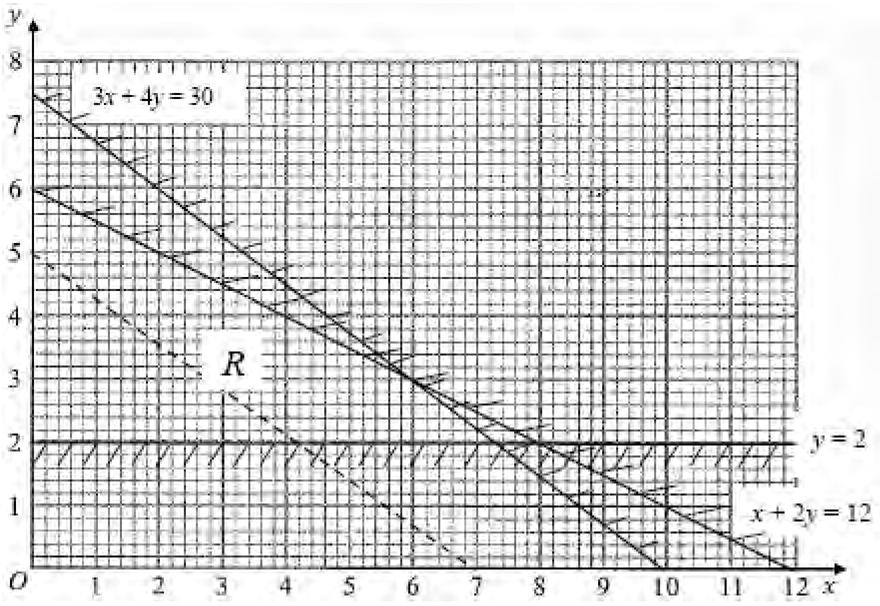
Simplify to 7*y* + 5*x* (*) A1cso 2

Note

1M1 – '1.4' or '5 × 40%'maybe '5+2' seen, they **must** be **seen** to engage with 140% in some way.

1A1 – cso **answer given.**

(e)



Graph:

$y \geq 2$ B1

$0.9x + 1.2y \leq 12$ $(3x + 4y \leq 30)$ B1

$x + 2y \leq 12$ $(150x + 300y \leq 1800)$ B1

Lines labelled & drawn with a ruler B1

Shading, Region identified B1, B1 6

Note

Lines should be within 1 small square of correct point at axes.

1B1 – correctly drawing $y = 2$.

2B1 – correctly drawing $3x + 4y = 30$ [$0.9x + 1.2y = 12$]

3B1 – correctly drawing $x + 2y = 12$ [$150x + 300y = 1800$], **ft only** if swapped coefficients in (a) (6,0) (2,8).

These next 3 marks are only available for candidates who have drawn at least 2 lines, including at least one 'diagonal' line with negative gradient.

4B1 – Ruler used. At least 2 lines labelled including one 'diagonal' line.

5B1 – Shading, or R correct, b.o.d. on their lines.

6B1 – all lines and R correct.

(f) Consider points and value of $5x + 7y$: M1A1

Or draw a clear profit line

(7,2) → 49 or (7 ⅓,2) 50 ⅔, or (7.3, 2) → 50.5

(6,3) → 51

(0,6) → 42 A1

(0,2) → 14

Best option is to buy 6 standard cupboards and 3 large cupboards. A1 4

Note

1M1 At least 2 points tested **or** objective line drawn with correct m or 1/m, minimum intercepts 3.5 and 2.5.

1A1 – 2 points correctly tested **or** objective line correct.

2A1 – 3 points correctly tested **or** objective line correct and distinct/labelled.

3A1 – 6 standard and 3 large, accept (6,3) if very clearly selected in some way.

[17]

4.	(a)	$7x + 5y \leq 350$	M1 A1	2

Note

1M1: Coefficients correct (condone swapped x and y coefficients) need 350 and any inequality

1A1: cso.

(b)	$y \leq 20$ e.g. make at most 20 small baskets	B1	
	$Y \leq 4x$ e.g. the number of small (y) baskets is at most 4 times the number of large baskets (x).	B1	2

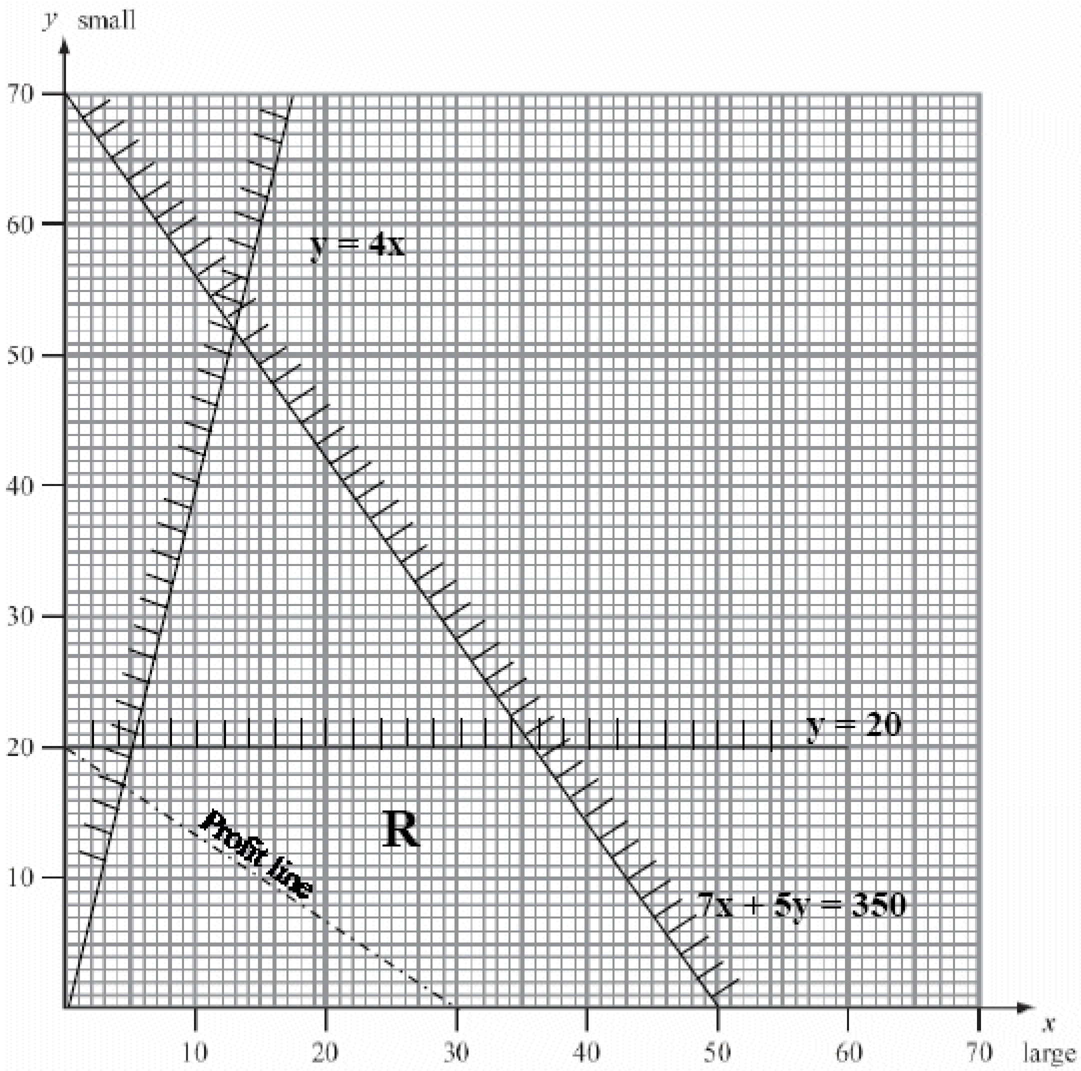
{E.g if y = 40, x =10, 11, 12 etc. or if x = 10, y = 40, 39, 38}

Note

1B1: cao

2B1: cao, test their statement, need both = and < aspects.

(c)



Draw three lines correctly	B3 2 1 0	
Label R	B1	4

Note

1B1: One line drawn correctly

2B1: Two lines drawn correctly

3B1: Three lines drawn correctly. Check (10, 40) (0, 0) and axes

4B1: R correct, but allow if one line is slightly out (1 small square).

(d) (P=) $2x + 3y$ B1 1

Note

1B1: cao accept an expression.

(e)	Profit line or point testing.	M1 A1	
	x = 35.7 y = 20 precise point found.	B1	
	Need integers so optimal point in R is (35, 20); Profit (£)130	B1;B1	5

Note

1M1: Attempt at profit line or attempt to test at least two vertices in their feasible region.

1A1: Correct profit line or correct testing of at least three vertices.

Point testing: (0,0) P= 0; (5,20) P = 70; (50,0) P = 100

$$\left(35\frac{5}{7},20\right)=\left(\frac{250}{7},20\right)\text{P}=131\frac{3}{7}=\frac{920}{7}$$

also (35, 20) P = 130. Accept (36,20) P = 132 for M but not A.

Objective line: Accept gradient of 1/m for M mark or line close to correct gradient.

1B1: cao – accept x co-ordinates which round to 35.7

2B1: cao

3B1: cao

[14]

5.	Maximise (P=) $0.2a + 0.15b$ or $20a + 15b$ **o.e.**	B1B1	2
	Subject to		
	$a + b \leq 800$	B1	
	$a \geq 2b$	B2,1,0	
	$50 \leq b \leq 100$	B1	
	$a \geq 0$	B1	5

1B1: 'Maximise'

2B1: ratio of coefficients correct

3B1: cao

4B1: ratio of coefficients of a and b correct

5B1: inequality correct way round i.e. $a \geq b$

6B1: cao accept < – accept two separate inequalities here

7B1: cao

- Penalise < and > only once with last B mark earned
- Be generous on letters a, b, A, B, x, y etc and mixed, but remove last B mark earned if consistent or 3 letters in the ones marked.

[7]

6. (a) Maximise, (P =) $15x + 15y$ B1, B1

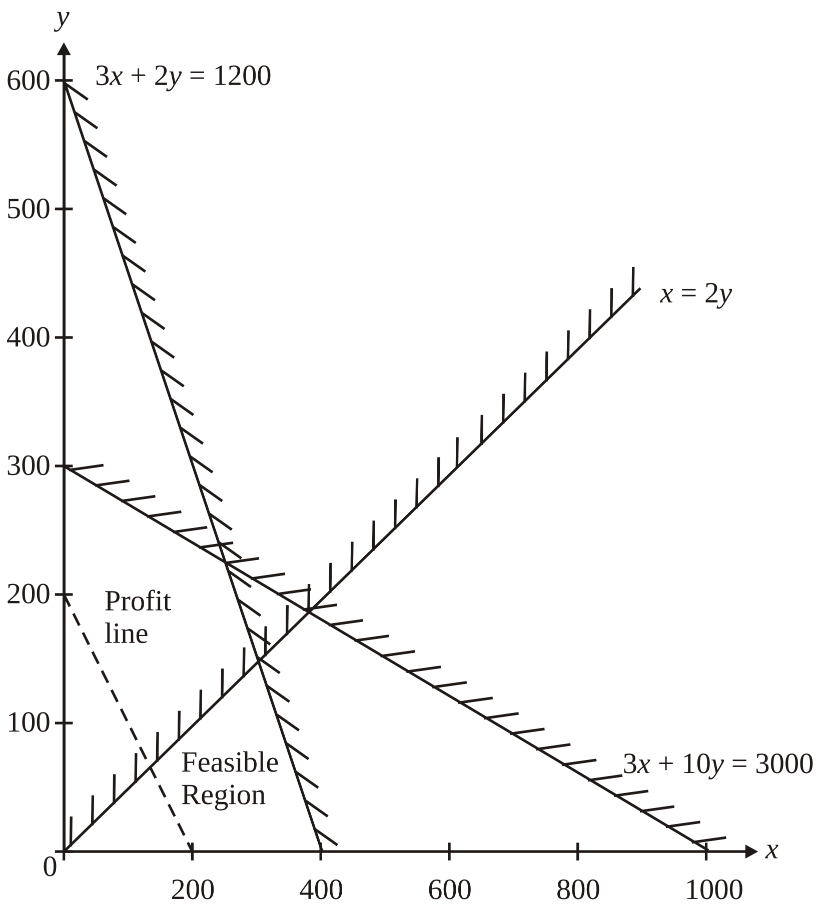
Subject to $3x + 10y \leq 3000$ B3, 2, 1, 0 5

$3x + 2y \leq 1200$ B3, 2, 1, 0

$x \geq 2y$

$x, y \geq 0$

(b)



B6, 5, 4, 3, 2, 1, 0 6

(c) Profit line or vertex testing, (300, 150). Profit = £67.50 M1 A1ft A1ft 3

(d) Production of stickers should be increased since this would be more the intersection point further from the origin. B2, 1ft, 0 2

(e) e.g. The constraint line, would be for outside the feasible region – so they would not effect it. B2, 1, 0 2

[18]

7. (a) Maximise P = $50x + 80y + 60z$ B1

subject to $x + y + 2z \leq 30$

$x + 2y + z \leq 40$

$3x + 2y + z \leq 50$ B3, 2, 1,0 4

where $x, y, z \geq 0$

(b) Initialising tableau B1ft M1

bv	x	y	z	r	s	t	value
r	1	1	2	1	0	0	30
s	1	(2)	1	0	1	0	40
t	3	2	1	0	0	1	50
p	–50	–80	–60	0	0	0	0

chooses correct pivot, divides R_2 by 2 A1 ft

states correct row operation $R_1 - R_2$, $R_3 - 2R_2$, $R_4 + 80R_2$, $R_2 \div 2$ A1 4

(c) The solution found after one iteration has a stack of 10 units of black per day B2, 1, 0 2

(d) (i)

bv	x	y	z	r	s	t	value	
r	$\frac{1}{2}$	0	($\frac{3}{2}$)	1	$-\frac{1}{2}$	0	10	
y	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	20	(given)
t	2	0	0	0	–1	1	10	
p	–10	0	–20	0	40	0	1600	

bv	x	y	z	r	s	t	value
z	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	$6\frac{2}{3}$
y	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	$16\frac{2}{3}$
t	2	0	0	0	–1	1	10
p	$-3\frac{1}{3}$	0	0	$13\frac{1}{3}$	$33\frac{1}{3}$	0	$1733\frac{1}{3}$

$R_1 \div \frac{3}{2}$ M1 A1

$R_2 - \frac{1}{2}R_1$

R_3 – no change M1 A1 4

$R_4 + 20R$

(ii) not optimal, a negative value in profit row B1ft

(iii) $x = 0$ $y = 16\frac{2}{3}$ $z = 6\frac{2}{3}$ M1 A1ft

$p = £1733.33$ $r = 0, s = 0, t = 10$ A1ft 4

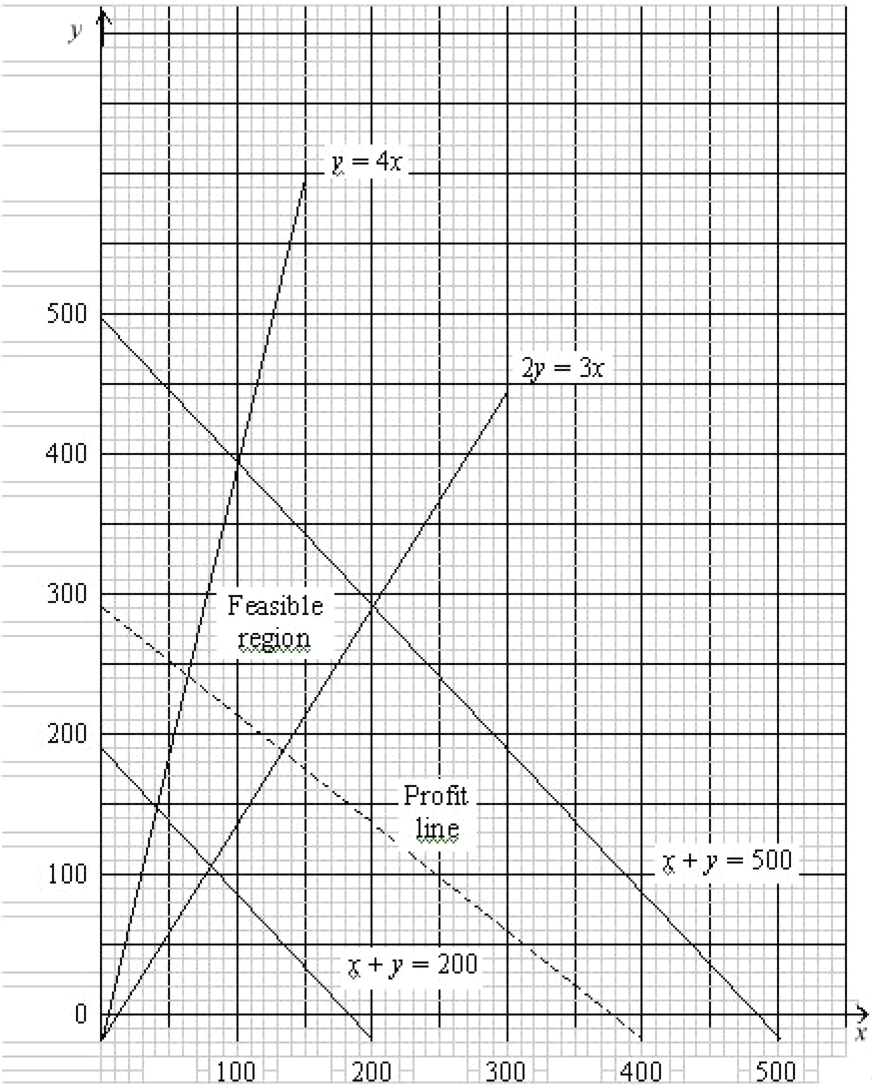
[18]

8.	(a)	Maximum (P=) $0.4x + 0.2y$	B1	
		accept 40x + 20y		
		Subject to $x \le 6.5$		
		$y \le 8$		
		$x + y \le 12$	B5,4,3,2,1,0	6
		$y \le 4 - x$		
		$y \ge 0$		
	(b)	Point testing or Profit line		
		(6.5, 5.5) $\Rightarrow$ 65ω type x and 55ω type y	M1A1A1	3
	(c)	P = 0.4(65ω) + 0.22(55co)	M1	
		= £3.7 co	A1	2

[11]

9.	(a)	Maximise $\underline{P = 30x + 40y}$ (or $P = 0.3x + 0.4y$)	B1	
		subject to $\underline{x + y \ge 200}$	B1	
		$\underline{x + y \le 500}$	B1	
		$x \ge \frac{20}{100}(x + y) \Rightarrow 4x \ge y$	M1 A1	
		$x \le \frac{40}{100}(x + y) \Rightarrow 3x \ge 2y$	A1	6

(b)



B5,4,3,2,1,0 5

(x + y = 200, x + y = 500)	B1 ft
(y = 4x)	B1 ft
(2y = 3x)	B1 ft
(labels)	B1 ft
FR	B1

(NB: Graph looks OK onscreen at 75% magnification but may print out misaligned)

(c) Point testing or profit line — M1
Intersection of $y = 4x$ and $x + y = 500$ — A1
(100, 400) Profit = £190 (units must be clear) — A1 — 3

[14]

10. (a) $x + 2y + 4z \leq 24$ — B1

(b) (i) $x + 2y + 4z + s = 24$ — B1f.t.

(ii) s(≥ 0) is the slack time on the machine in hours — B1

(c) 1 Euro — B1 — 4

(d)

b.v	x	y	z	r	s	value			
r	3/2	2	0	1	–3/2	14	$R_1–6R_2$	M1	
z	¼	½	1	0	¼	6	$R_2÷4$	A1 ft	
p	0	–1	0	0	1	24	R_3+4R_2	A1	3

b.v	x	y	z	r	s	value			
y	¾	1	0	½	–3/4	7	$R_1 ÷ 2$	M1	
z	–1/8	0	1	–¼	5/8	5/2	R_2- ½ R_1	A1 ft	
p	¾	0	0	½	¼	31	R_3+R_1	A1	3

Profit = 31 Euros $y = 7$ (medium) $z = 2.5$ (large) $x = r = s = 0$ — M1 A1 ft A1 ft — 3

(e) Cannot make ½ a lamp — B1 — 1

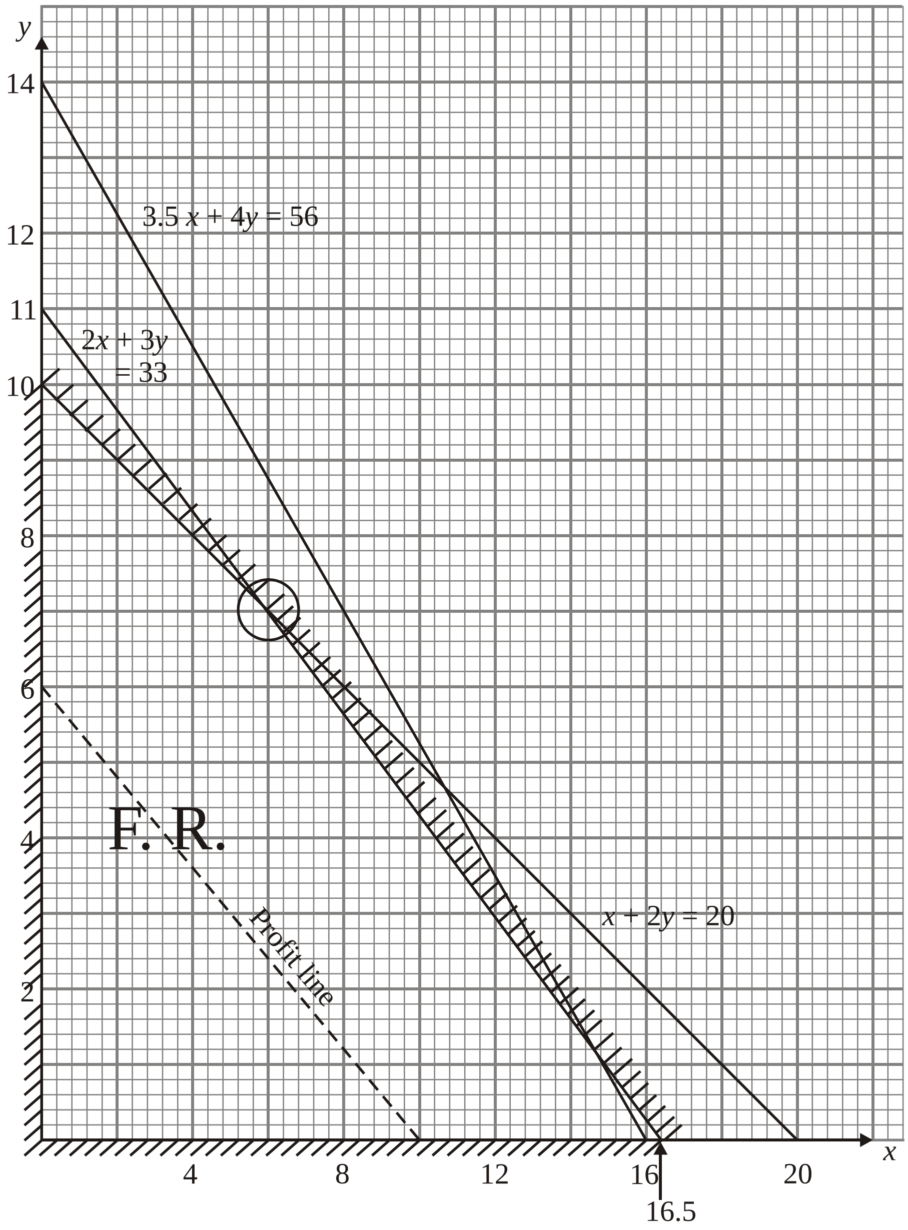
(f) e.g. (0, 10, 0) or (0, 6, 3) or (1, 7, 2) — B1
checks in **both** inequalities — B1 — 2

[16]

11. (a) (P =) $300x + 500y$ — B1

(b) Finishing $3.5x + 4y \leq 56 \Rightarrow 7x + 8y \leq 112$ (or equivalent) — B1
Packing $2x + 4y \leq 40 \Rightarrow x + 2y \leq 20$ (or equivalent) — B1 — 3

(c)



B4, 3, 2, 1, 0 4

(d) e.g.:

Point testing: test corner points in feasible region
find profit at each and select point yielding maximum

Profit line: draw profit lines
select point on profit line furthest from the origin B2,1,0 2

(e) Using a correct, complete method M1
make 6 Oxford and 7 York Profit = £5300 A1 ft A1 ft 3

(f) The line $3.5x + 4y = 49$ passes through (6, 7)
so reduce finishing by 7 hours M1 A1 ft A1 3

[15]

12.	$y + z \leq \frac{1}{2}x$	$\Rightarrow$	$2(y + z) \leq x$	B1	1
	$y \geq \frac{10}{100}(x + y + z)$	$\Rightarrow$	$x + z \leq 9y$	M1 A1	2
	$y \geq \frac{20}{100}(x + y + z)$	$\Rightarrow$	$x + z \geq 4y$	M1 A1	2
	$z \geq \frac{1}{2}y$	$\Rightarrow$	$2z \geq y$	B1	
	$x \geq 0, y \geq 0, z \geq 0,$				
	$x + y + z \geq 250$			B1	
	objective function: minimise; $c = 20x + 26y + 36z$			B1; B1	4

[9]

1. No Report available for this question.

2. This question gave rise to a good spread of marks. Most candidates completed part (a) correctly although some very lengthy responses were seen. $5x + 4y = 80$ was drawn correctly more often than $3x = 2y$ in part (b), with many candidates drawing the latter with a negative gradient. Pleasingly most candidates used a ruler to draw their lines, a great improvement on previous years. The feasible region was often incorrectly identified and labels were often absent.

Most were able to complete part (c) correctly.

Those who used the objective line method in (d) usually gained more marks than those who used the point testing method. Some of those using the latter method seemed confused by the y-axis scale and only considered vertices with even values of y, many tested points by reading from the graph rather than solving simultaneous equations.

A large number of solutions had $y = 6$ despite answering part (a) correctly. Some found the maximum solution. Many did not make their method clear.

3. Most candidates were able to score at least 12 out of 17 marks on this question. Parts (a), (b) and (c) were usually correct, with only a very few making slips with the inequality in (a) or muddling ‘small’ with ‘large’ in part (c). The units in part (b) caused difficulty for some candidates, but most changed all lengths into cm and proceeded correctly. Many candidates struggled with part (d). When the answer is printed on the paper candidates must ensure that their reasoning is both clear and convincing, disappointingly, many candidates were not able to derive the given result and in particular many ‘derivations’ attempted to start with $1.4y = x$. There were many fully correct graphs, helped by widespread use of rulers, a big improvement from past papers. Three correct lines almost always invariably led to the correct region. As always, some lost a mark because they did not label their lines and/or R. In (f) both the vertex testing and profit line methods were often successful. As always it is vital that the method is clearly seen, some lost all 4 marks in (f) because they merely described the use of a profit line but failed to draw it. Others drew a very short profit line – candidates should use sufficiently large values for the axes intercepts to ensure an accurate gradient. Those using the vertex method should be reminded that all vertices should be tested, a number of candidates only tested one or two vertices.

4. There were some very good, and very poor, solutions seen to this question. Almost all candidates were able to write down the correct inequality in part (a) with only a very few getting the wrong coefficients or replacing the inequality with an equals sign. Part (b) proved challenging for many candidates. Candidates struggled in particular to interpret $y \leq 4x$. The usual error was to confuse ‘small’ with ‘large’ but many failed to refer to, or reversed, the inequality. The most able described the inequality in terms of percentages; where this was seen it was almost always correct. Most candidates drew $5x + 7y = 350$ and $y = 20$ correctly. Most candidates used a ruler and most plotted the axes interceptions accurately. Unsurprisingly $y = 4x$ caused the most difficulty, often replaced by $x = 4y$. Most candidates used shading sensibly although some shaded so scruffily that they obscured their line. Most candidates labelled R correctly; most candidates did not label their lines. Most candidates were able to write down the correct objective function. Part (e) was often poorly done with many candidates failing to make their method clear; if using the objective line method candidates MUST draw an objective line, and of a sensible length, so that its accuracy can be checked; if using point testing then the points and their values must be stated. As always those who use the objective line method are more successful than those who use point testing. When point testing, all vertices in the feasible region must be tested. Many candidates assumed that the point (36, 20) was a vertex; it was pleasing to see a small number of scripts where this was tested and found to be outside the feasible region. Others found the precise point but then did not seek integer solutions to complete their answer.

5. Many candidates omitted 'maximise' here, other common errors were omitting the non-negativity constraint on a, and getting the 2 on the wrong side of the second inequality. A number of candidates tried to combine several conditions into one inequality, a frequently seen one being $2a + b \le 800$. A number of candidates wasted time by starting to solve the LP problem.

6. Most candidates were able to make some progress with part (a), most correctly stated the objective function but often the objective was omitted. The non-negativity constraints were often omitted and many had difficulty in finding the $x \ge 2y$ inequality. The examiners were all disappointed by the standard of the graph work seen in part (b). Lines were often imprecisely drawn or omitted, $x = 2y$ (if found in part (a)) was often incorrectly drawn. Labels, scales and/or shading were often omitted and the feasible region was not always indicated. Not all candidates used sharp pencils and rulers. If candidates are going to use the profit line method in part (c) they must draw in, and label, a profit line which should long enough to enable examiners to check the gradient. If candidates are going to use the point testing method they must state and test **every** vertex point in the feasible region, not just the most likely point. Many candidates did not state the profit, and of those that did, some did not state units. Those who drew a correct graph generally answered part (d) well. Part (e) was often well-answered but there were many irrelevant comments seen.

7. Many omitted the instruction to maximise the objective. Most candidates were able to write down the 3 constraints correctly, although few remembered to include $x, y, z \ge 0$. Most of the candidates were able to form an initial tableau, although the value in the profit row was often left blank. Many candidates were able to state their row operations correctly, although some only wrote expressions such as – R2 rather than R1 –R2 and many forgot to state R2 /2. The practical meaning of part (c) was not understood by many candidates. Part (d) (i) was often well-attempted, but there were many calculation slips. In part (ii) candidates needed to expressly refer the presence of negatives in the final/profit/objective **row**. Very few stated the values of all seven variables in part (iii).

8. Many omitted the instruction to maximise the objective. Most candidates were able to write down at least 3 constraints correctly, although few obtained them all. $y=0$, was often omitted and there were often errors in writing down $x + y = 12$ and $y = 4x$. The solution of those candidates using point testing in part (b) was often spoilt by using incorrect coordinates. The y coordinate of the point with x coordinate 6.5 was frequently misstated. Those using the objective line method must draw the line clearly on the diagram, and in such a way that the gradient can be see to be correct. The clearest way in this case is to draw the line from e. g. (2, 0) to (0, 4). A number of candidates did not pick up that the values of x and y represented 1000's of batteries and this together with problems with pounds and pence caused much confusion in part (c), although many completely correct answers were seen.

9. This question was often poorly done. Poor algebra was often seen in part (a). Most candidates were able to state the objective function but did not state that this was to be maximised. The $x + y$ inequalities were better handled. Only the better candidates were able to correctly handle the 20% and 40% inequalities. A number tried to combine the four inequalities into two constraints. In part (b) the lines $x + y = 200$ and $x + y = 500$ were plotted correctly but the other lines very poorly plotted. Many candidates drew vertical or horizontal lines. Candidates should use rulers and sharp pencils to draw lines. Labels were frequently omitted both of the lines and the feasible region. Most who used the 'profit line' method got the correct answer although a few drew lines with the reciprocal gradient. Some candidates using the 'vertex method' considered that they only need check two points and not all four.

10. Almost all the candidates were able to complete parts (a), (b) (i) and (e) correctly. Many candidates were able to define a slack variable but fewer were able to explain it in practical terms. Many candidates wrote an expression in part (c) or gave the profit as -1. Part (d) was well attempted by the majority of the candidates: the basic variables were competently dealt with this time, but the usual arithmetical errors were seen. A number of candidates did not always select a correct pivot. Few candidates stated the values of all of the variables from their final tableau, with P occasionally omitted and r and s frequently omitted. Many found part (f) too challenging and few were able to find a practical solution and check its feasibility.

11. Parts (a) and (b) were well done by the vast majority of candidates. Most candidates were able to draw the lines correctly in part (c) but many did not label their lines or the axis. Some very poor choices of scale were often seen, so that sometimes the whole feasible region was not shown, or was too small to be useful. Most candidates were able to score some credit in part (d) but few were able to give complete, clear explanations. A surprisingly large number of candidates failed to show any evidence of working in part (e), no point testing or drawing of a profit line giving no marks. Part (f) was often omitted, but well done by those who did attempt it, with candidates either using their graphs or testing in all three inequalities.

12. This was probably the question that candidates found to be the hardest. Only the most able candidates were able to make a good attempt at it. Most were able to write down an expression for the objective function, and some of these correctly stated that it was to be minimised. Some candidates were unable to make progress with the parts that referred to the total number of seats. Those who correctly used $x + y + z$ however were usually able to make some progress. Many candidates did not make all the coefficients integers or simplify their inequalities.