Solutionbank D1
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Draw a connected graph with

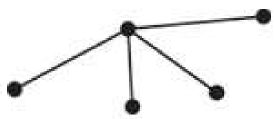
a one vertex of degree 4 and 4 vertices of degree 1,

b three vertices of degree 2, one of degree 3 and one of degree 1,

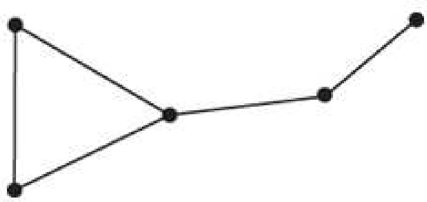
c two vertices of degree 2, two of degree 3 and one of degree 4.

Solution:

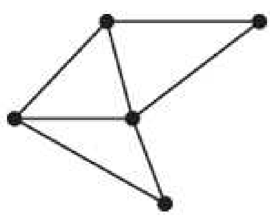
a For example



b



c



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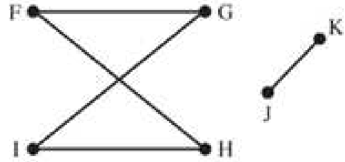
Exercise A, Question 2

Question:

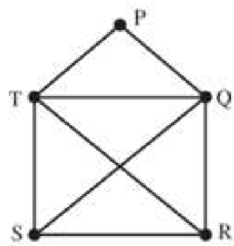
Which of the graphs below are not simple?

a

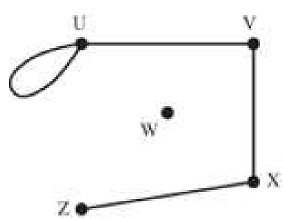
b



c



d



Solution:

a is not simple. There are two edges connecting C with D.
b and **c** are simple.
d is not simple. There is a loop attached to U.

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Exercise A, Question 3

Question:

In question 2, which graphs are not connected?

Solution:

a and **c** are connected.
b is not connected, there is no path from J to G, for example.
d is not connected, there is no path from W to Z, for example.

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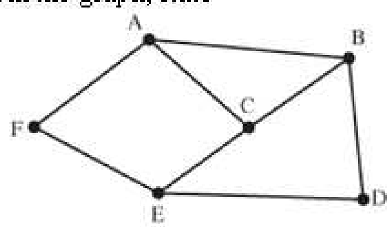
Exercise A, Question 4

Question:

From the graph, state



There are many correct answers to these questions.

a four paths from F to D,
b a cycle passing through F and D,
c the degree of each vertex.

Use the graph to

d draw a subgraph,
e confirm the handshaking lemma (that the sum of the degrees is equal to twice the number of edges).

A lemma is a mathematical fact used as a stepping stone to more important results.

Solution:

a Any four of these

F A B D	F E D
F A C B D	F E C B D
F A B C E D	F E C A B D
F A C E D	

b Here are examples. (These all start at F, but you could start at any point.)
There are others.

F A B D E F	F A B D B A F	F A C E D E C A F
F E D B A F	F E D E F	F A C E D E F
F A C B D E F	F A C B D B C A F	F E D E C A F
F E D B C A F	F E C B D B C E F	F E C B D E F
		F E D B C E F

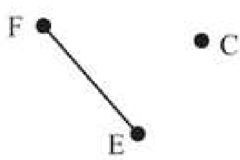
c

Vertex	A	B	C	D	E	F
Degree	3	3	3	2	3	2

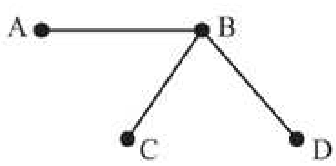
d Here are examples. (There are many others.)

i

ii



iii



e Sum of degrees $= 3+3+3+2+3+2 = 16$
number of edges $= 8$
sum of degrees $= 2 \times$ number of edges for this graph

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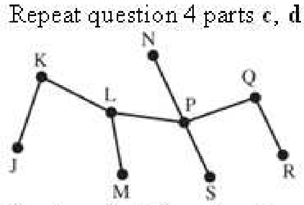
Exercise A, Question 5

Question:

a Repeat question 4 parts **c**, **d** and **e** using this graph.



b Confirm that there is only one path between any two vertices.

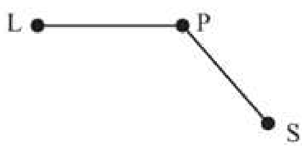
Solution:

a

Vertex	J	K	L	M	N	P	Q	R	S
Degree	1	2	3	1	1	4	2	1	1

Here are some possible subgraphs (there are many others).

i



ii

iii

Sum of degrees $= 1+2+3+1+1+4+2+1+1=16$

number of edges $= 8$

sum of degrees $= 2\times$number of edges for this graph

b This graph is a tree so there will only be one path between any two vertices.

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Exercise A, Question 6

Question:

Show that it is possible to draw a graph with

a an even number of vertices of even degree,

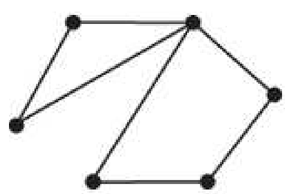
b an odd number of vertices of even degree.

It is not possible to draw a graph with an odd number of vertices of odd degree. Explain why not.

Use the handshaking lemma.

Solution:

a For example



6 vertices all even
5 of degree 2
1 of degree 4

b For example

3 vertices all even, all of order 2

The sum of degrees $= 2 \times$ number of edges, so the sum of degrees must be even. Any vertices of odd degree must therefore 'pair up'. So there must be an even number of vertices of odd degree.

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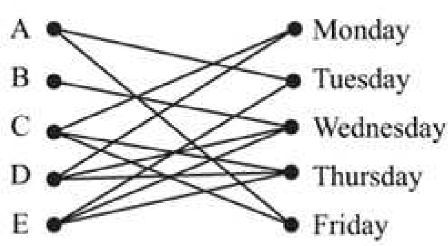
Exercise A, Question 7

Question:

Five volunteers, Ann, Brian, Conor, Dave and Eun Jung are going to run a help desk from Monday to Friday next week.
One person is required each day.
Ann is available on Tuesday and Friday,
Brian is available on Wednesday,
Conor is available on Monday, Thursday and Friday,
Dave is available on Monday, Wednesday and Thursday,
Eun Jung is available on Tuesday, Wednesday and Thursday.
Draw a graph to model this situation.

Look at Example 3.

Solution:



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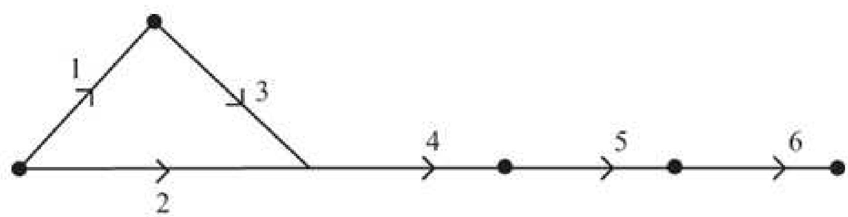
Exercise A, Question 8

Question:

A project consists of six activities 1, 2, 3, 4, 5 and 6.
1 and 2 can start immediately, but 3 cannot start until 1 is completed.
4 cannot start until both 2 and 3 are complete, 5 cannot start until 4 is complete and 6 cannot start until 5 is complete.
Draw a digraph to model this situation.

Look at Example 5.

Solution:



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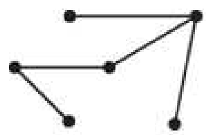
Exercise B, Question 1

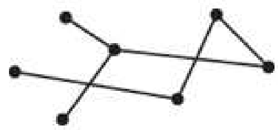
Question:

State which of the following graphs are trees.

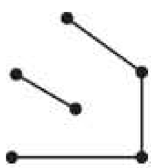
a



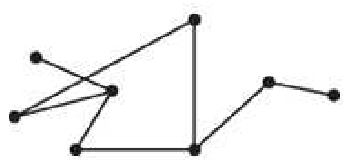
b



c



d



Solution:

a and **b** are trees.
c is not a tree, it is not a connected graph.
d is not a tree, it contains a cycle.

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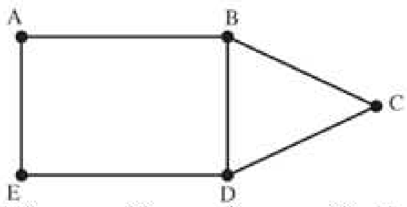
Exercise B, Question 2

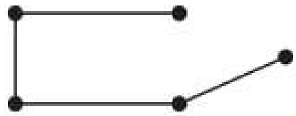
Question:



There are 11 spanning trees for the graph above. See how many you can find.

Solution:

i



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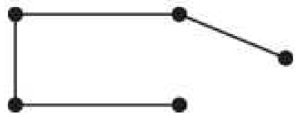

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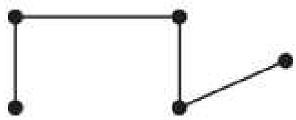
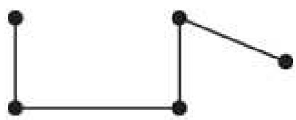


iv

v



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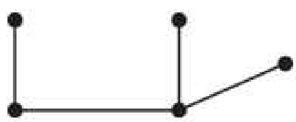
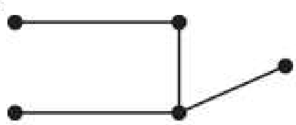
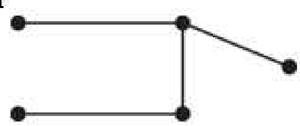


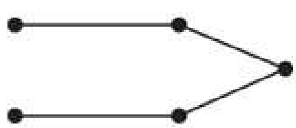
vii



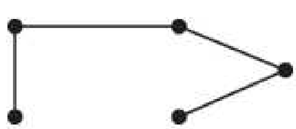
viii



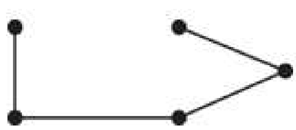
ix



x



xi



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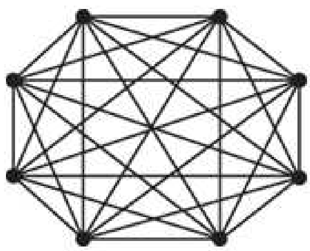
Exercise B, Question 3

Question:

Draw k_8.

What is the degree of each vertex in the graph k_n?

Solution:



k_8

Each vertex will have a degree of $(n-1)$.

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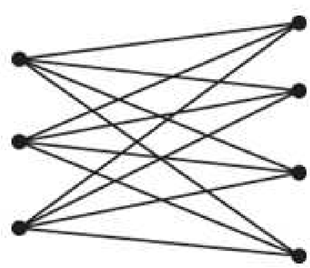
Exercise B, Question 4

Question:

Draw $k_{3,4}$.

How many edges would be in $k_{n,m}$?

Solution:



$k_{3,4}$

There will be nm edges.

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Exercise B, Question 5

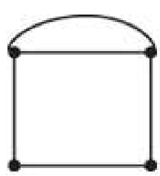
Question:

Which of these graphs are isomorphic?

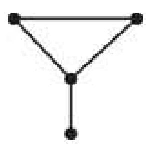
a

b

c

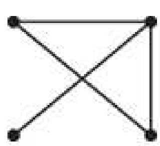


d



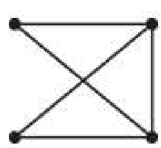
e

f



g

h



i

Solution:

a, **e** and **h** are isomorphic.
b and **i** are isomorphic.
c and **g** are isomorphic.
d and **f** are isomorphic.

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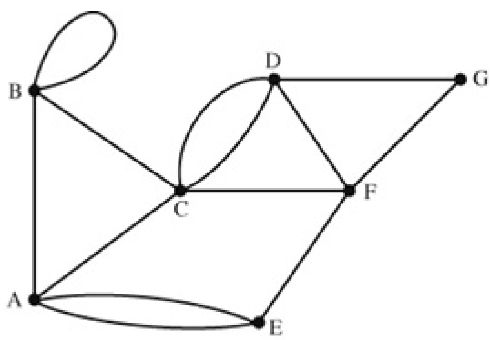
Exercise B, Question 6

Question:



Use an adjacency matrix to represent the graph above.

Solution:

	A	B	C	D	E	F	G
A	0	1	1	0	2	0	0
B	1	2	1	0	0	0	0
C	1	1	0	2	0	1	0
D	0	0	2	0	0	1	1
E	2	0	0	0	0	1	0
F	0	0	1	1	1	0	1
G	0	0	0	1	0	1	0

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Exercise B, Question 7

Question:

Draw a graph corresponding to each adjacency matrix.

a

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	1	1
C	0	1	0	2	0
D	1	1	2	0	1
E	0	1	0	1	0

b

	A	B	C	D
A	0	1	0	1
B	1	0	0	1
C	0	0	2	1
D	0	0	1	1

c

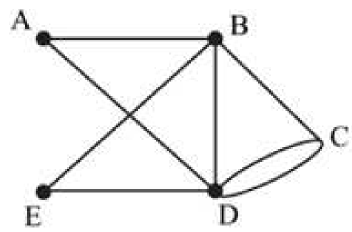
	A	B	C	D
A	0	1	0	1
B	1	0	1	1
C	0	1	0	1
D	1	1	1	0

d

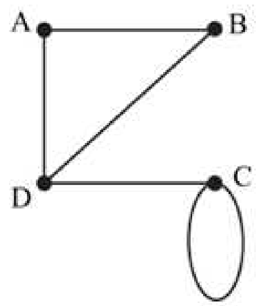
	A	B	C	D
A	0	2	0	1
B	2	0	1	0
C	0	1	0	1
D	1	0	1	0

Solution:

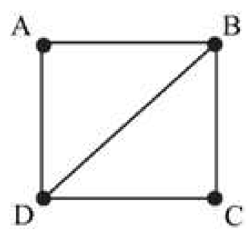
a



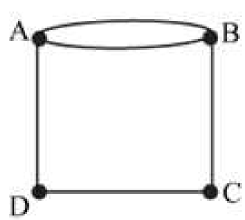
b



c



d



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Exercise B, Question 8

Question:

Which graphs in Question **5** could be described by the adjacency matrices in Question **7c** and **d**?

Solution:

7**c** could describe 5**a**, **e** and **h**.
7**d** could describe 5**c** and **g**.

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Exercise B, Question 9

Question:

Draw the network corresponding to each distance matrix.

a

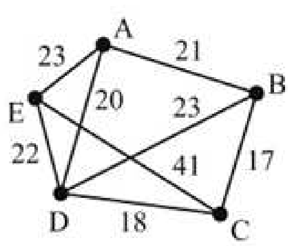
	A	B	C	D	E
A	–	21	–	20	23
B	21	–	17	23	–
C	–	17	–	18	41
D	20	23	18	–	22
E	23	–	41	22	–

b

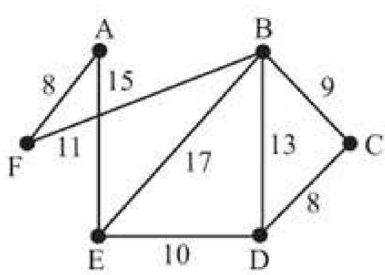
	A	B	C	D	E	F
A	–	–	–	–	15	8
B	–	–	9	13	17	11
C	–	9	–	8	–	–
D	–	13	8	–	10	–
E	15	17	–	10	–	–
F	8	11	–	–	–	–

Solution:

a



b



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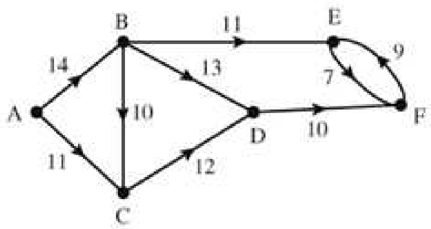
Exercise B, Question 10

Question:



Use a distance matrix to represent the directed network above.

Solution:

	A	B	C	D	E	F
A	–	14	11	–	–	–
B	–	–	10	13	11	–
C	–	–	–	12	–	–
D	–	–	–	–	–	10
E	–	–	–	–	–	7
F	–	–	–	–	9	–

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Exercise C, Question 1

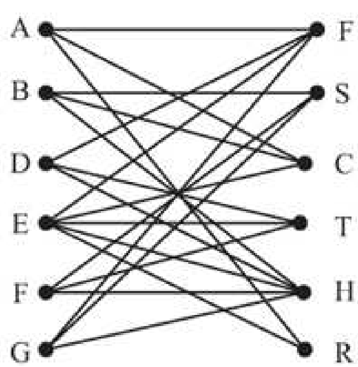
Question:

A group of 6 children, Ahmed, Bronwen, Di, Eddie, Fiona and Gary, were asked which of 7 sports, football, swimming, cricket, tennis, hockey and rugby, they enjoyed playing. The results are shown in the table below.

Ahmed	football	cricket	rugby		
Bronwen	swimming	cricket	hockey		
Di	football	tennis	hockey		
Eddie	football	cricket	tennis	hockey	rugby
Fiona	tennis	swimming	hockey		
Gary	football	swimming	hockey		

Draw a bipartite graph to show this information.

Solution:



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Exercise C, Question 2

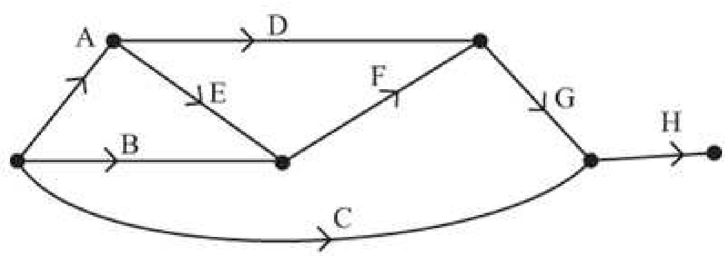
Question:

A project involves eight activities A, B, C, D, E, F, G and H, some of which cannot be started until others are completed. The table shows the tasks that need to be completed before the activity can start. For example, activity F cannot start until both B and E are completed.

Activity	Activity that must be completed
A	–
B	–
C	–
D	A
E	A
F	B E
G	D F
H	C G

Draw a digraph to represent this information.

Solution:



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Exercise C, Question 3

Question:

a Draw a graph with eight vertices, all of degree 1.

b Draw a graph with eight vertices, all of degree 2, so that the graph is

i connected and simple

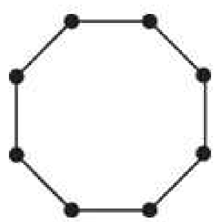
ii not connected and simple

iii not connected and not simple.

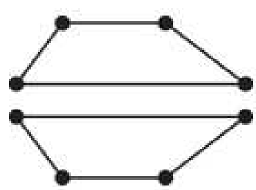
Solution:

a

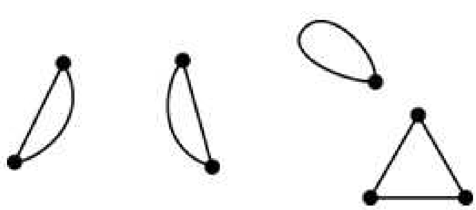
b **i**



ii



iii



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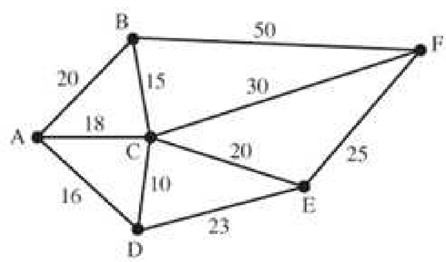
Exercise C, Question 4

Question:

Use a distance matrix to represent the network below.



Solution:

	A	B	C	D	E	F
A	–	20	18	16	–	–
B	20	–	15	–	–	50
C	18	15	–	10	20	30
D	16	–	10	–	23	–
E	–	–	20	23	–	25
F	–	50	30	–	25	–

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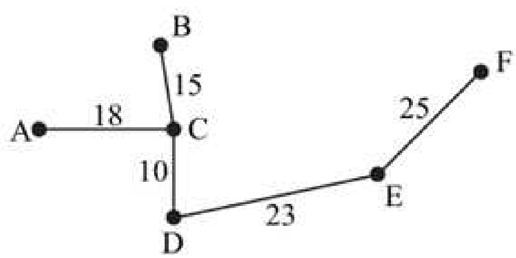
Exercise C, Question 5

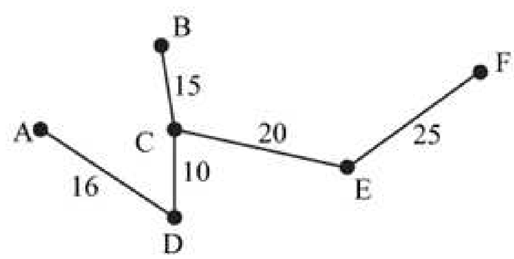
Question:

Find two spanning trees for the graph in Question **4**.

Solution:

Here are examples. (There are many other solutions.)





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Exercise C, Question 6

Question:

Write down a formula connecting the number of edges, E, in a spanning tree with V vertices.

Solution:

$E = V - 1$ (or $V = E + 1$)