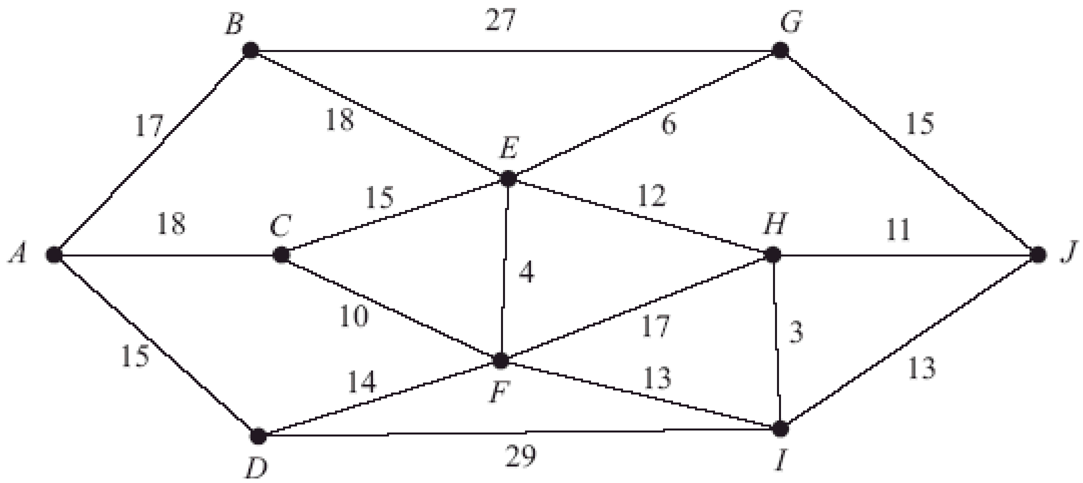
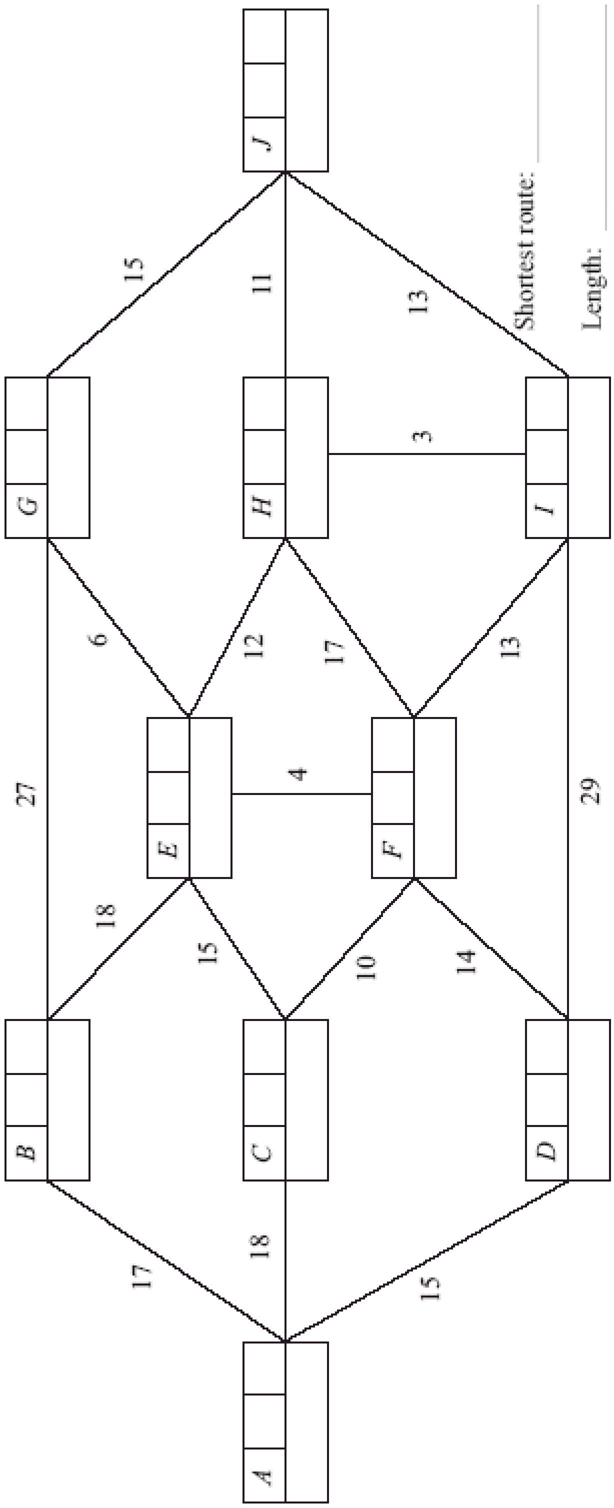
1.



The figure above shows a network of roads. The number on each arc represents the length of that road in km.

(a) Use Dijkstra's algorithm to find the shortest route from *A* to *J*. State your shortest route and its length.



(5)

(b) Explain how you determined the shortest route from your labelled diagram.

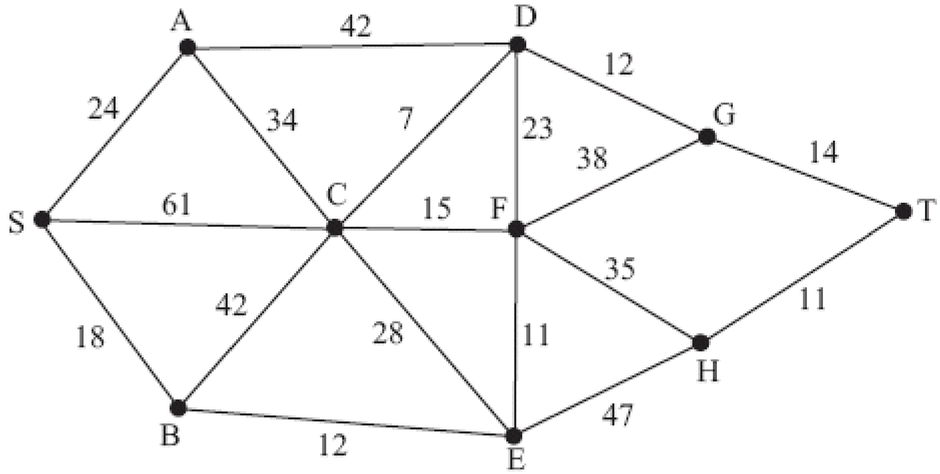
(2)

The road from *C* to *F* will be closed next week for repairs.

(c) Find a shortest route from *A* to *J* that does not include *CF* and state its length.

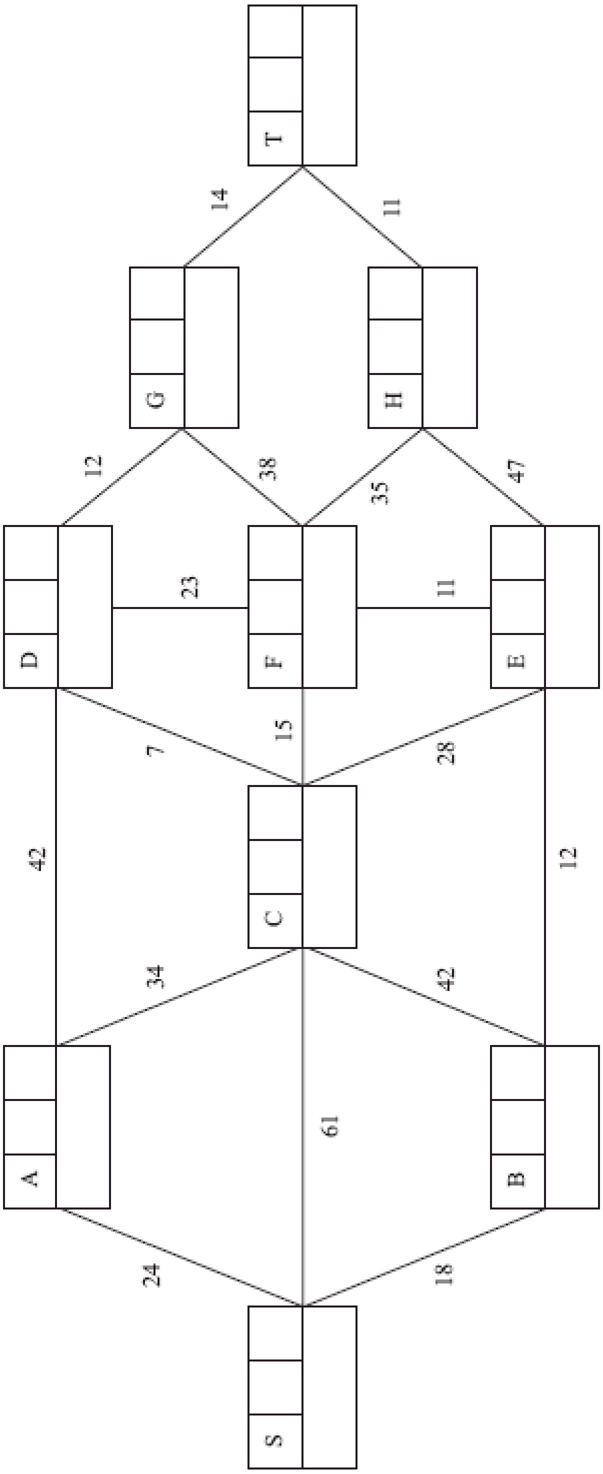
(2)
(Total 9 marks)

2.



The diagram above shows a network of cycle tracks within a national park. The number on each arc represents the time taken, in minutes, to cycle along the corresponding track.

(a) Use Dijkstra's algorithm to find the quickest route from S to T. State your quickest route and the time it takes.



(6)

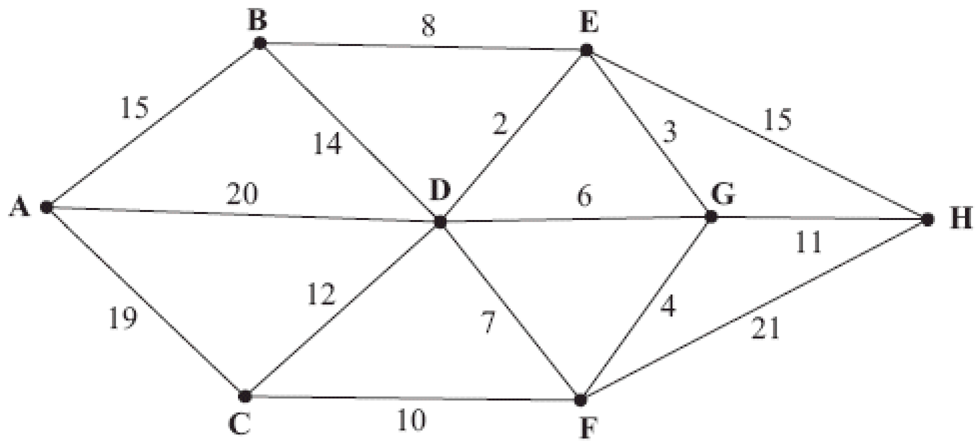
(b) Explain how you determined your quickest route from your labelled diagram.

(2)

(c) Write down the quickest route from E to T.

(1)
(Total 9 marks)

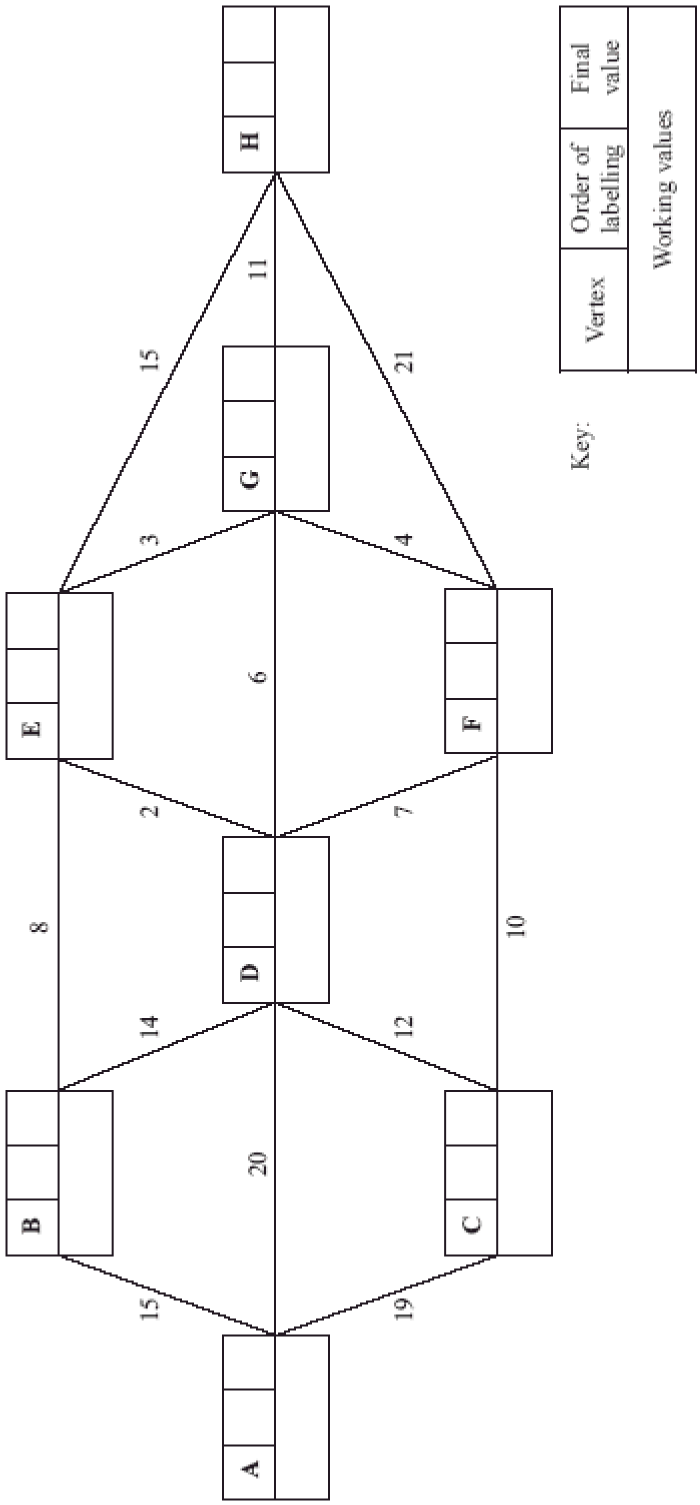
3.



[*The total weight of the network is 167*]

The diagram above represents a network of paths. The number on each arc gives the time, in minutes, to travel along that path.

(a) Use Dijkstra's algorithm to find the quickest route from A to H. State your quickest route and the time taken.



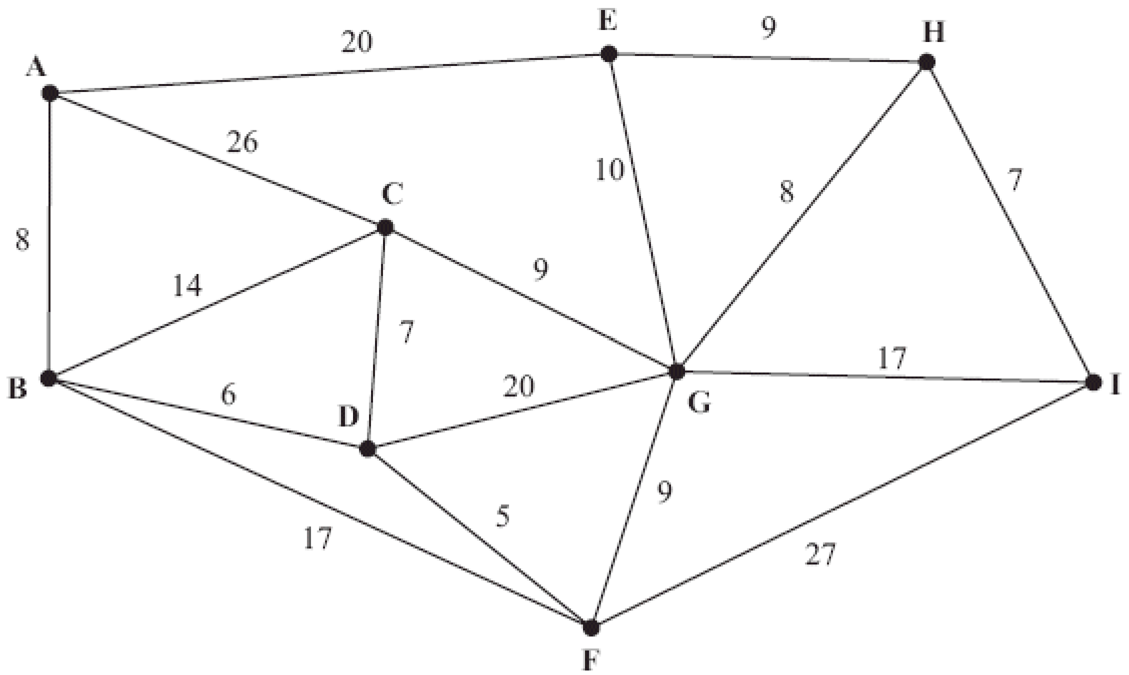
(5)

Kevin must walk along each path at least once and return to his starting point.

(b) Use an appropriate algorithm to find the time of Kevin's quickest possible route, starting and finishing at A. You should make your method and working clear.

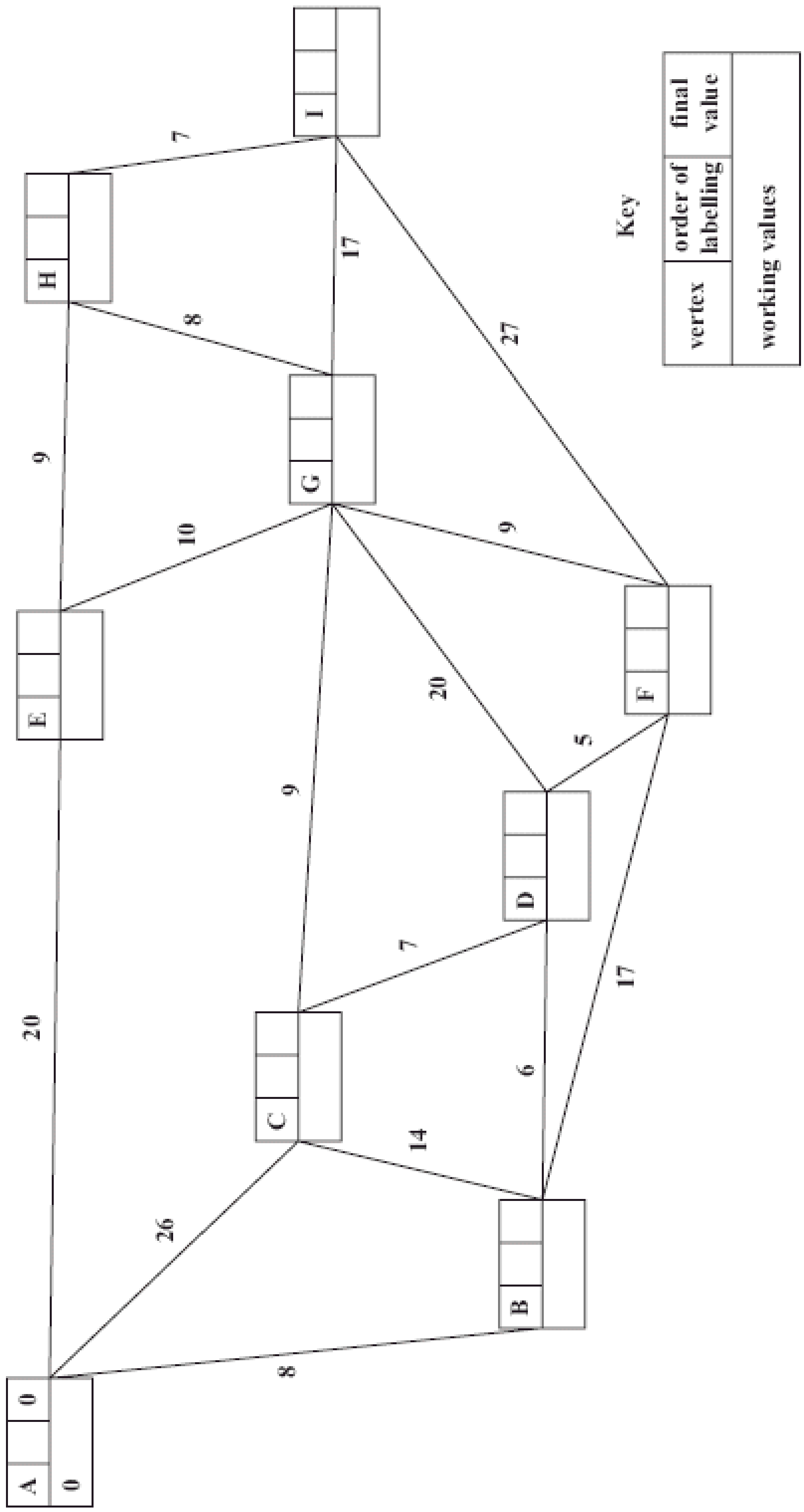
(5)
(Total 10 marks)

4.



The diagram above represents a network of roads. The number on each arc gives the length, in km, of that road.

(a) Use Dijkstra's algorithm to find the shortest distance from A to I. State your shortest route.

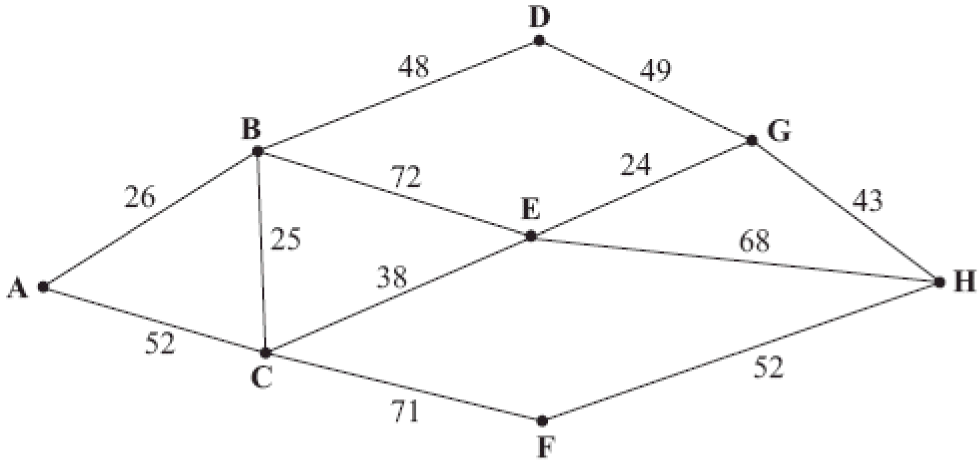


(6)

(b) State the shortest distance from A to G.

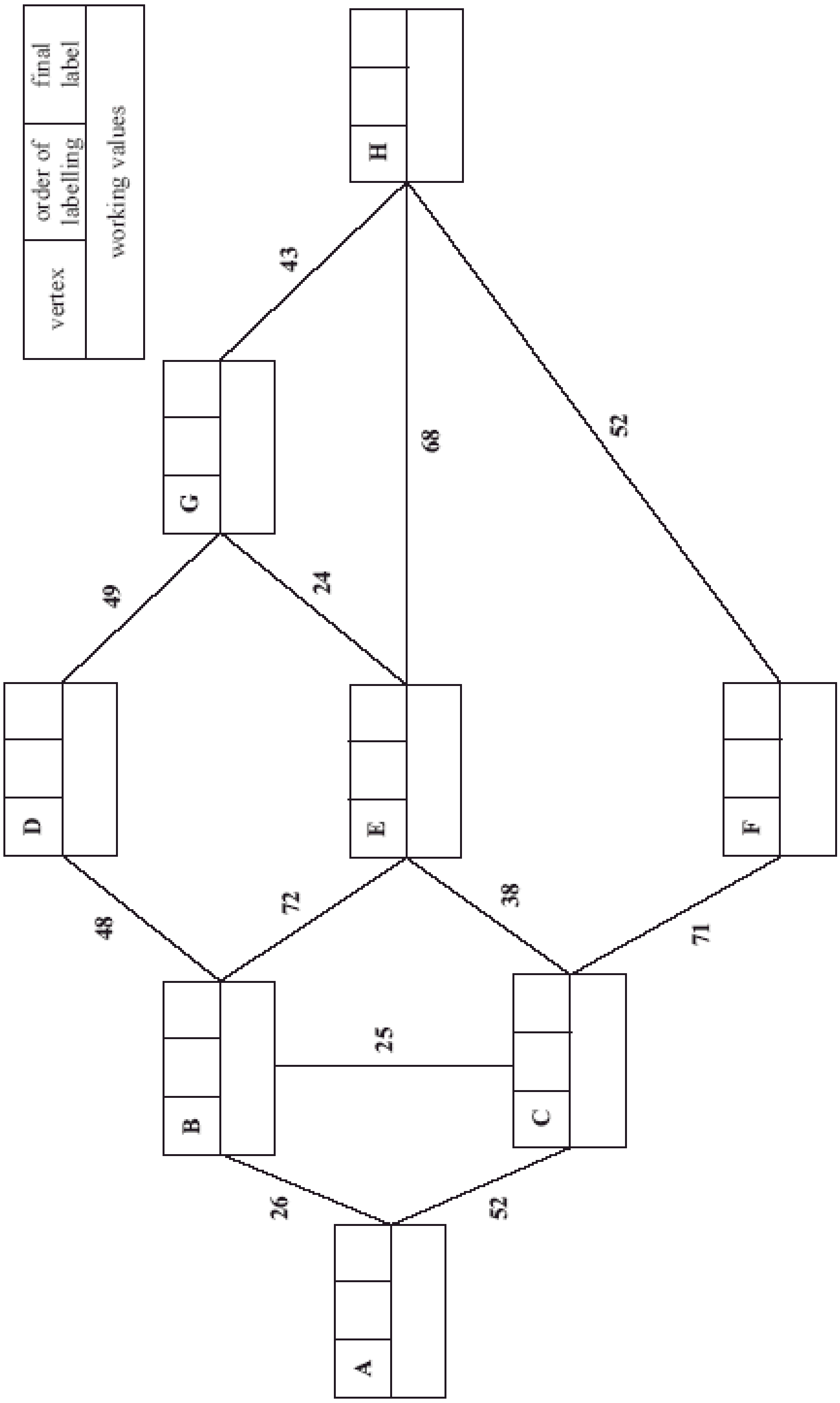
(1)
(Total 7 marks)

5.



The diagram above shows a network of roads through eight villages, A, B, C, D, E, F, G and H. The number on each arc is the length of that road in km.

(a) Use Dijkstra's algorithm to find the shortest route from A to H. State your shortest route and its length.



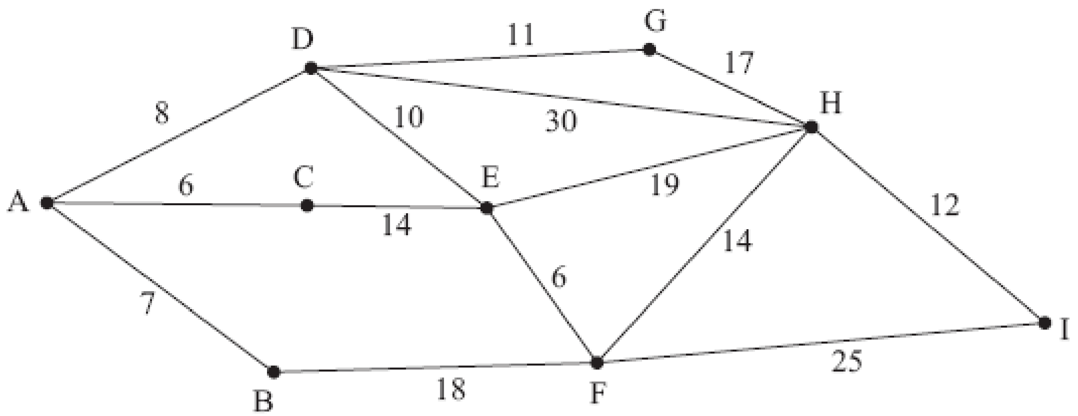
(5)

There is a fair in village C and you cannot drive through the village. A shortest route from A to H which avoids C needs to be found.

(b) State this new minimal route and its length.

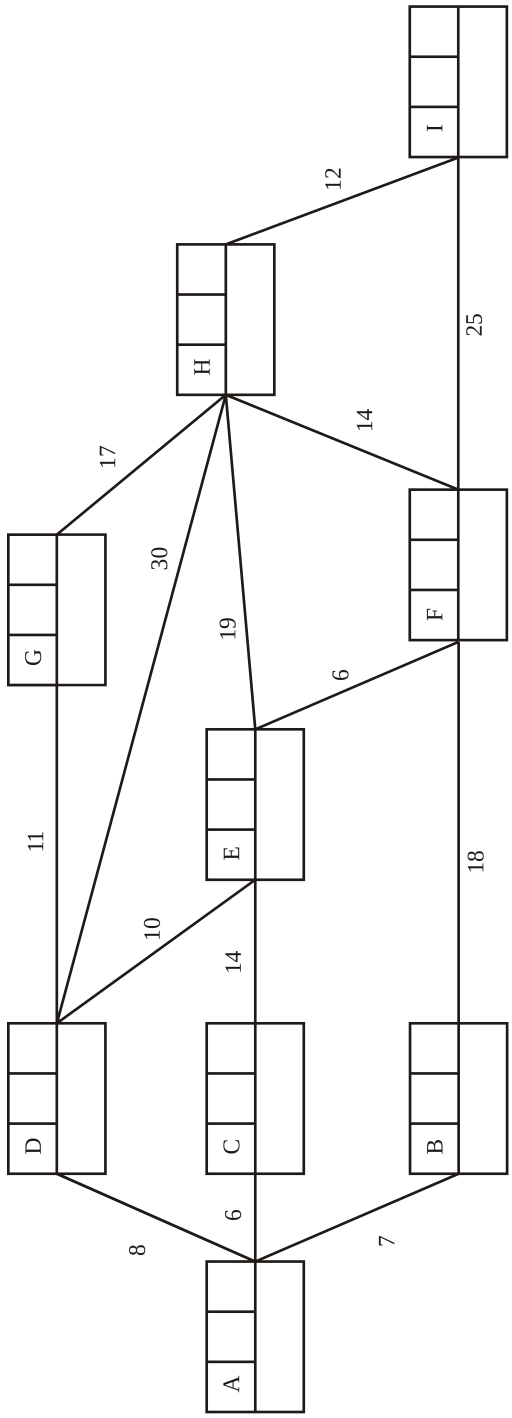
(2)
(Total 7 marks)

6.



The diagram above shows a network of roads. The number on each arc represents the length, in km, of that road.

(a) Use Dijkstra's algorithm to find the shortest route from A to I. State your shortest route and its length.



(5)

Sam has been asked to inspect the network and assess the condition of the roads. He must travel along each road at least once, starting and finishing at A.

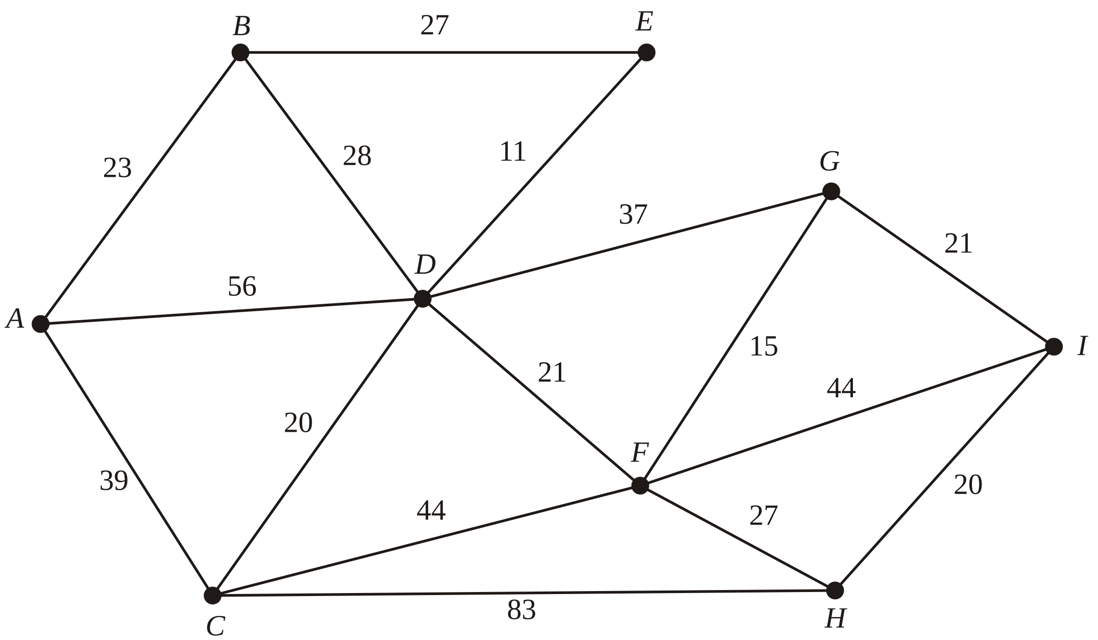
(b) Use an appropriate algorithm to determine the length of the shortest route Sam can travel. State a shortest route.

(The total weight of the network is 197 km.)

(4)
(Total 9 marks)

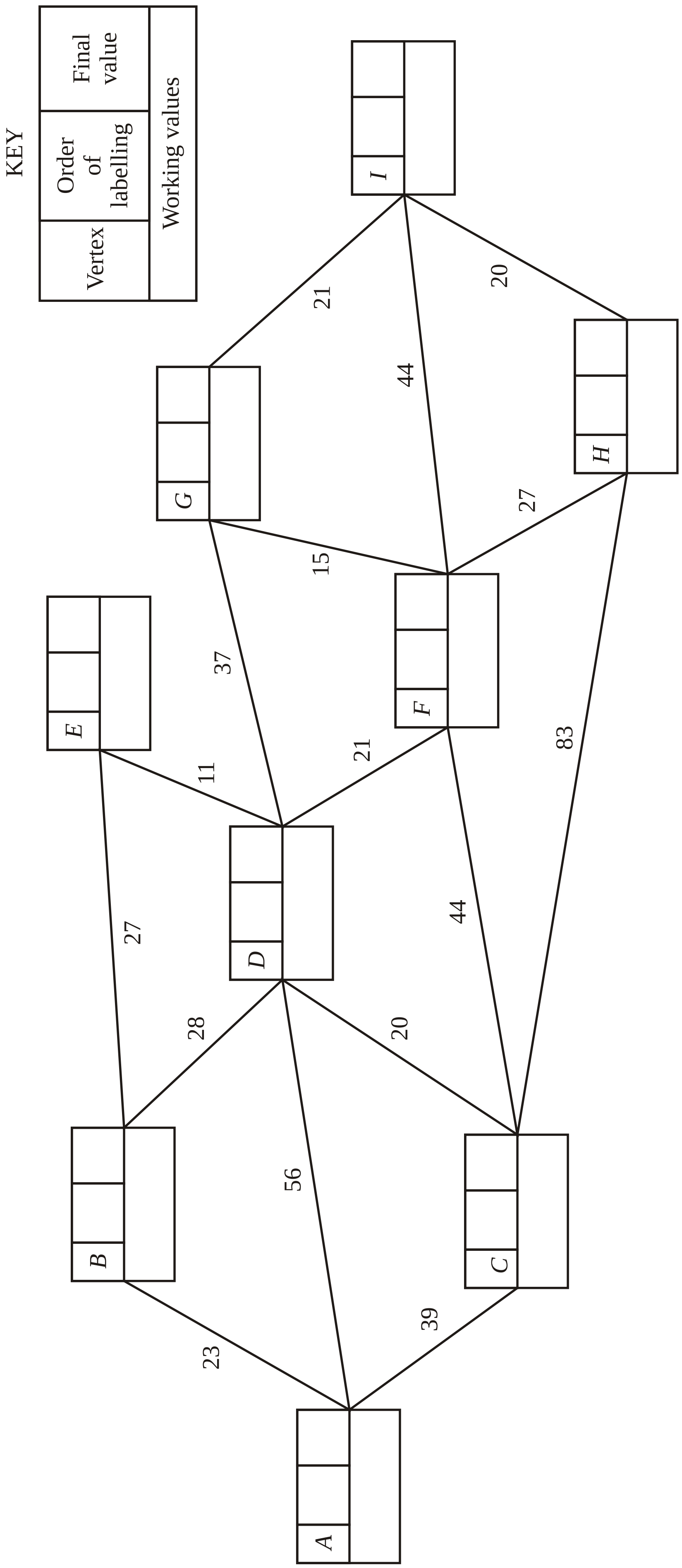
7. (a) Explain what is meant by the term 'path'.

(2)



The figure above shows a network of cycle tracks. The number on each edge represents the length, in miles, of that track. Mary wishes to cycle from *A* to *I* as part of a cycling holiday. She wishes to minimise the distance she travels.

(b) Use Dijkstra's algorithm to find the shortest path from *A* to *I*. Show all necessary working in the boxes in the diagram below in the answer book. State your shortest path and its length.


KEY
Vertex
Order of labelling
Final value
Working values
A
B
C
D
E
F
G
H
I
23
56
39
27
28
20
44
83
11
37
21
15
27
21
44
20

Shortest path ....

Length...

(6)

(c) Explain how you determined the shortest path from your labelling.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(2)

Mary wants to visit a theme park at E.

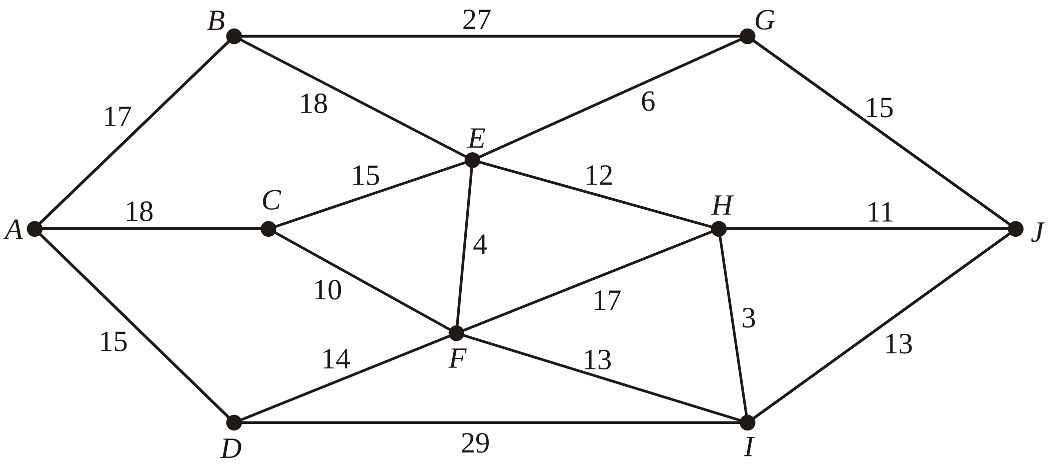
(d) Find a path of minimal length that goes from A to I via E and state its length.

Route..

Length...

(2)
(Total 12 marks)

8.



This figure shows a network of roads. The number on each arc represents the length of that road in km.

(a) Use Dijkstra's algorithm to find the shortest route from A to J. State your shortest route and its length.

(5)

(b) Explain how you determined the shortest route from your labelled diagram.

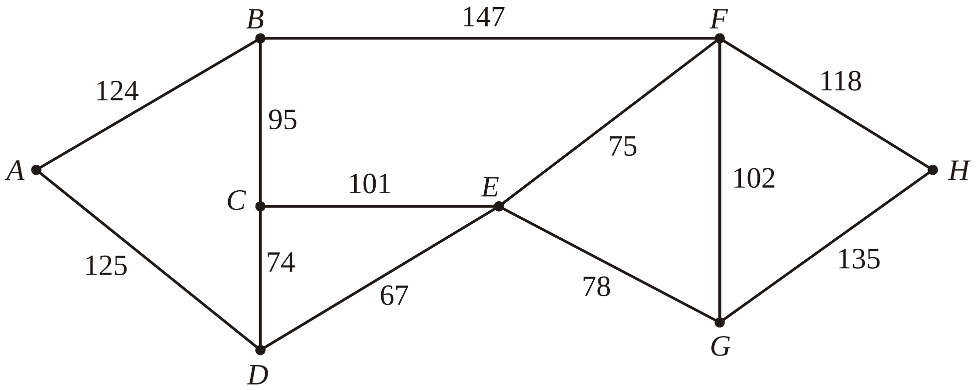
(2)

The road from C to F will be closed next week for repairs.

(c) Find the shortest route from A to J that does not include CF and state its length.

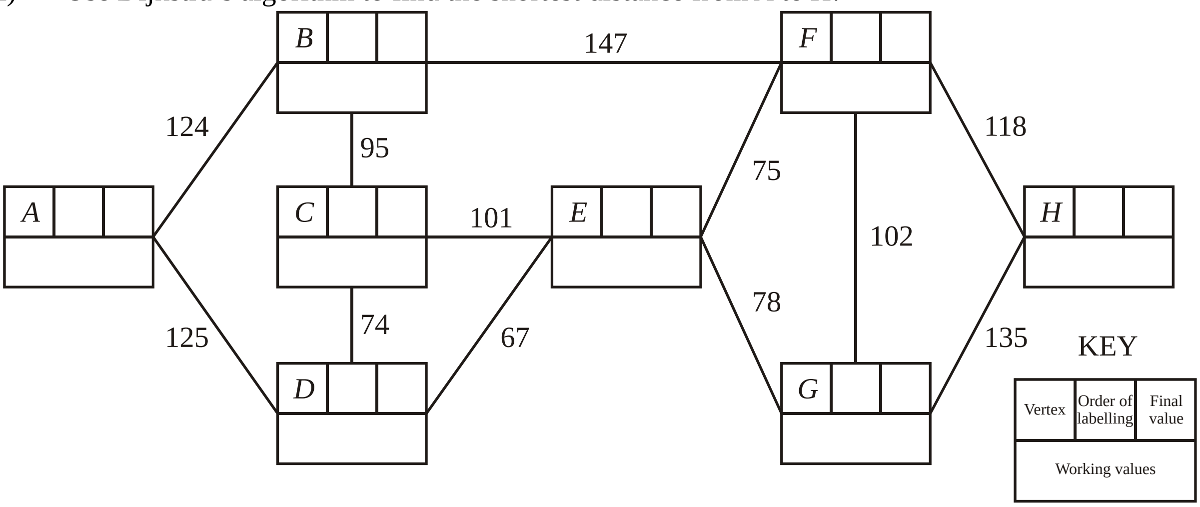
(3)
(Total 10 marks)

9.



The diagram above shows a network of paths. The number on each arc gives the distance, in metres, of that path.

(i) Use Dijkstra's algorithm to find the shortest distance from A to H.



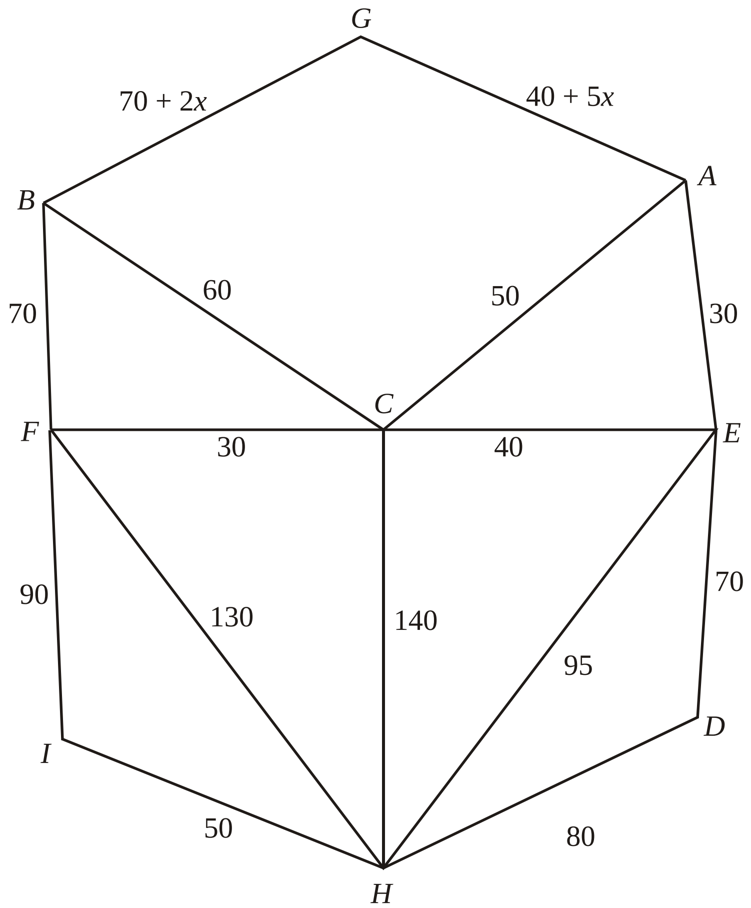
(ii) Solve the route inspection problem for the network shown in the diagram. You should make your method and working clear. State a shortest route, starting at A, and find its length.

[The total weight of the network is 1241]

(6)
(Total 11 marks)

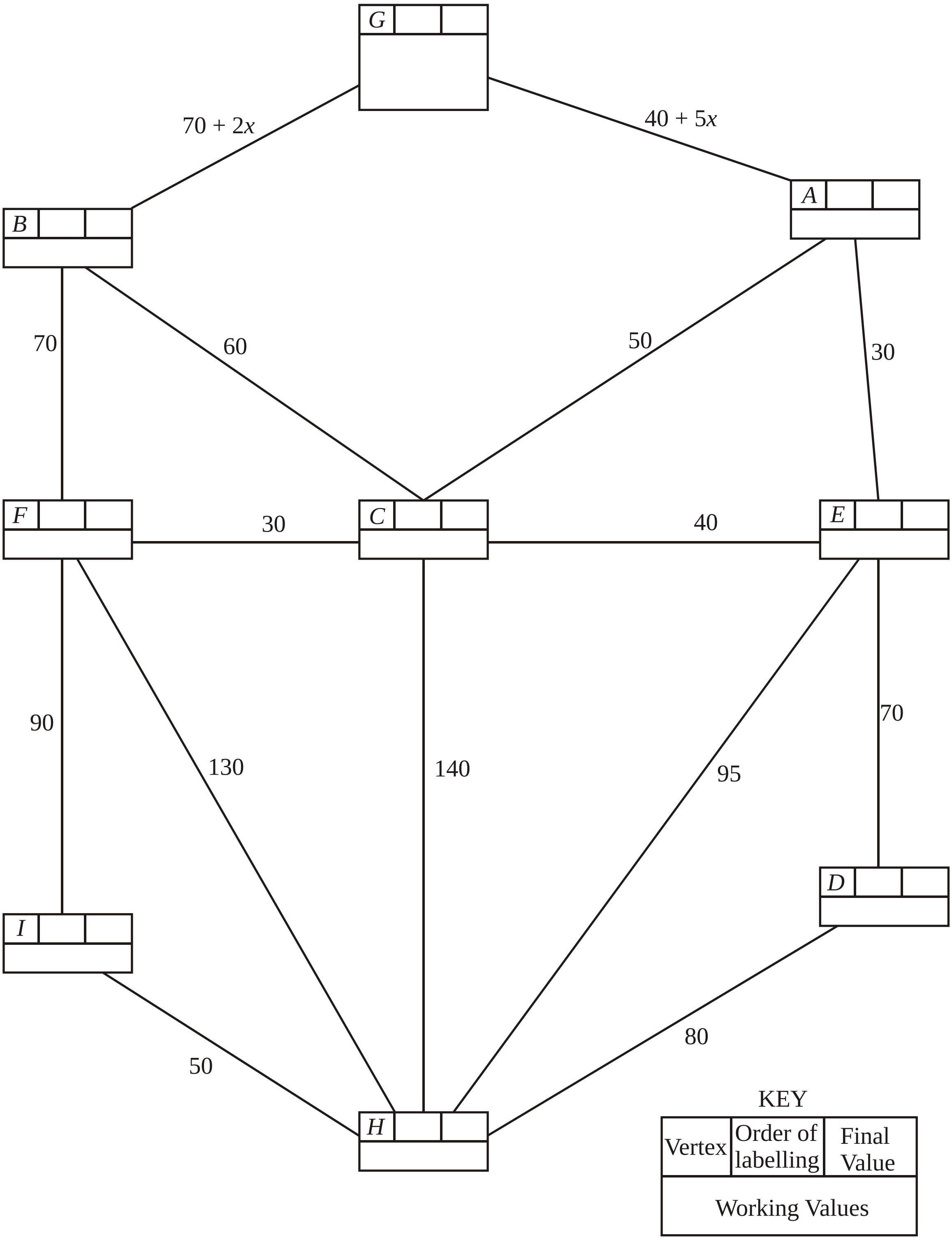
10.

Figure 1



Peter wishes to minimise the time spent driving from his home *H*, to a campsite at *G*. Figure 1 shows a number of towns and the time, in minutes, taken to drive between them. The volume of traffic on the roads into *G* is variable, and so the length of time taken to drive along these roads is expressed in terms of *x*, where $x \geq 0$.

(a) On the diagram below, use Dijkstra's algorithm to find two routes from *H* to *G* (one via *A* and one via *B*) that minimise the travelling time from *H* to *G*. State the length of each route in terms of *x*.



(7)

(b) Find the range of values of x for which Peter should follow the route via A.

(3)
(Total 10 marks)

11.

Figure 1

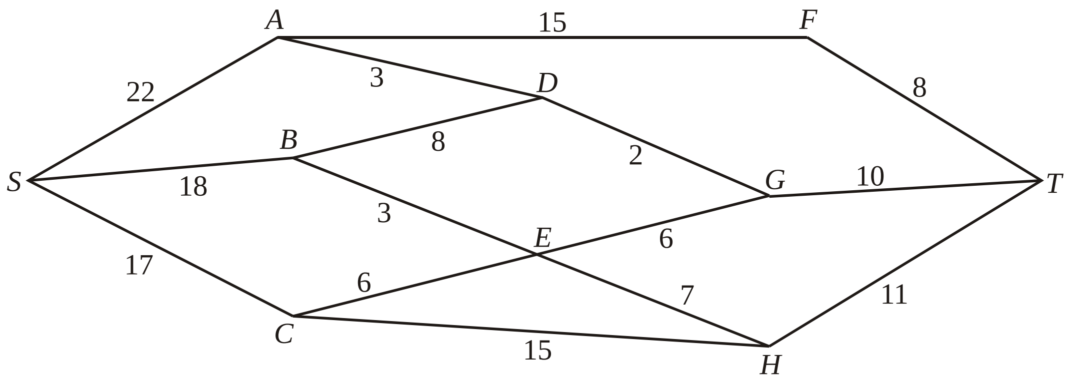
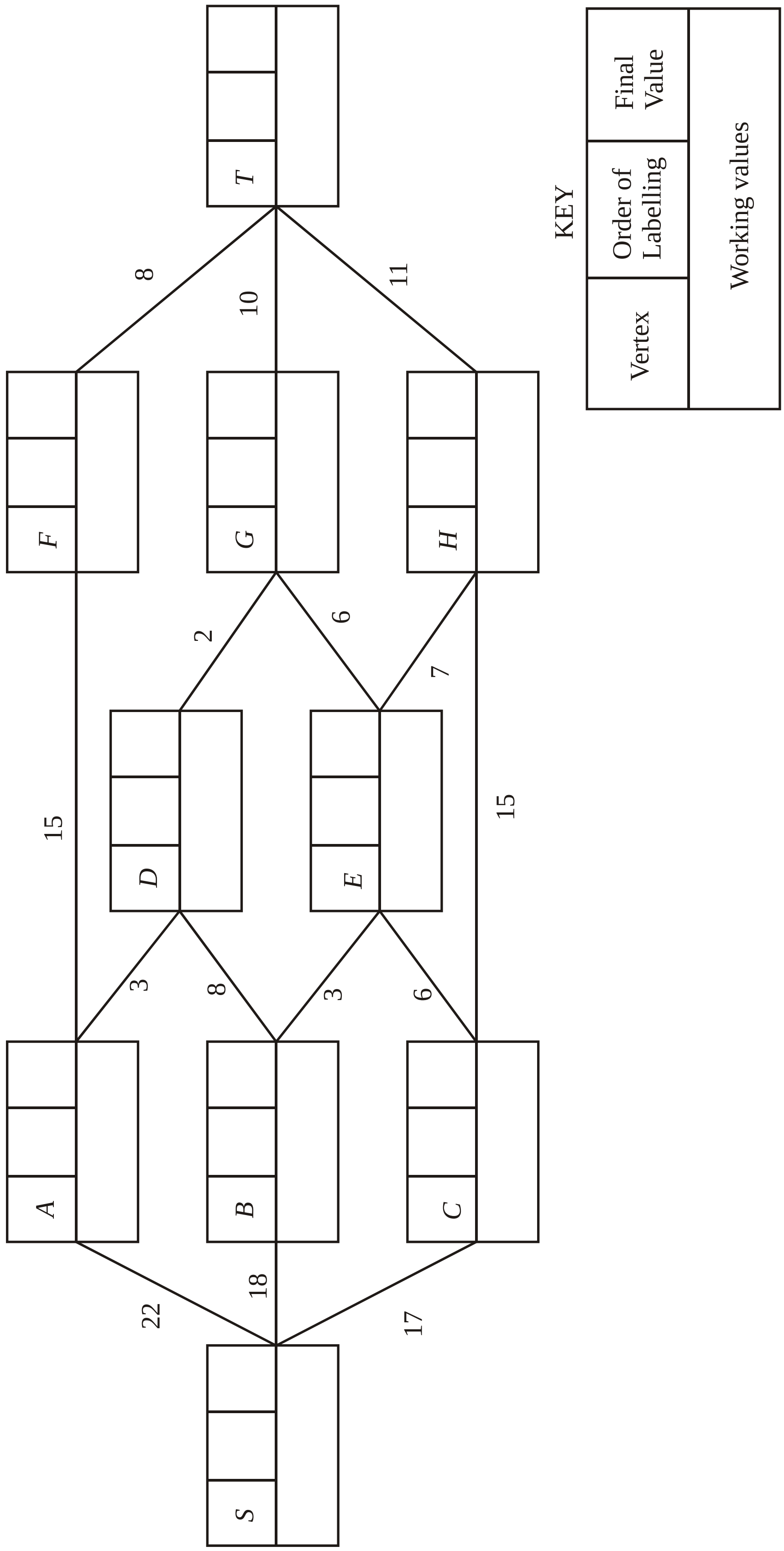


Figure 1 shows a network of roads. The number on each edge gives the time, in minutes, to travel along that road. Avinash wishes to travel from S to T as quickly as possible.

(a) Use Dijkstra’s algorithm to find the shortest time to travel from S to T.


S
A
B
C
D
E
F
G
H
T
22
18
17
3
8
3
6
15
15
2
6
7
8
10
11
KEY
Vertex
Order of Labelling
Final Value
Working values

(4)

(b) Find a route for Avinash to travel from S to T in the shortest time. State, with a reason, whether this route is a unique solution.

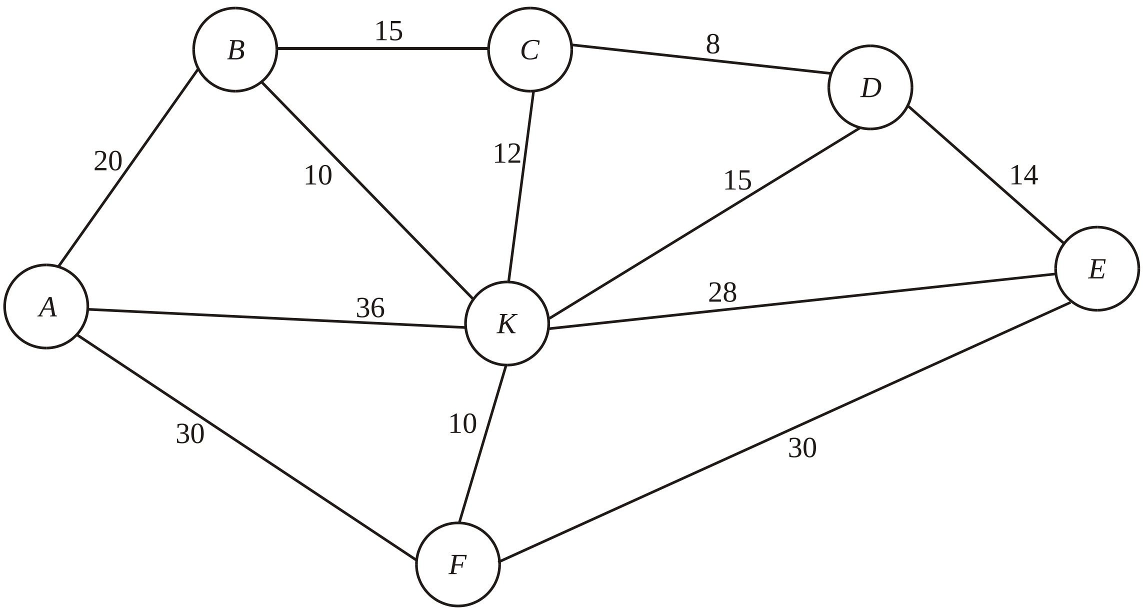
(2)

On a particular day Avinash must include C in his route.

(c) Find a route of minimal time from S to T that includes C, and state its time.

(2)
(Total 8 marks)

12. *This question should be answered on the page below.*



The diagram above shows a number of satellite towns A, B, C, D, E and F surrounding a city K. The number on each edge give the length of the road in km.

(a) Use Dijkstra's algorithm to find the shortest route from A to E in the network. Show your working in the boxes provided on the answer sheet.

(8)

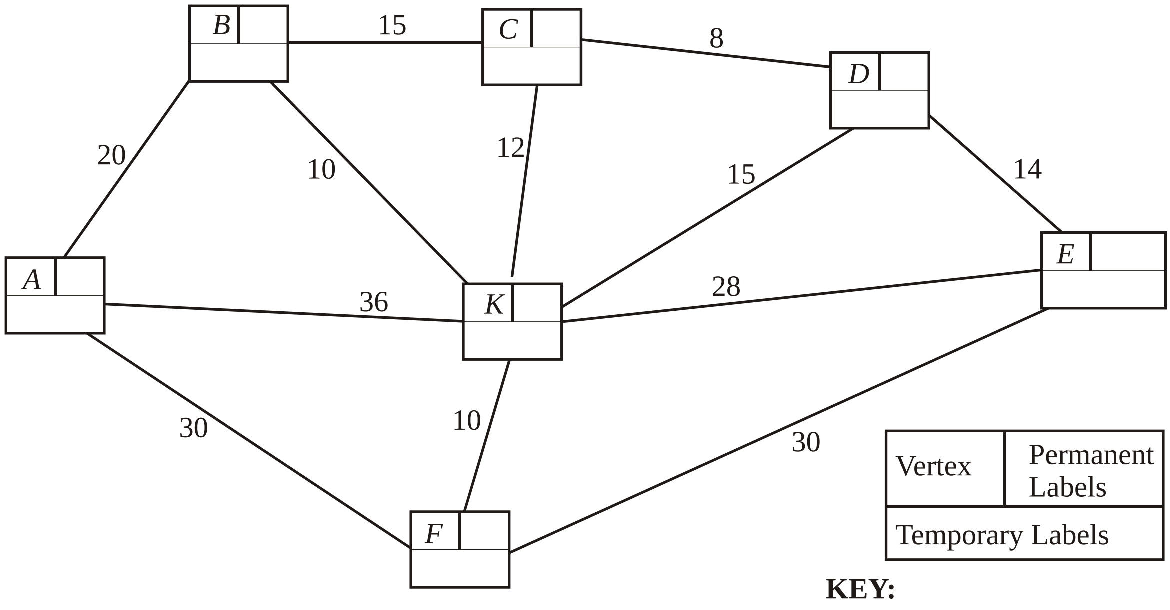
It is planned to link all the sites A, B, C, D, E and F and K by telephone lines laid alongside the roads.

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network and hence obtain the minimum total length of cable required. Draw your tree.

(6)
(Total 14 marks)

Sheet for use in answering this question

(a)



Length of Shortest Path:

...……………………………………………………………………………………

Determination of shortest route:

...……………………………………………………………………………………

...……………………………………………………………………………………

...……………………………………………………………………………………

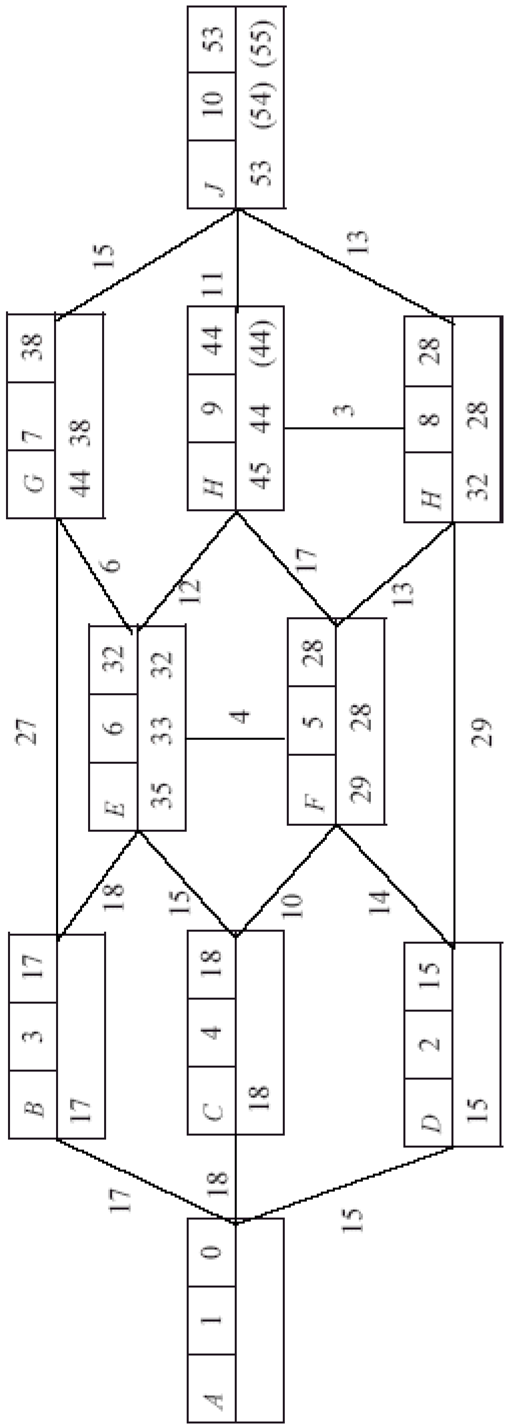
...……………………………………………………………………………………

...……………………………………………………………………………………

(b) Minimum spanning tree

Total weight of minimum spanning tree: ……………………………………………

1. (a)



M1 A1 A1 ft A1 ft

Route: *AC F EG J*

Length: 53 km A1 5

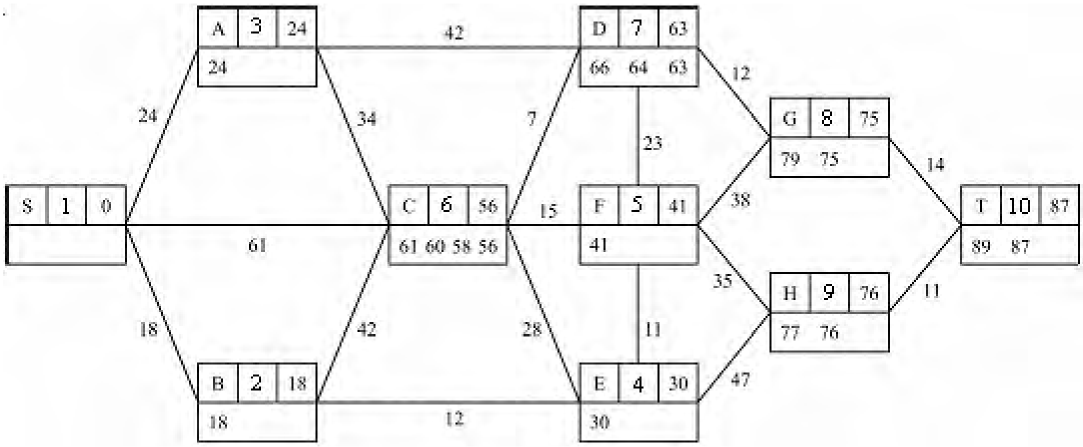
(b) General explanation – trace back from *J*
– Include arc *XY* if *Y* is already on path and if difference in trial labels equals length of arc.

Specific explanation 53 – 15 = 38GJ
38 – 6 = 32£G
32 – 4 = 28 *FE*
28 – 10 = 18CF
18 – 18 = 0^C B2ft 1ft 2

(c) Eg *A D F E G J* or *A C E G J*; length 54 km B1 B1ft 2

[9]

2. (a)



M1 A1 A1 ft A1

Route: SBEFHT B1
Time: 87 minutes B1 ft 6

<u>Note</u>

1M1: Smaller number replacing larger number in the working values at C or D or G or H or T. (generous – give bod)
1A1: All values in boxes S, A, B, E and F correct
2A1ft: All values in boxes C and D (ft) correct. Penalise order of labelling errors just once.
3A1: All values in boxes G, H and T correct
1B1: CAO (not ft)
2B1ft: Follow through from their T value, condone lack of units here.

(b) Accept demonstration of relevant subtractions, or general explanation. B2ft, 1ft, 0 2

<u>Note</u>

1B1ft: Partially complete account, maybe muddled, bod gets B1
2B1ft: Complete, clear account.

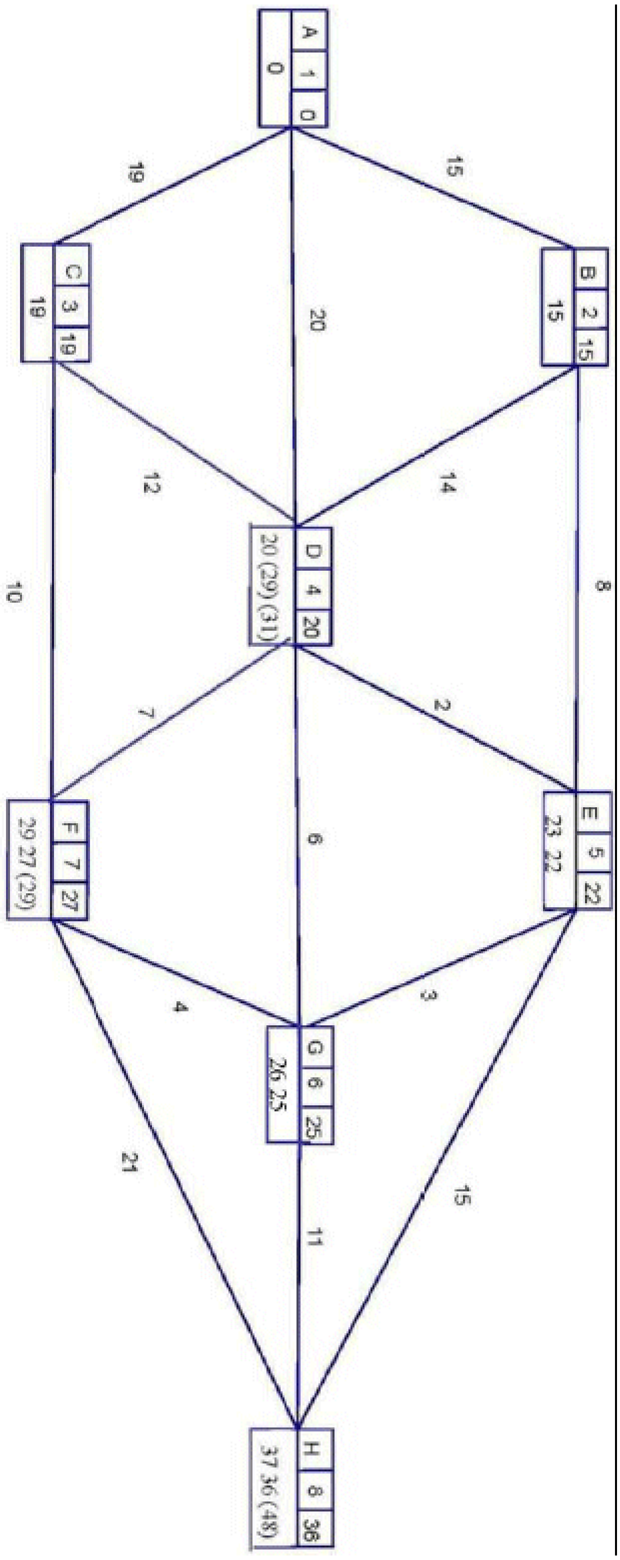
(c) Route: EFHT B1 1

Note

1B1: CAO

[9]

3. (a)



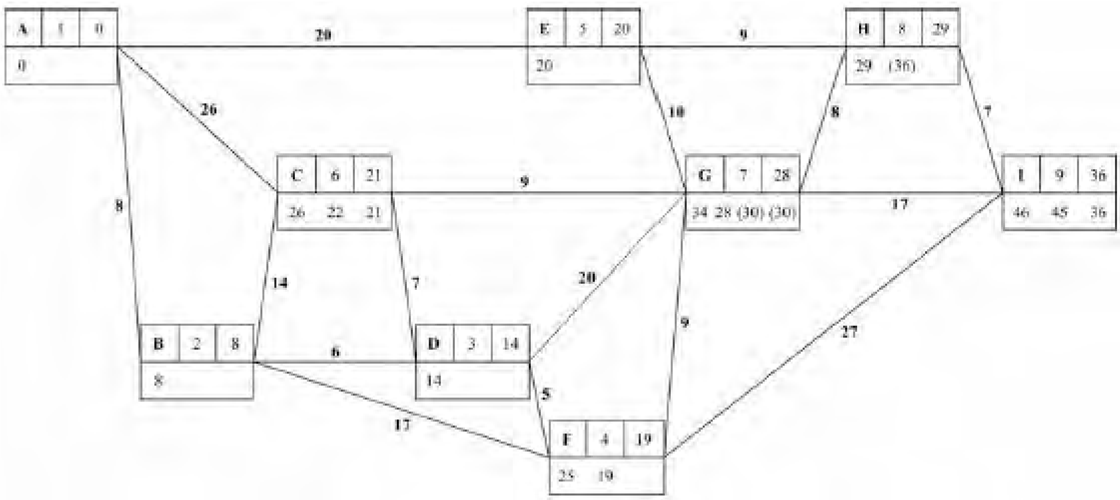
Clear method to include at least 1 update		
(look at E, F, G, or H)	M1	
BCDE correct	A1	
FGH correct	A1ft	
Route ADEGH	A1	
Total time 36 Minutes	A1ft	5

(b)

Odd nodes are A, B, C, H	M1	
AB + CH = 15 + 25 = 40	A1	
AC + BH = 19 + 22 = 41	A1	
AH + BC = 36 + 22 = 58	A1	
(40 is the shortest, repeating AB and CF + FG + GH)		
Must be choosing from at least two pairings for this last mark		
Shortest time = 167 + 40 = 207 minutes.	A1ft	5
167 + their shortest		

[10]

4. (a)



M1

1A1

2A1ft

3A1ft

4A1ft

Route: A E H I 5A1

Note

1M1: Small replacing big in the working values at C or F or

G or I

1A1: Everything correct in boxes at A, B, D and F

2A1ft: ft boxes at E and C handled correctly but penalise order of labelling only once

3A1ft: ft boxes at G and H handled correctly but penalise order of labelling only once

4A1ft: ft boxes at I handled correctly but penalise order of labelling only once

5A1: route cao A E H I

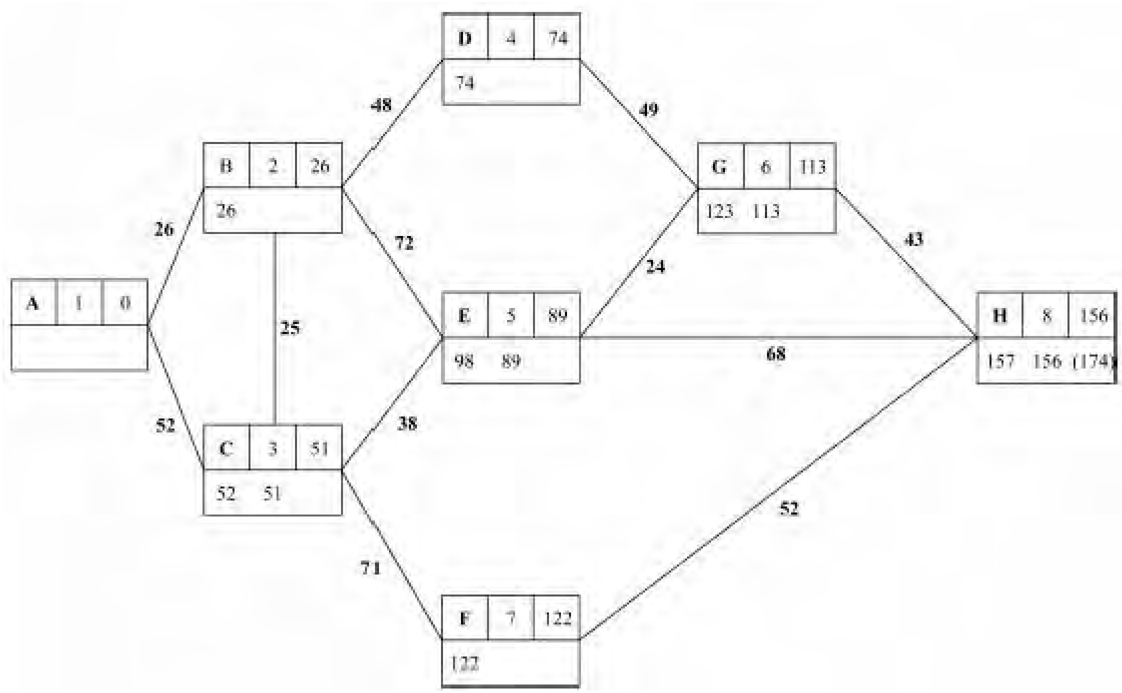
(b) Shortest distance from A to G is 28 km B1ft

Note

1B1ft: ft their final label at G condone lack of km

[7]

5. (a)



M1A1A1ft

Shortest route: A B C E G H A1

Length: 156 (km) A1ft 5

Note

1M1: Dijkstra's algorithm, small replacing larger in at least one of the sets of working values at C, E, G or H

1A1: Values correct at vertices A to E.

2A1ft: Values correct at vertices F to H, penalise order only once.

3A1: cao

4A1ft: 156ft

(b) New route: A B E G H B1

Length: 165 (km) B1 2

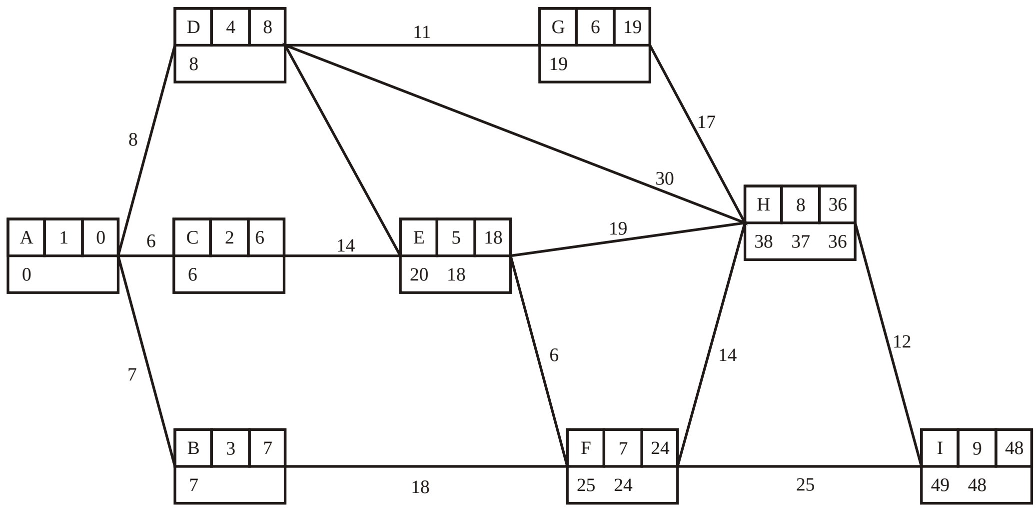
<u>Note</u>

1B1: cao ABEGH

2B1: 165 Special Case Accept 166 if ABDGH listed as the path.

[7]

6. (a)



M1A1A1ft

Route: ADGH A1

Length: 48 (km) A1ft 5

1M1: Smaller number replacing larger number in the working values at E or F or H or I. (generous – give bod)

1A1: All values in boxes A to E and G correct

2A1ft: All values in boxes F, H and I correct (ft). Penalise order of labelling just once.

3A1: CAO (not ft)

4A1ft: Follow through from their I value, condone lack of units here.

(b) Odd vertices are A and H B1

Attempt to find shortest route from A to H = ADGH M1

New length: 197 + 36 = 233 A1ft

Route: e.g. ADGHGDACEDHIFHEFBA (18) A1 4

1B1: A and H identified in some way – allow recovery from M mark.

1M1: Accept, if correct, path, or its length. Accept attempt if finding shortest.

1A1ft: 197 + their shortest A to H (36)

2A1: A correct route.

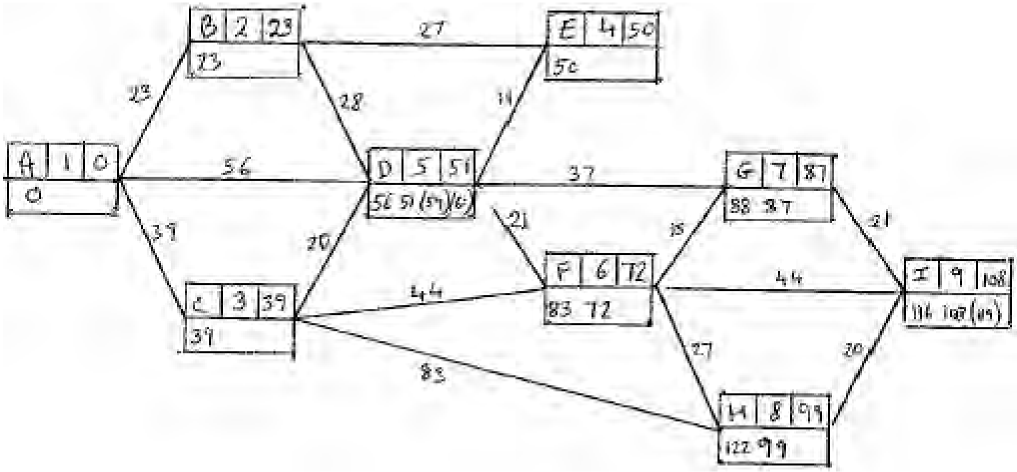
[9]

7. (a) A path is a (finite) sequence of edges, such that the end vertex of one edge is the start vertex of the next and in which no vertex appears more than once/no cycles. B2,1,0 2

B2 A good, complete answer

B1 Close – mostly there. "Bad" sets B1, "route" series may be OK

(b) M1, A1, A1, A1ft



Shortest path: ABDFGI length: 108 miles A1, A1ft 6

M1 In D, F, G, H, or I working values large replaced by small

A1 A, B, C, E corrects labels in a rising sequence

A1 D, F correct labels ft

A1ft G, H, I correct labels ft } *penalise order of labelling only once*

A1 Path c.a.o.

A1ft Length ft from I – accept 108 if a correct path

(c) e.g.

108 – 21 = 87	GI	<u>or</u>	trace back from I
87 – 15 = 72	FG		include arc XY if Y already on the path
72 – 21 = 51	DF		and if the difference in final labels equals the
51 – 28 = 23	BD		length of arc
23 – 23 = 0	AB		

B2ft1ft0 2

B2ft complete version of one of the two given explanations

B1ft all there bar one step "bad" sets B1 – easy mark

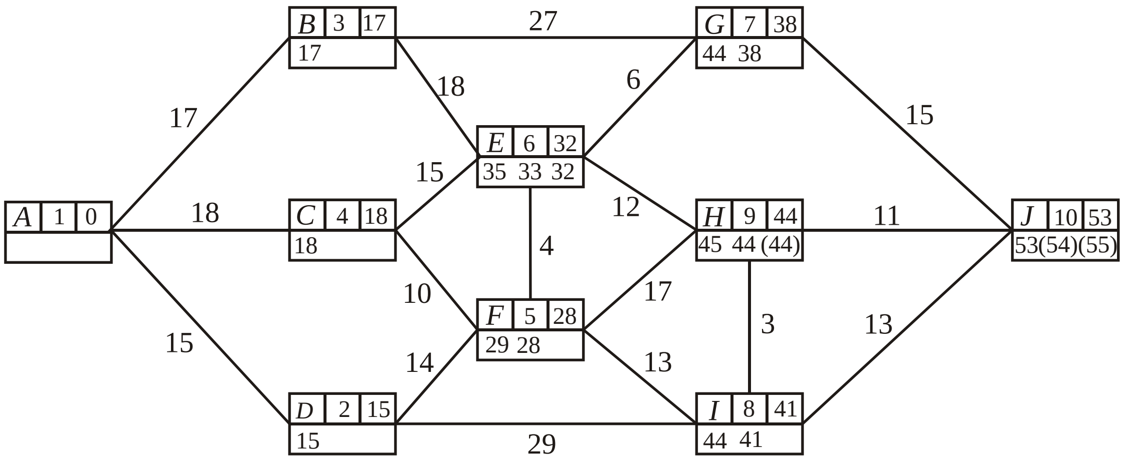
(d) A B E D F G I length: 118 miles M1 A1 2

M1 Route A to I including E

A1 c.a.o.

[12]

8. (a)



M1 A1 A1ft A1ft

Route: *A C F E G J*
length: 53 km A1 3

M1 in E or F or G or H or I w.v. large replaced by small

A1 A, B, C, D, F correct (order in rising sequence)

A1ft E G I correct + labelling (penalise order of labelling only once)

A1ft H, J correct + labelling (penalise order of labelling once

A1 Route + length (<u>both</u>) condone lack of km

(b) General explanation – trace back from *J*
– Include arc *XY* if *Y* is already on path <u>and</u>
if difference in final labels equals length of arc

Specific explanation – 53 – 15 = 38 *GJ* B2ft,1ft,0 2
38 – 6 = 32 *EG*
32 – 4 = 28 *FE*
28 – 10 = 18 *CF*
18 – 18 = 0 *AC*

B2ft complete version of one of the 2 given explanations

B1ft all these bar one stop. "bad" gets B1

(c) e.g. *A D F E G J* or *A C E G J*; length 54 k, M1A1; A1 3

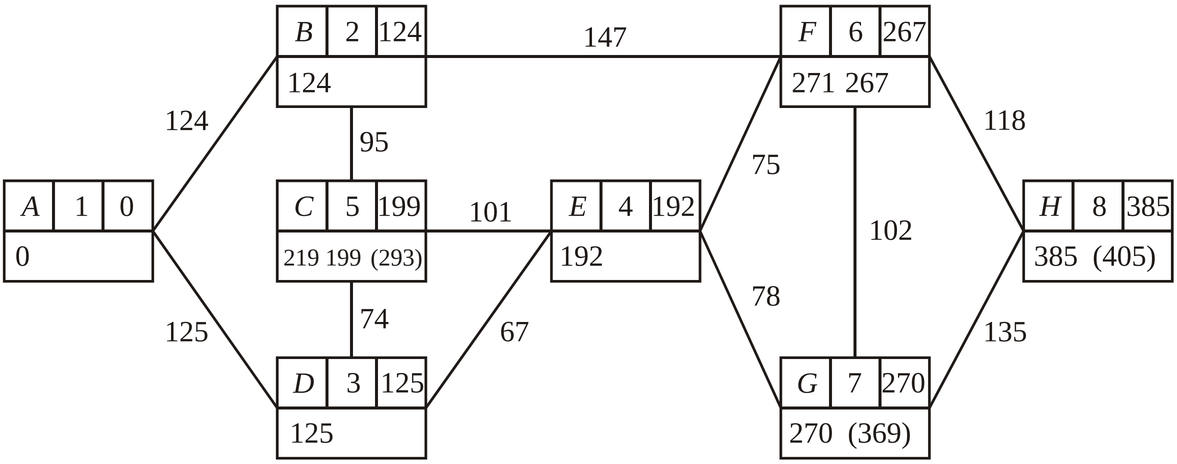
M1 Route A to J avoiding CF

A1 c.a.o. or a description

A1 54 (condone lack of km)

[10]

9. (i)



M1 A1 A1ft A1ft

<u>shortest distance is 385 m</u> A1 5

(ii) Odd vertices B, C, D, G m1

BC + DG = 95 + 145 = 240 (*) A1

BD + CG = 169 + 179 = 348 A1

BG + CD = 249 + 74 = 323 A1 4

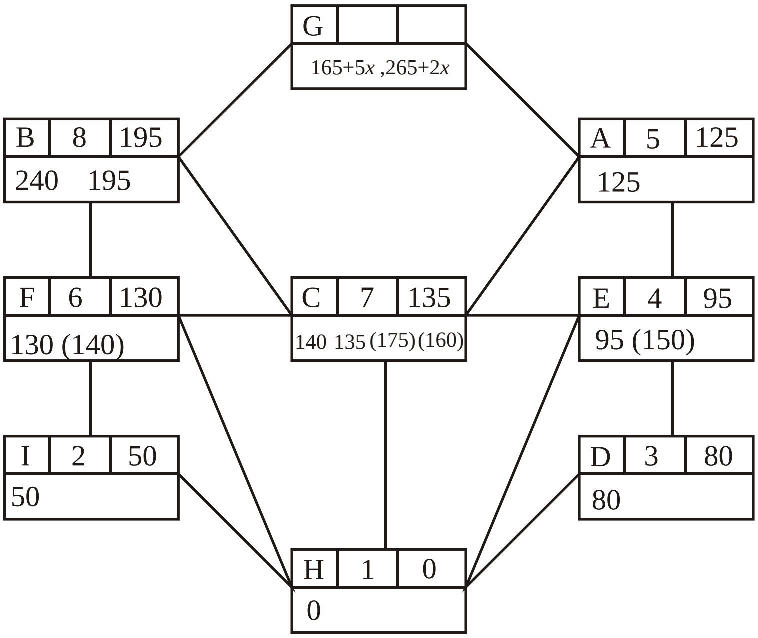
Repeat BC, DE and EG

eg. A B C B F H G F E G E C D E D A B1

length 1241 + 240 = <u>1481 m</u> B1 2

[11]

10. (a)



M1A1A1ftA1ft 4

Via A HEAG length $165 + 5x$ M1A1

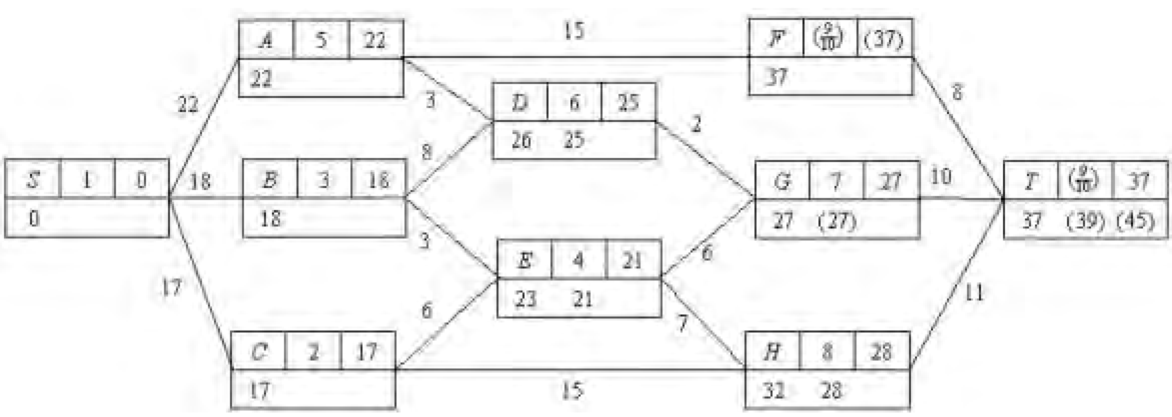
Via B HECBG length $265 + 2x$ A1 3

(b) $165 + 5x < 265 + 2x \Rightarrow x , 33\frac{1}{3}$ M1A1ft

so range is $0 \leq x < 33\frac{1}{3}$ A1ft 3

[10]

11. (a) M1 A1 A1 ft

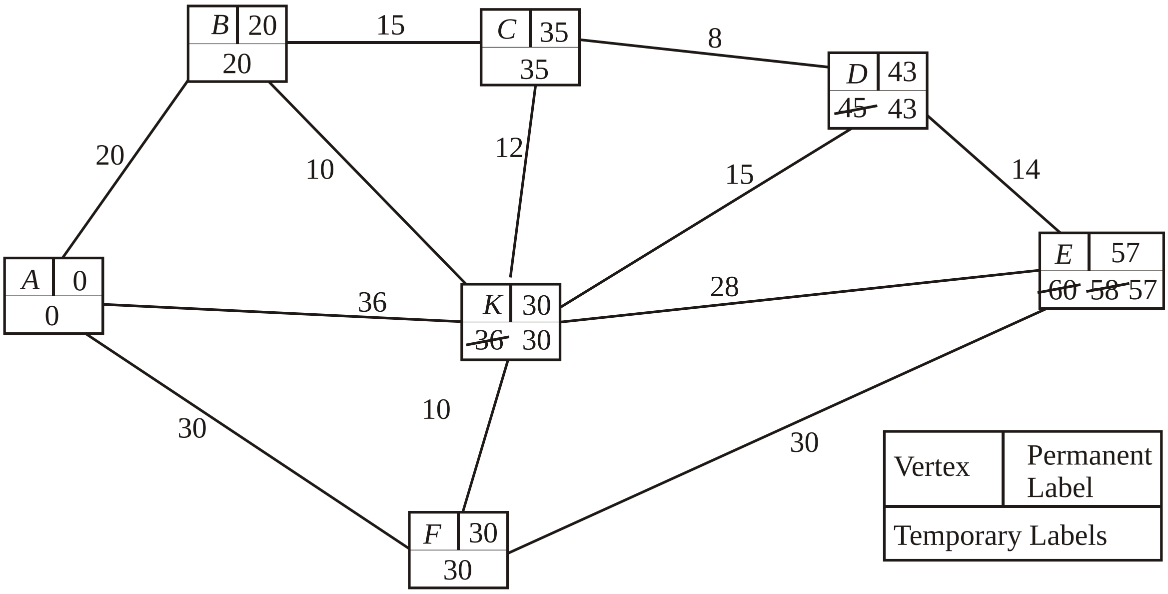


Time = 37 minutes A1 ft 4

(b)	<u>Either</u> $S – A – D – G – T$ or $S – B – E – G – T$	A1 ft	
	Not unique, e.g. gives other path	A1 ft	2
(c)	$S – C – E – G – T$ 39 minutes	M1 A1	2

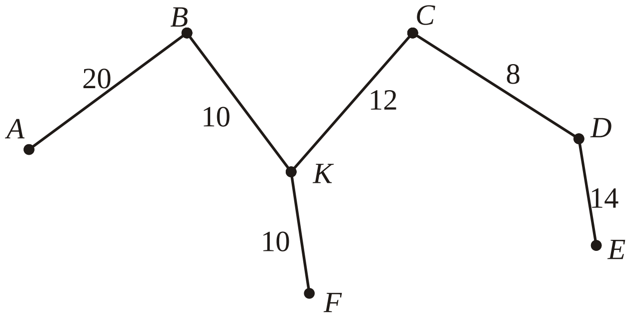
[8]

12. (a)



Length of Shortest Path: 57 km	B1	
Determination of shortest route:		
Label E – Label D = 57 – 43 = 14 (DE)		
Label D – Label C = 43 – 35 = 8 (CD)	M1 A1	
Label C – Label B = 35 – 20 = 15 (BC)		
Label B – Label A = 20 – 0 = 20 (AB)		
Hence shortest route is $A\ B\ C\ D\ E$	A1	8

(b) Minimum spanning tree



CD (8)
BK (10)
KF (10)
CK (12)
DE (14)

Not *KD* (15) Cycle
Not *BC* (15) Cycle
AB (20)

Total weight of minimum spanning tree: 74 km (tree) B1
(weight) B1 6

[14]

1. No Report available for this question.

2. Most candidates were able to gain some marks in part (a) but many made errors and only the most able gained full marks. As always, candidates are reminded that it is the **order** of the working values that is key for examiners to determine if the algorithm has been applied correctly. Many candidates made errors at C, either labelling it fourth due to its position or omitting at least one of the four working values. Most candidates were able to determine the correct route.

A full demonstration of the appropriate calculations in part (b) tended to earn both marks, whereas more generalised explanations tended to omit some key point.

Part (c) was usually answered correctly though EFT was also seen.

3. This was generally well done, with most candidates using the boxes sensibly to make their working clear, however some candidates listed their working values in the wrong order and others did not state the correct order of labeling, with F and G often incorrectly ordered. The correct shortest path and its length were frequently seen. The method was usually correct, in part (b), with three pairs being found. However, only the better candidates got all three values correct. Few seemed to make use of the result found in part (a) with 37 frequently stated as the value of AH. This part also saw a proliferation of basic arithmetical errors. Values seen were AB + CH = 43, 44, 45, 46, 59; AC + BH = 42; AH + BC = 55, 59, 61, 62, 63, 64. Some went to the trouble of listing a possible route but then sometimes did not state its length.

4. Many candidates found part (a) straightforward and gained full marks, but candidates are reminded that it is the **order** of the working values that is key for examiners to determine if the algorithm has been applied correctly. Many candidates missed the shortcut to C, giving the third (and smallest) value of 21, coming from D and it appeared that some candidates were working ‘geographically’ left to right through the network rather in order of their arithmetic nearness to A. This isconception often led to incorrect, additional or incorrectly ordered working values particularly at C, D, G and I. Many candidates did not read part (b) of the questions carefully and stated the shortest distance from A to I rather than A to G.

5. Most candidates were able to make progress in part (a) and there were many fully correct responses. There were a lot of errors seen in the order of, and calculation of, the working values and in the order of labelling. It is essential that the working values are listed in the correct order if the candidates are to gain full credit. Many candidates found the correct new route in (b) although a few found one of the slightly longer routes (ABEH or ABDGH) of length 166.

6. Most candidates found part (a) straightforward and gained full marks, but candidates are reminded that it is the order of the working values that is key for examiners to determine if the algorithm has been applied correctly. Many gained full, or nearly full marks in (b) with errors mostly due to stating an incorrect inspection route (C was frequently omitted). Some wasted time finding the shortest route from A to H from scratch, rather than using their working for part (a), Dijkstra's algorithm finds the length of the shortest route from the start to all intermediate points. Others wasted time finding the weight of the network even though it was given to them.

7. Once again the use of technical terms was very confused and very few candidates were able to give a full, clear definition in part (a), but most were able to gain some credit. The rest of the question proved accessible to most candidates, with some weaker candidates gaining a lot of their marks here. There were, of course, the usual problems with candidates listing the working values in any order in their working for part (b). Candidates must list the working values in the order in which they occur if they are to demonstrate that they are using the algorithm properly. This carelessness was a major source of mark loss for some candidates. Mistakes often appeared at D and E, 62 often being seen as an extra working value at E with the orders of these 2 vertices reversed. In part (c) candidates who listed the subtractions they had used, together with listing the arcs this indicated, were usually much more successful than those who attempted a more general explanation, but it was noted that the general quality of the response to this part of the question has improved.

8. Candidates had to apply Dijkstra's algorithm very carefully to obtain full marks in part (a) and many were able to do this. There are still a few candidates who treat this as a sort of 'minimising critical path' forward pass however. The examiners use the working values and the order in which they occur in the box as the main confirmation that Dijkstra's algorithm has been correctly applied, thus it is important that they are legible and that candidates do write them in order. The order of the working values at E, G, I and H were often incorrect, and the working values at E, G and H were often incorrect. Most candidates were able to find the correct route and length. Those that gave a numerical demonstration in part (b) gained full marks most easily, but those who gave a general explanation often missed out part of the process. Many candidates were able to score full marks in part (c).

9. This was mostly well done, but Dijkstra's algorithm had to be very carefully applied to gain full marks. Common errors were to award a final label to vertex C before vertex E and an incorrect order of working values. Some candidates did not write down the shortest distance, but wrote down the shortest route instead. Some candidates did not realise what was being asked in part (ii) and explained how they achieved their shortest route from their labelled diagram. The majority, who did attempt the route inspection, were usually fairly successful. The commonest error was to state that the pairing BG + CD = 348. Most were able to state a correct route and its length.

10. Part (a) was mostly well done, but a large number of candidates did not make their use of Dijkstra's algorithm clear, writing down the working values in any order, and even returning to vertices already awarded their final label. There were also errors in labelling the vertices e.g. with two vertices given the same number. There were often errors in listing the routes and the expressions, with 270 + 2x and HCBG often seen. In part (b) disappointingly few candidates set up the correct inequality and solved it. Many used trial and improvement with little success. Many candidates omitted 0 = x from their final answer.

11. This was often well answered by the majority of the candidates. It is essential that the working values are in written in the order in which they occur, if correct use of Dijkstra's algorithm is to be inferred. The values at D, E, H and T were often in the wrong order and the order of labelling of A and R was often transposed. Parts (b) and (c) were usually well answered. Some candidates clearly did not understand the word 'unique'.

12. No Report available for this question.