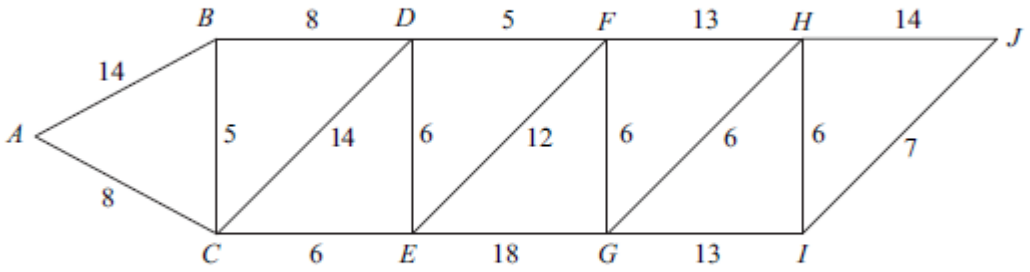
Decision 1 Shortest Path Questions

5 [**Figure 1**, printed on the insert, is provided for use in this question.]

The network shows the times, in minutes, to travel between 10 towns.



(a) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to travel from A to J. *(6 marks)*

(b) State the corresponding route. *(1 mark)*

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7 A connected graph **G** has m vertices and n edges.

(a) (i) Write down the number of edges in a minimum spanning tree of **G**. *(1 mark)*

(ii) Hence write down an inequality relating m and n. *(2 marks)*

(b) The graph **G** contains a Hamiltonian cycle. Write down the number of edges in this cycle. *(1 mark)*

(c) In the case where **G** is Eulerian, draw a graph of **G** for which $m = 6$ and $n = 12$. *(2 marks)*

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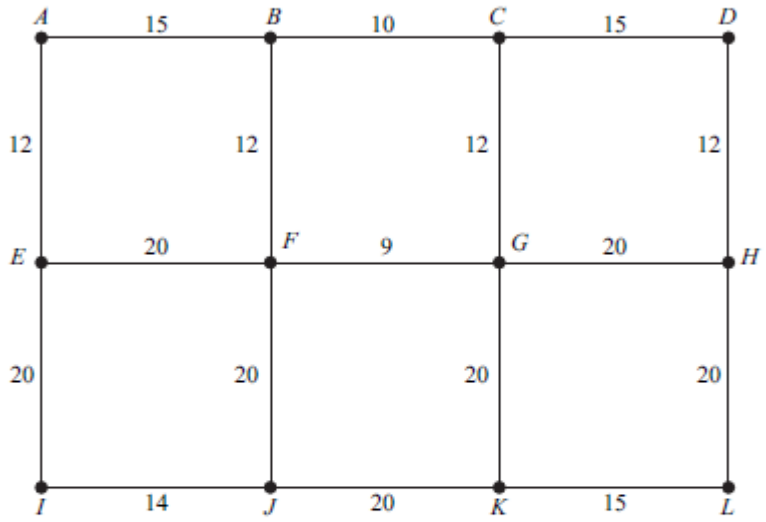
3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The following network represents the footpaths connecting 12 buildings on a university campus. The number on each edge represents the time taken, in minutes, to walk along a footpath.



(a) (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from A to L. *(7 marks)*

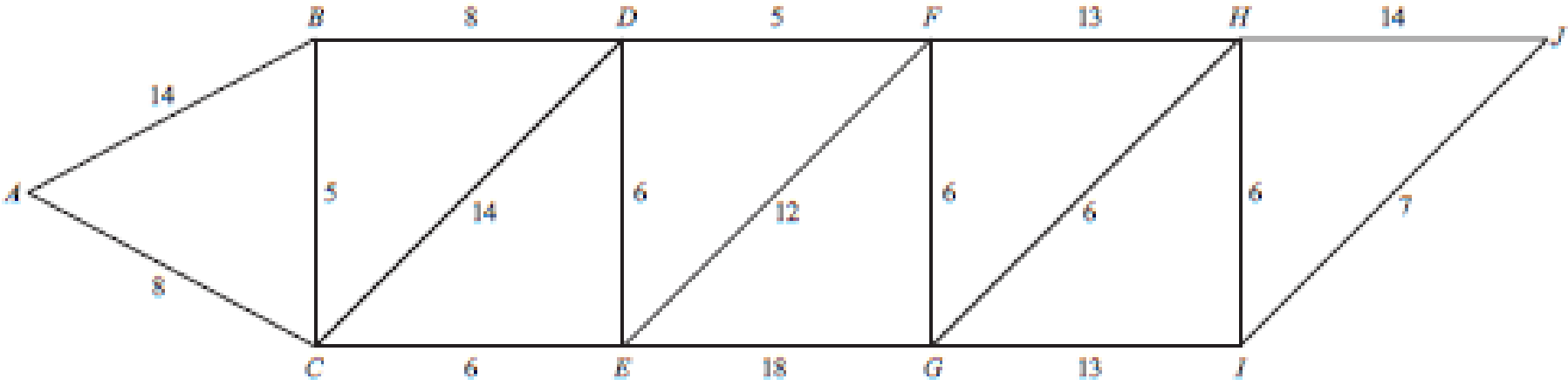
(ii) State the corresponding route. *(1 mark)*

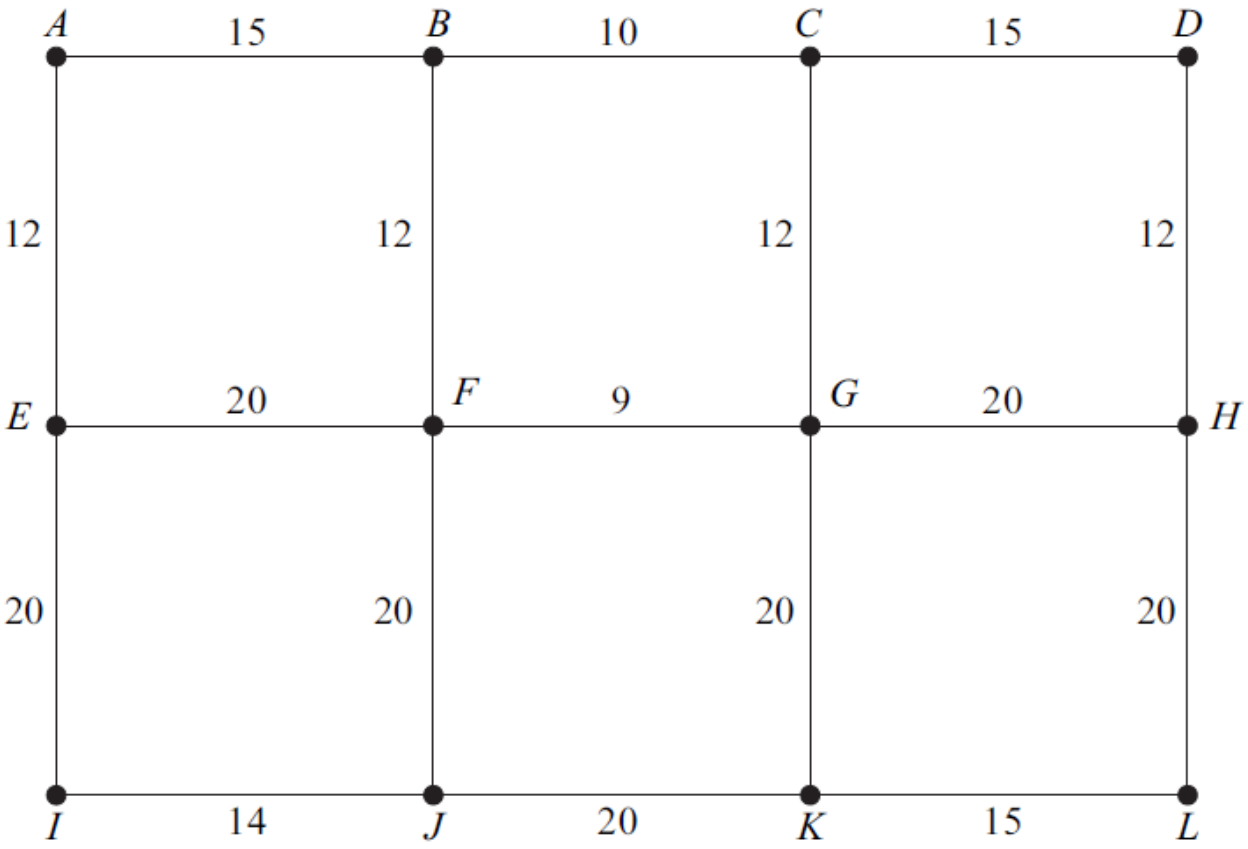
(b) A new footpath is to be constructed. There are two possibilities:

from A to D, with a walking time of 30 minutes; or

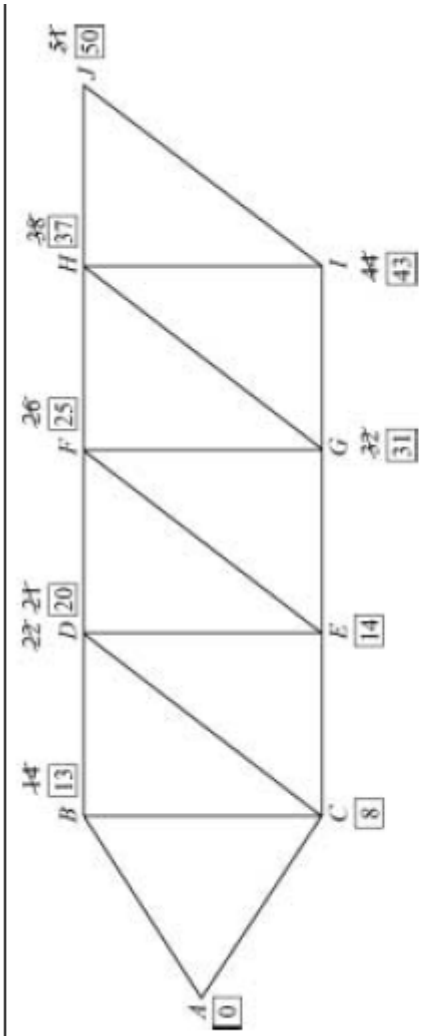
from A to I, with a walking time of 20 minutes.

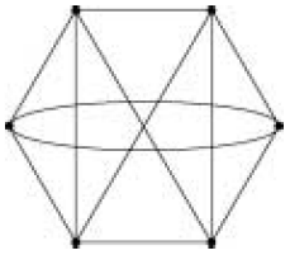
Determine which of the two alternative new footpaths would reduce the walking time from A to L by the greater amount. *(3 marks)*

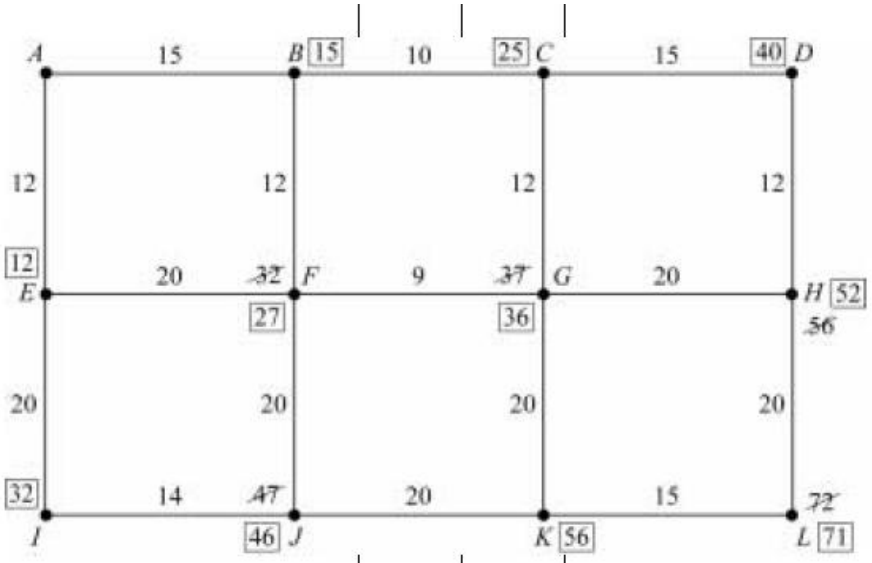

A
B
C
D
E
F
G
H
I
J
14
8
5
14
6
12
6
6
6
7
8
6
18
13
14


A
15
B
10
C
15
D
12
12
12
12
E
20
F
9
G
20
H
20
20
20
20
I
14
J
20
K
15
L

Decision 1 Shortest Path Answers

5(a)		M1 A1 M1 M1 A1 B1	 6	SCA 2 correct values at B 3 values at D 2 values at I all correct for 50 at J
(b)	Route $A\,C\,E\,D\,F\,G\,H\,I\,J$	B1	1	

7(a)(i)	$m-1$	B1	1	
(ii)	$n \geq m-1$	B2	2	B1 for $>$ or $(n>m)$ OE
(b)	$m(=n)$	B1	1	
(c)		M1 A1	 2	$m=6$ and eulerian All correct
	Total		**6**	

3(a)(i)				
			M1	SCA
			A1	Correct at *F*
			M1	2 values at *G*
			M1	2 values at *J*
			M1	2 values at *H*
			A1	All correct
	71		B1	7
	OR			
(a)(i)	Working back from L			
	35 at *G* 47 at *C* 44 at *F* 49 at *I* 56 at *B* 64 at *E* 71 at *A* } B1 × 7			
(ii)	*A B F G K L*		B1	1
(b)	*ADL* gives 62		M1	OE
	AIL gives 69		A1	OE Either, considering routes *ADL* or *AIL*
	∴ *A* to *D*		A1	3 CSO
		Total		**11**