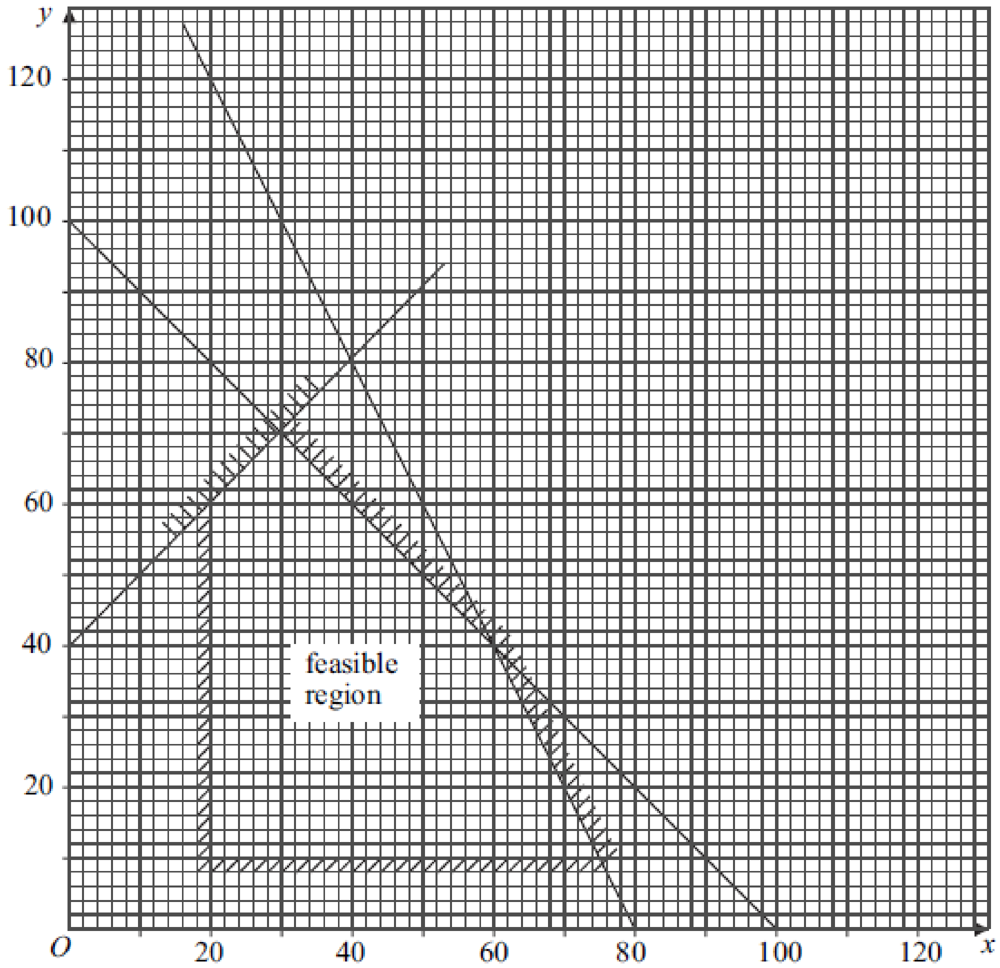
Decision 1 Linear Programming Questions

4 The diagram shows the feasible region of a linear programming problem.



(a) On the feasible region, find:

(i) the maximum value of $2x + 3y$; *(2 marks)*

(ii) the maximum value of $3x + 2y$; *(2 marks)*

(iii) the minimum value of $-2x + y$. *(2 marks)*

(b) Find the 5 inequalities that define the feasible region. *(6 marks)*

6 [**Figure 3**, printed on the insert, is provided for use in this question.]

Ernesto is to plant a garden with two types of tree: palms and conifers.

He is to plant at least 10, but not more than 80 palms.

He is to plant at least 5, but not more than 40 conifers.

He cannot plant more than 100 trees in total.

Each palm needs 20 litres of water each day and each conifer needs 60 litres of water each day. There are 3000 litres of water available each day.

Ernesto makes a profit of £2 on each palm and £1 on each conifer that he plants and he wishes to maximise his profit.

Ernesto plants x palms and y conifers.

(a) Formulate Ernesto's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 3**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(7 marks)*

(c) Find the maximum profit for Ernesto. *(2 marks)*

(d) Ernesto introduces a new pricing structure in which he makes a profit of £1 on each palm and £4 on each conifer.

Find Ernesto's new maximum profit and the number of each type of tree that he should plant to obtain this maximum profit. *(2 marks)*

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

Dino is to have a rectangular swimming pool at his villa.

He wants its width to be at least 2 metres and its length to be at least 5 metres.

He wants its length to be at least twice its width.

He wants its length to be no more than three times its width.

Each metre of the width of the pool costs £1000 and each metre of the length of the pool costs £500.

He has £9000 available.

Let the width of the pool be x metres and the length of the pool be y metres.

(a) Show that one of the constraints leads to the inequality

$$2x + y \leqslant 18$$ *(1 mark)*

(b) Find four further inequalities. *(3 marks)*

(c) On **Figure 1**, draw a suitable diagram to show the feasible region. *(6 marks)*

(d) Use your diagram to find the maximum width of the pool. State the corresponding length of the pool. *(3 marks)*

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

The Jolly Company sells two types of party pack: excellent and luxury.

Each excellent pack has five balloons and each luxury pack has ten balloons.

Each excellent pack has 32 sweets and each luxury pack has 8 sweets.

The company has 1500 balloons and 4000 sweets available.

The company sells at least 50 of each type of pack and at least 140 packs in total.

The company sells x excellent packs and y luxury packs.

(a) Show that the above information can be modelled by the following inequalities.

$x + 2y \leqslant 300$, $\quad 4x + y \leqslant 500$, $\quad x \geqslant 50$, $\quad y \geqslant 50$, $\quad x + y \geqslant 140$ *(4 marks)*

(b) The company sells each excellent pack for 80p and each luxury pack for £1.20. The company needs to find its minimum and maximum total income.

(i) On **Figure 2**, draw a suitable diagram to enable this linear programming problem to be solved graphically, indicating the feasible region and an objective line. *(8 marks)*

(ii) Find the company's maximum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

(iii) Find the company's minimum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

Figure 3 (for use in Question 6)

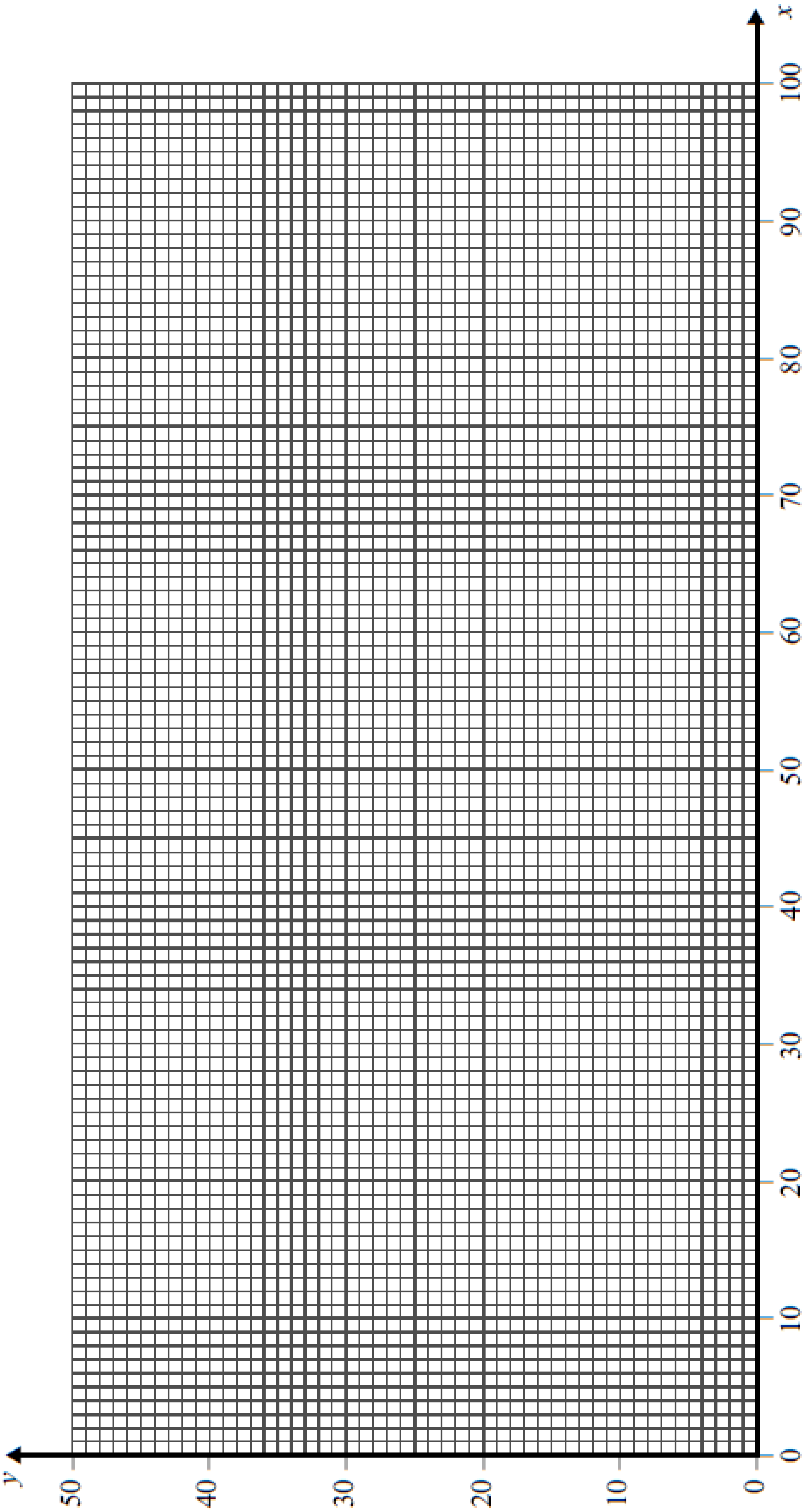


Figure 1 (for use in Question 6)

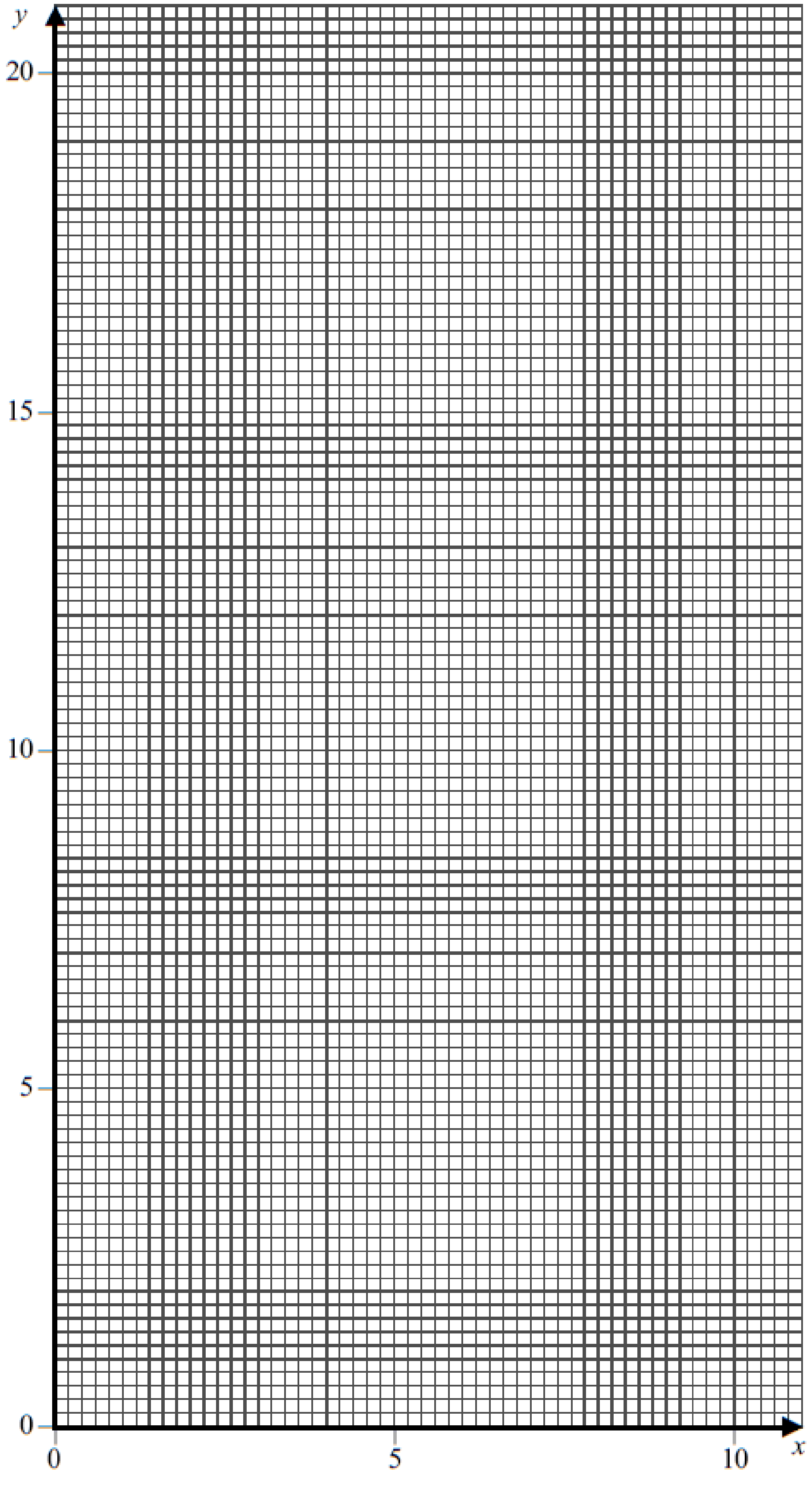
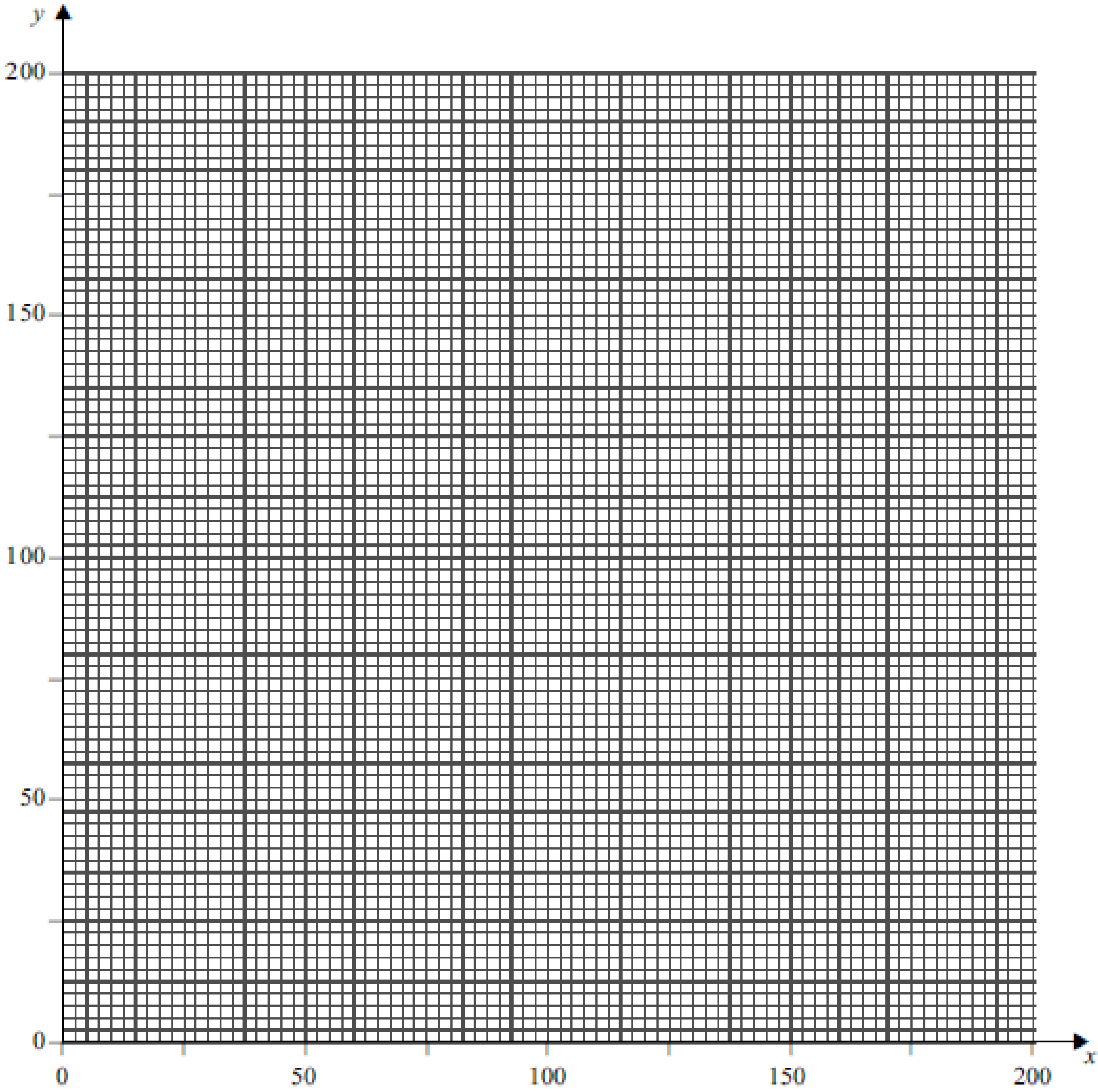
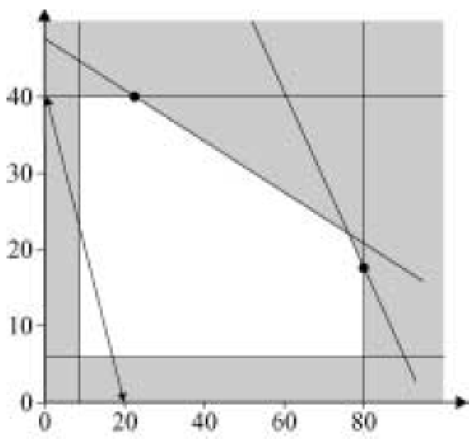


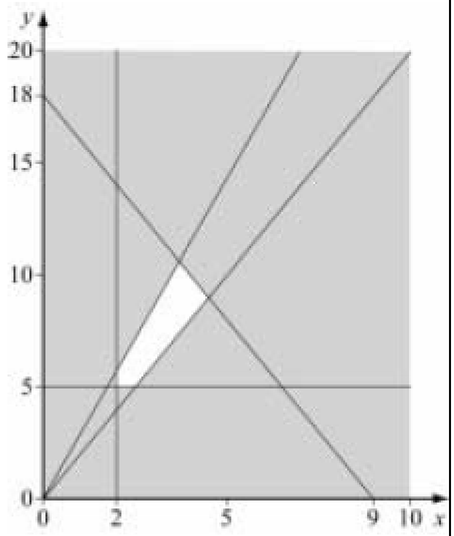
Figure 2 (for use in Question 5)

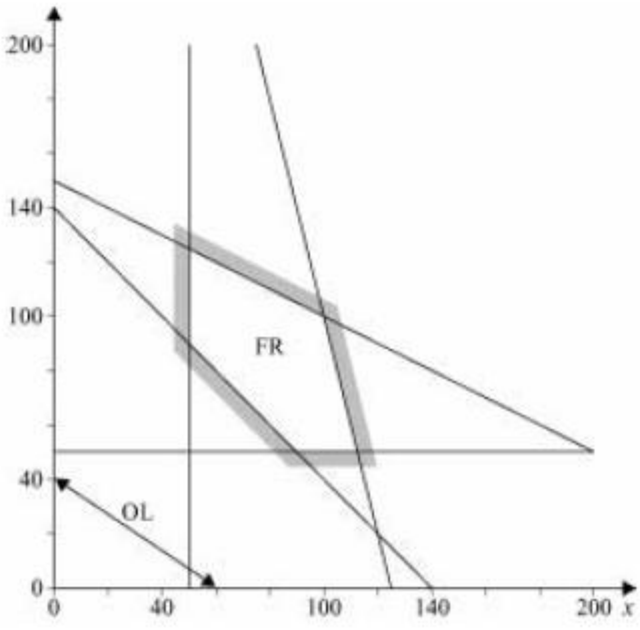


Decision 1 Linear Programming Answers

4(a)(i)	Max $2x+3y$ at (30, 70)	M1		Extreme
	$=270$	A1	2	
(ii)	Max $3x+2y$ at (60, 40)	M1		Extreme
	$=260$	A1	2	
(iii)	Min $-2x+y$ at (75, 10)	M1		$x=75$
	$=-140$	A1	2	
(b)	$x \geq 20, \quad y \geq 10$	B1		OE
	$x+y \leq 100$	B1		OE
	$2x+y \leq 160$ OE	M1		for gradient of - 2
		A1		
	$y \leq x+40$ OE	M1		for positive gradient
		A1	6	
	Total		**12**	

6(a)	$10 \leq x \leq 80$	B1		Strict inequalities -1 (or using p, c)
	$5 \leq y \leq 40$	B1		
	$x+y \leq 100$	B1		
	$20x+60y \leq 3000$ OE	B1		
	(maximise)(P =) $2x+y$	B1	5	May be seen in (b) or (c)
(b)		B1 M1A1 M1A1		For "x lines" and "y lines" For each other line M1 – ve gradient $(0,50)$ M1 – ve gradient $(100,0)$
		B1		Feasible region correct to within 1 square
		B1	7	Objective line
(c)	Max at $(80, 20)$	M1		Considering an extreme point in their region
	P = £180	A1	2	
(d)	$P = x+4y$			
	Max at (30, 40)	M1		Using (30,40) ($\pm$square)
	P = £190	A1	2	
	Total		**16**	

6(a)	$1000x + 500y \leq 9000$ $(2x + y \leq 18)$	B1	1	
(b)	$x \geq 2,\ y \geq 5$ $y \geq 2x$ $y \leq 3x$	B1 B1 B1	3	–1 for strict inequalities –1 for 'w's and 'l's
(c)		B1 B1 M1 A1 A1 B1	6	$x = 2,\ y = 5$ $2x + y = 18$ Line $y = mx$ $y = 2x$ $y = 3x$ Feasible region
(d)	Considering an extreme point on their f.r. $x = 4.5$ $y = 9$	M1 A1 A1	3	Extreme point - vertex
	Total		**13**	

5(a)	$5x+10y \le 1500$ (balloons)			
	$\Rightarrow x+2y \le 300$	E1		
	$32x+8y \le 4000$ (sweets)	E1		
	$\Rightarrow 4x+y \le 500$			
	$x \ge 50, y \ge 50$, at least 50 of each	E1		
	$x+y \ge 140$, at least 140 in total	E1	4	
(b)(i)				
		B1		$x = 50, y = 50$
		B1		$x + y = 140$
		M1		Negative gradient (either)
		A1		$4x + y = 500$
		A1		$x + 2y = 300$
		B1		Feasible region
		M1		Objective line drawn
		A1	8	
(ii)	Maximum$(100,100)$ $= £200$	M1 A1	 2	Considering extreme point on their region
(iii)	Minimum $(90,50)$ $= £132$	M1 A1	 2	Considering extreme minimum point on their region
	Total		**16**	