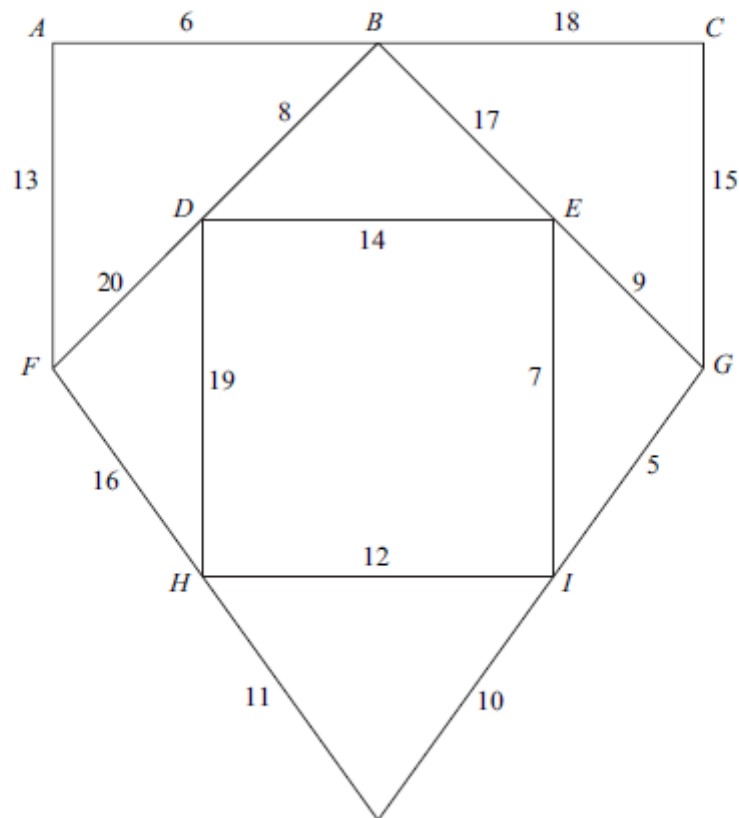


Decision 1 Minimum Spanning Tree Questions

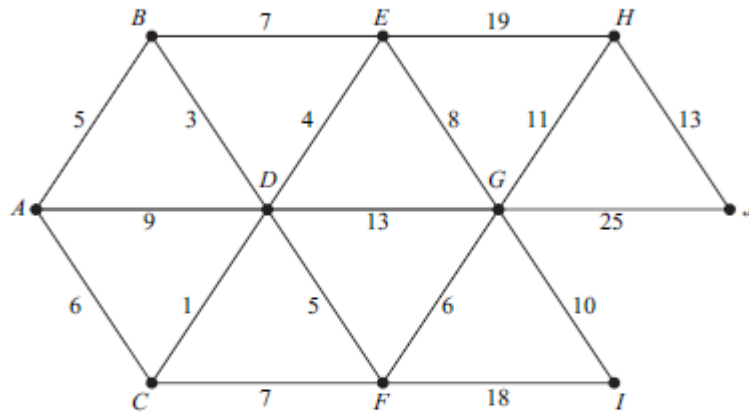
- 3 (a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices. *(1 mark)*
- (ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*
- (b) The following network has 10 vertices: A, B, \dots, J . The numbers on each edge represent the distances, in miles, between pairs of vertices.



- (i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*
- (ii) State the length of your spanning tree. *(1 mark)*
- (iii) Draw your spanning tree. *(2 marks)*
-

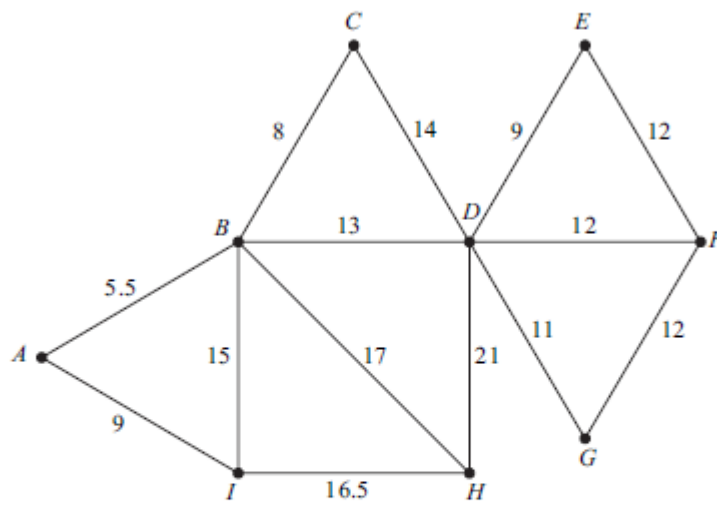
3 [Figure 1, printed on the insert, is provided for use in part (b) of this question.]

The diagram shows a network of roads. The number on each edge is the length, in kilometres, of the road.



- (a) (i) Use Prim's algorithm, starting from A , to find a minimum spanning tree for the network. *(5 marks)*
- (ii) State the length of your minimum spanning tree. *(1 mark)*
- (b) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest distance from A to J . *(6 marks)*
- (ii) A new road, of length x km, is built connecting I to J . The minimum distance from A to J is reduced by using this new road. Find, and solve, an inequality for x . *(2 marks)*
-

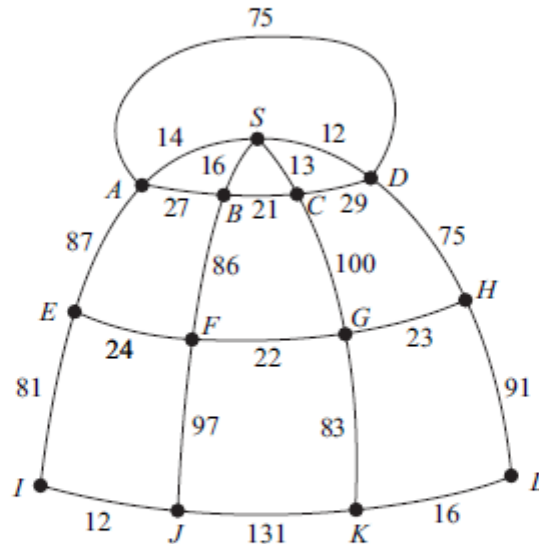
1 The following network shows the lengths, in miles, of roads connecting nine villages.



- Use Prim's algorithm, starting from A , to find a minimum spanning tree for the network. *(5 marks)*
 - Find the length of your minimum spanning tree. *(1 mark)*
 - Draw your minimum spanning tree. *(3 marks)*
 - State the number of other spanning trees that are of the same length as your answer in part (a). *(1 mark)*
-

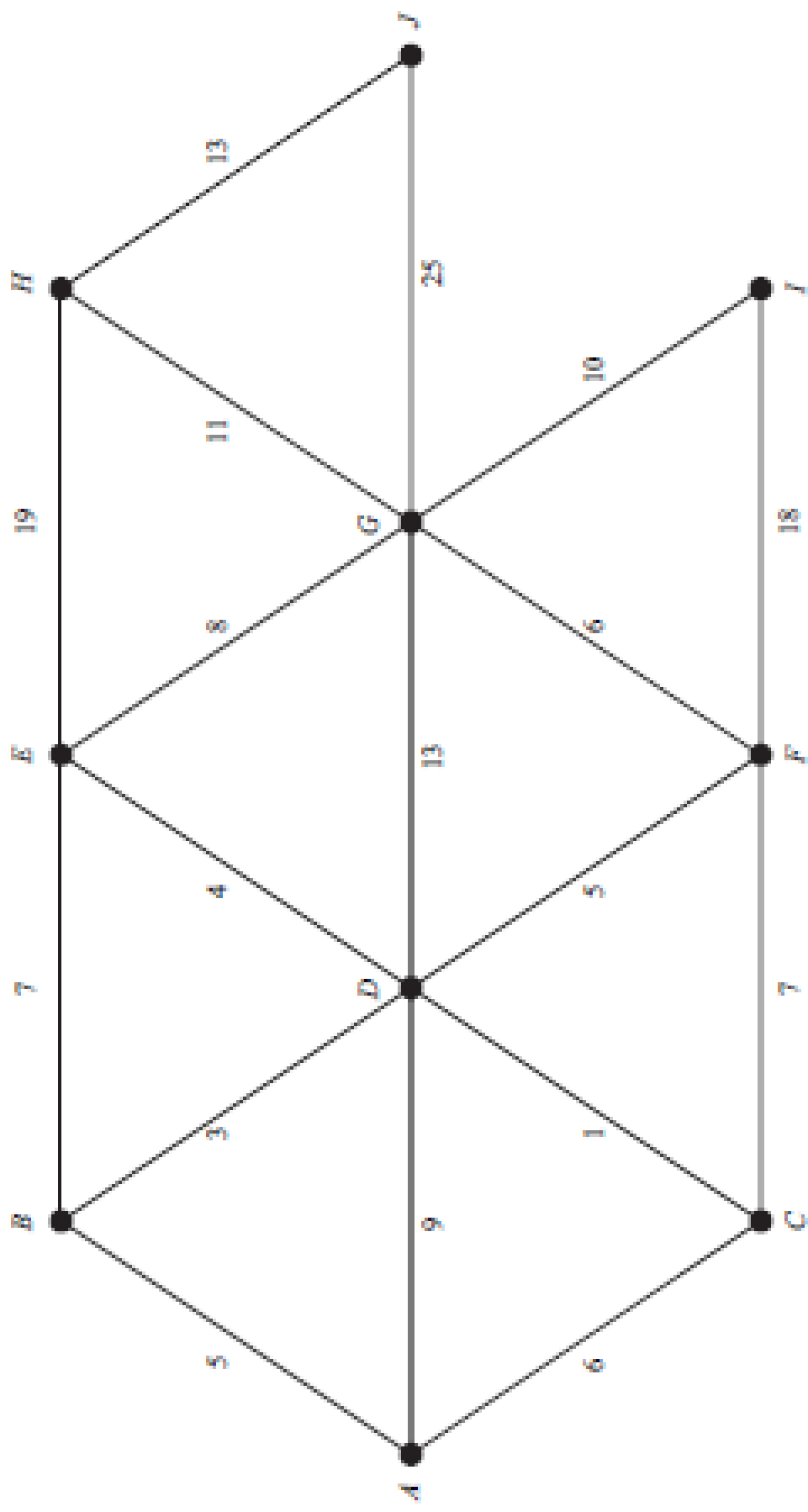
- 4 The diagram shows the various ski-runs at a ski resort. There is a shop at S . The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points A, B, \dots, L and at the shop at S .

The number on each edge represents the distance, in metres, between two points.

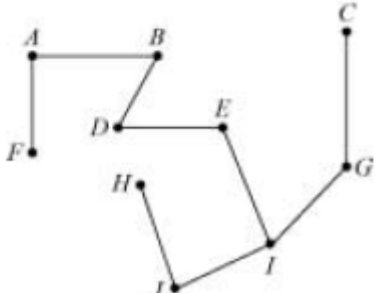


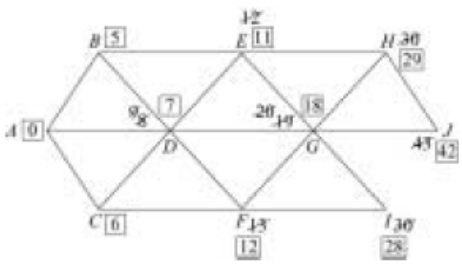
Total of all edges = 1135

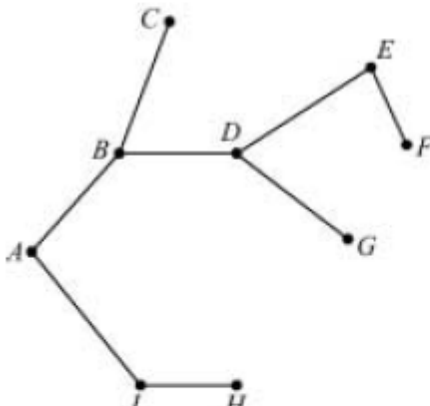
- (a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points A, B, \dots, L and the shop at S .
- Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points. *(5 marks)*
 - State the length of your minimum spanning tree. *(1 mark)*
 - Draw your minimum spanning tree. *(3 marks)*
 - The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree. *(2 marks)*
- (b) At the end of each day a snow plough has to drive at least once along each edge shown in the diagram in preparation for the following day's skiing. The snow plough must start and finish at the point L .
- Use the Chinese Postman algorithm to find the minimum distance that the snow plough must travel. *(6 marks)*
-



Decision 1 Minimum Spanning Trees Answers

3(a)(i)	9	B1	1	
(ii)	$n - 1$	B1	1	
(b)(i)	GI 5	B1		9 edges
	AB 6	M1		SCA
	EI 7	A1		start with GI
	BD 8			
	EG 9			
	LJ 10	A1		LJ fifth
	HJ 11			
	HI 12			
	AF 13			
	DE 14	A1	5	all correct
CG 15				
(ii)	89	B1	1	
(iii)		M1		9 edges
		A1	2	
		Total		10

3(a)(i)	AB 5	M1		SCA	
	BD 3	B1		9 edges	
	DC 1	A1		DC 3 rd	
	DE 4	A1		DE 4 th	
	DF 5				
	FG 6				
(ii)	GI 10				
	GH 11	B1	5	All correct	
	HJ 13				
	(ii)	58	B1	1	
	(b)(i)		M1		SCA
			M1		3 values at D
A1				All correct at D	
M1				3 values at G	
A1				All correct	
B1			6	42 at J – or in script	
(ii)	$28 + x < 42$ O.E.	M1		Allow \leq	
	$x < 14$ ISW	A1	2	SC $x \leq 13$ B1	
Total			14		

1(a)	AB 5.5 BC 8 AI 9 BD 13 DE 9 DG 11 DF, EF, GF 12 IH 16.5	B1 M1 A1 A1 A1	 5	8 edges SCA AI 3rd BD 4th All correct
(b)	84	B1	1	
(c)		M1 B1 A1	 3	Minimum spanning tree 8 edges All correct including labelling (or including DF or GF instead of EF)
(d)	2	B1	1	
Total			10	

4(a)(i)	<i>SD</i>	12	M1		Prim's (first 4 edges, allow 1 slip)
	<i>SC</i>	13			
	<i>SA</i>	14	B1		12 edges
	<i>SB</i>	16			
	<i>DH</i>	75			
	<i>HG</i>	23	A1		<i>HG</i> 6 th
	<i>GF</i>	22			
	<i>FE</i>	24			
	<i>EI</i>	81	A1		<i>EI</i> 9 th
	<i>IJ</i>	12			
	<i>GK</i>	83			
	<i>KL</i>	16	B1	5	All correct
	(ii)		391	B1	1
(iii)			M1		MST (10 + edges)
			A1		12 edges
			A1	3	All correct
(iv)	<i>GF</i> 7 th (22) <i>HG</i> 8 th (23)		B1		
			B1	2	
(b)	Odd vertices (<i>E, H, J, K</i>) $EH + JK = 69 + 131 = (200)$ $EJ + HK = 93 + 106 = (199)$ $EK + JH = 129 + 142 = (271)$ Repeat $EJ + HK$ Total $1135 + 199 = 1334$		E1		PI
			M1		2 correct sets of pairings
			A3,2,		
			1,0		
			B1	6	
Total				17	