Decision 1 Matching Graph Questions

1 (a) Draw a bipartite graph representing the following adjacency matrix. *(2 marks)*

	U	*V*	*W*	*X*	*Y*	*Z*
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(b) Given that initially A is matched to W, B is matched to X, C is matched to V, and E is matched to Y, use the alternating path algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

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1 Five people, A, B, C, D and E, are to be matched to five tasks, 1, 2, 3, 4 and 5. The table shows which tasks each person can do.

Person	**Tasks**
A	1, 3, 5
B	2, 4
C	2
D	4, 5
E	3, 5

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially A is matched to task 3, B to task 4, C to task 2 and E to task 5.

Use an alternating path from this initial matching to find a complete matching. *(4 marks)*

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2 Five people A, B, C, D and E are to be matched to five tasks R, S, T, U and V.

The table shows the tasks that each person is able to undertake.

Person	Tasks
A	R, V
B	R, T
C	T, V
D	U, V
E	S, U

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task V, B to task R, C to task T, and E to task U.

Demonstrate, by using an alternating path from this initial matching, how each person can be matched to a task. *(4 marks)*

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1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	0	1	0	1	1
D	0	0	0	1	0	0
E	0	1	0	0	0	1
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

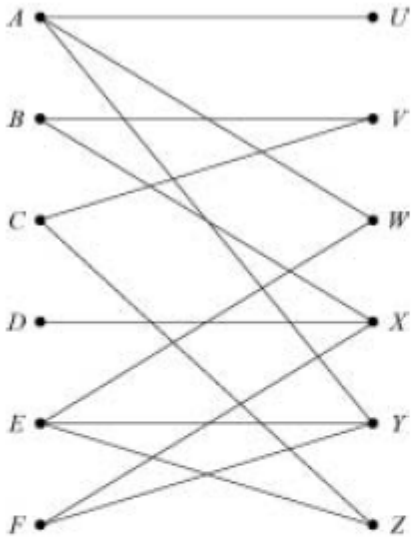
(b) At first F insists on being matched to task 4. Explain why, in this case, a complete matching is impossible. *(1 mark)*

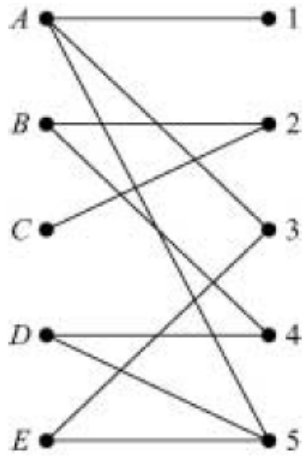
(c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

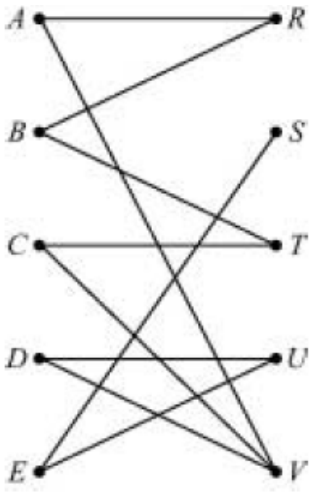
Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

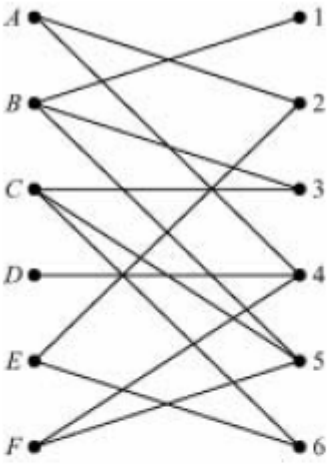
From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*

Decision 1 Matching Problems Answers

1(a)		M1 A1	 2	Must be in part (a)
(b)	$D-X+B-V+C$ $-Z$	M1 A1		Starting from D, F, Z, U
	$F-Y+E-W+A-U$	M1A1		Same
	Match: AU, BV, CZ, DX, EW, FY	B1	5	
	Total		7	

1(a)		M1 A1	 2	
(b)	Initial A3, B4, C2, E5 D – 4 + B – 2 + C No D – 5 + E – 3 + A – 1 Yes Complete A1, B4, C2, D5, E3	B1 M1 A1 B1	 4	Starting from D,1 Either Only solution
	Total		**6**	

2(a)		M1		Bipartite graph
		A1	2	All correct
(b)	Start with D (or S)	B1		
	$D-U+E-S$ **or** $D-V+A-R+B-T+C$ $-V+D-U+E-S$	M1 A1		For attempt at any path
	Match: AV, BR, CT, DU, ES **or** AR, BT, CV, DU, ES	B1	4	Must be 5 pairs
	Total		**6**	

1(a)		M1 A1	2	
(b)	D can only do 4	E1	1	Cannot be matched to task
(c)	$A-2+E-6+C-5$	M1A1		Starting with A, D, 5, 1 First pass
	$D-4+F-5+C-3+B-1$	M1A1 A1		Second pass All Correct **Alt:1** $A-4+F-5$ $D-4+A-2+E-6+C-3+B-1$
	Match $A2, B1, C3, D4, E6, F5$	B1	6	**Alt: 2** $D-4+F-5$ $A-2+E-6+C-3+B-1$
	Total		**9**	