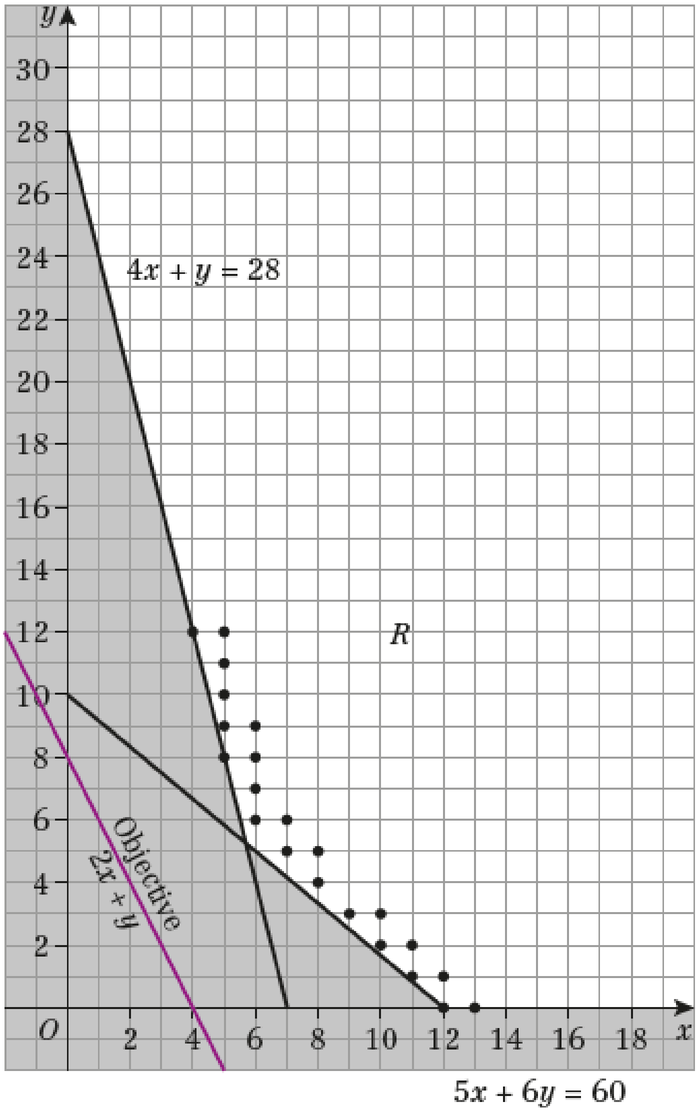
Exercise 7D

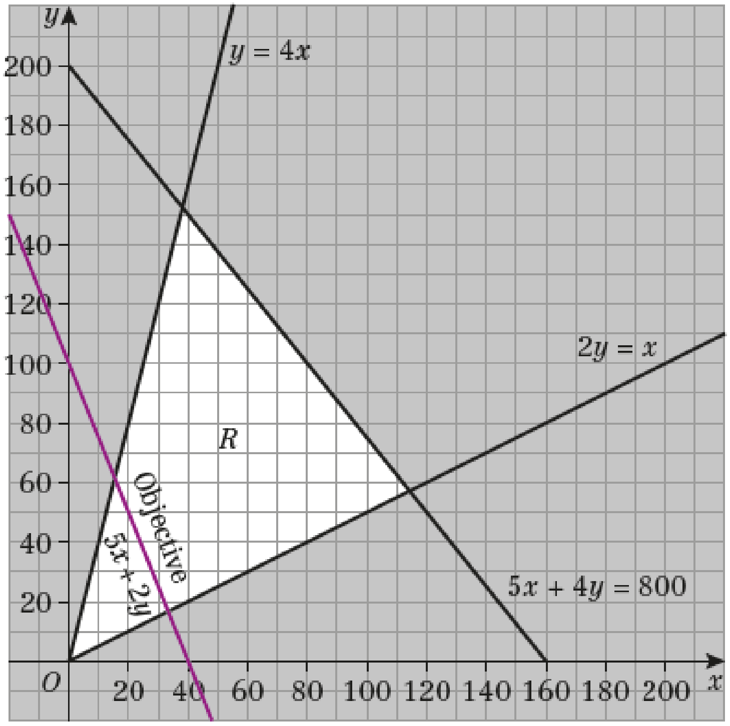
1 **a** Maximum integer value (5,1)
$3x + 2y = 17$

b



Minimum integer value (6,5)
$2x + y = 17$

1 c

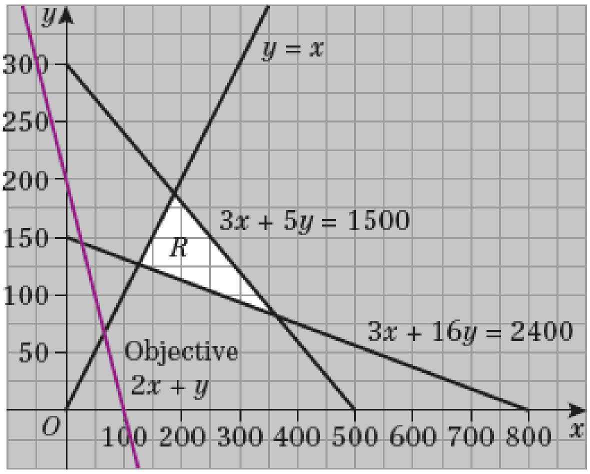


Solving $2y = x$ and $5x + 4y = 800$ simultaneously gives $114\frac{2}{7}, 57\frac{1}{7}$
Test integer values nearby.

Point	$2y \geqslant x$	$5x + 4y \leqslant 800$	$5x + 2y$
(114,57)	✓	✓	684
(114,58)	✓	✗	-
(115,57)	✗	✗	-
(115,58)	✓	✗	-

so optimal point is (114,57) value 684.

1 d



Solving $3x + 16y = 2400$
$3x + 5y = 1500$
simultaneously gives
$\left(363\frac{7}{11}, 81\frac{9}{11}\right)$

Taking integer point

Point	$3x+16y \geqslant 2400$	$3x+5y \leqslant 1500$	$2x+y$
(363,81)	✓	✗	-
(363,82)	✓	✓	808
(364,81)	✗	✓	-
(364,82)	✓	✗	-

So optimal integer point is (363,82) value 808

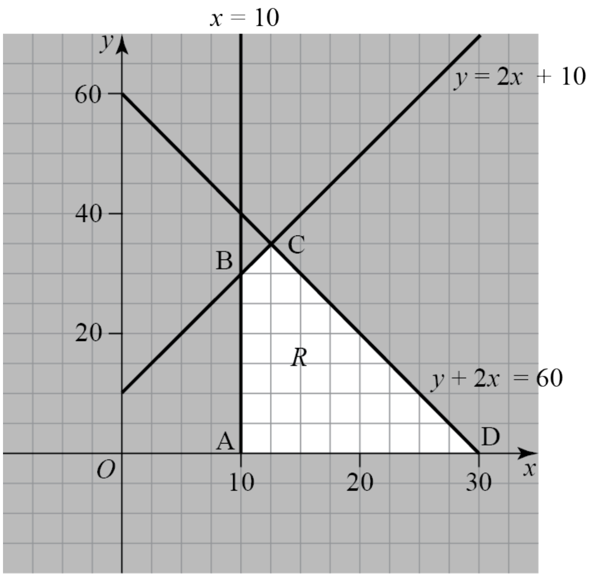
2 Identify vertices as intersections of appropriate lines.

$x = 10, y = 0 \Rightarrow A = (10, 0)$

$y = 2x + 10, x = 10 \Rightarrow B = (10, 30)$

$y = 2x + 10, y + 2x = 60 \Rightarrow C = (12.5, 35)$

$y + 2x = 60, y = 0 \Rightarrow D = (30, 0)$



As C is not an integer point so investigate integer points close to it.

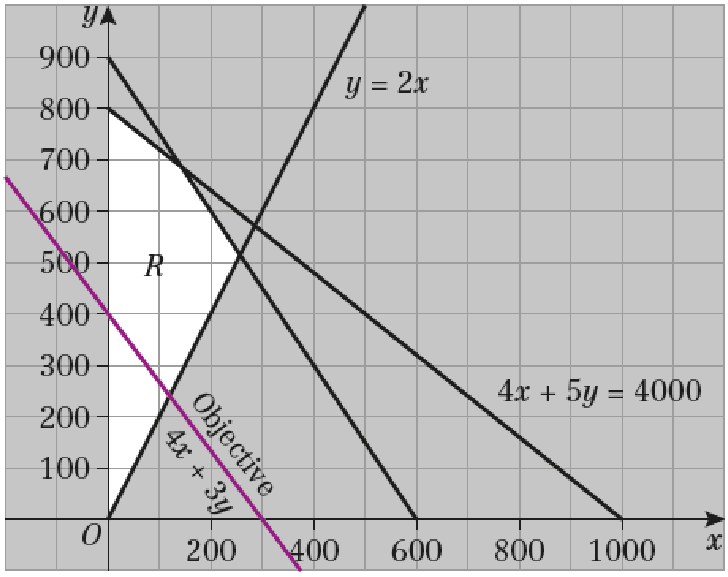
Point	$y \leqslant 2x+10$?	$y+2x \leqslant 60$?	In R?
(12,35)	$35 > 34$		No
(13,35)	$35 \leqslant 36$	$61 > 60$	No
(12,34)	$34 \leqslant 34$	$58 \leqslant 60$	Yes
(13,34)	$34 \leqslant 36$	$60 \leqslant 60$	Yes

Check values of the objective function at points A, B, D and $C' = (12,34)$, $C'' = (13,34)$

Point	Coordinates	Value of $P = 5x + 3y$
A	$x = 10, y = 0$	50
B	$x = 10, y = 30$	140
C'	$x = 12, y = 34$	162
C''	$x = 13, y = 34$	167
D	$x = 30, y = 0$	150

Maximal value of P we have found is 167 attained at point (13,34)

3 This is the problem formulated in Exercise 6A question **1**.



Intersection of $4x + 5y = 4000$ and $3x + 2y = 1800$

giving $\left(142\frac{6}{7}, 685\frac{5}{7}\right)$

Testing nearby integer points

Point	$4x+5y \leqslant 4000$	$3x+2y \leqslant 1800$	$80x+60y$
(142,685)	✓	✓	52460
(142,686)	✓	✓	52520
(143,685)	✓	✓	52540
(143,686)	X	X	

so maximum integer solution is 52 540 pennies at (143,685)

4 This is the problem formulated in Exercise 7A question **2**.

a Let x and y be the number of displays of type A and B, respectively.

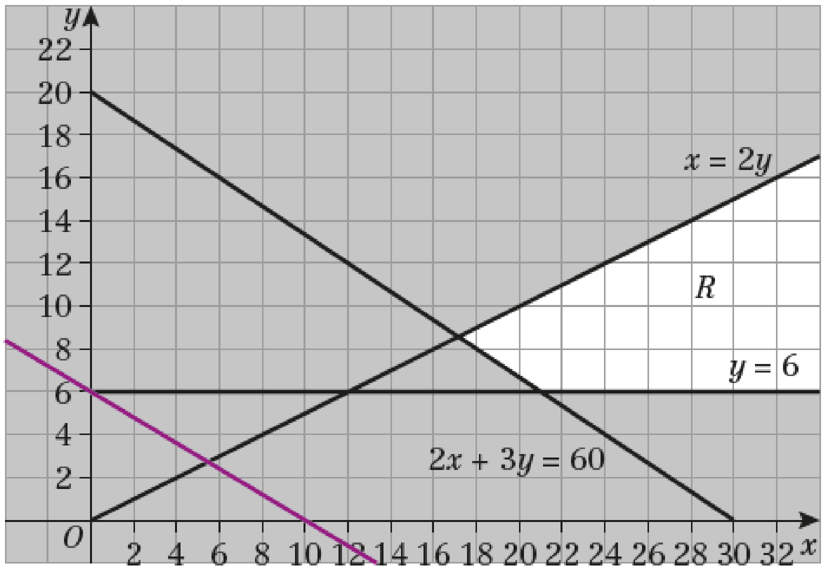
length of display at least 30m $\Rightarrow x + 1.5y \geqslant 30 \Rightarrow 2x + 3y \geqslant 60$

at least twice as many A as B $\Rightarrow 2y \leqslant x$

at least 6 of type B $\Rightarrow y \geqslant 6$

non-negativity $\Rightarrow x, y \geqslant 0$

4 b



c We aim to minimise cost $C = 6x + 10y$, in pounds.

By objective line method we see that the optimal point lies at the intersection of $y = 6$ and $2x + 3y = 60$, i.e. $x = 21$, $y = 6$. Optimal value is $6\times21 + 10\times6 = 186$

Since it is an integer point, it also solves the original problem. The cost is minimized when the client buys 21 displays of type A and 6 displays of type B. Optimal cost is £186.

5 This is the problem formulated in Exercise 7A question **3**.

a Let x and y be the number of games of Cludopoly and Trivscrab, respectively.

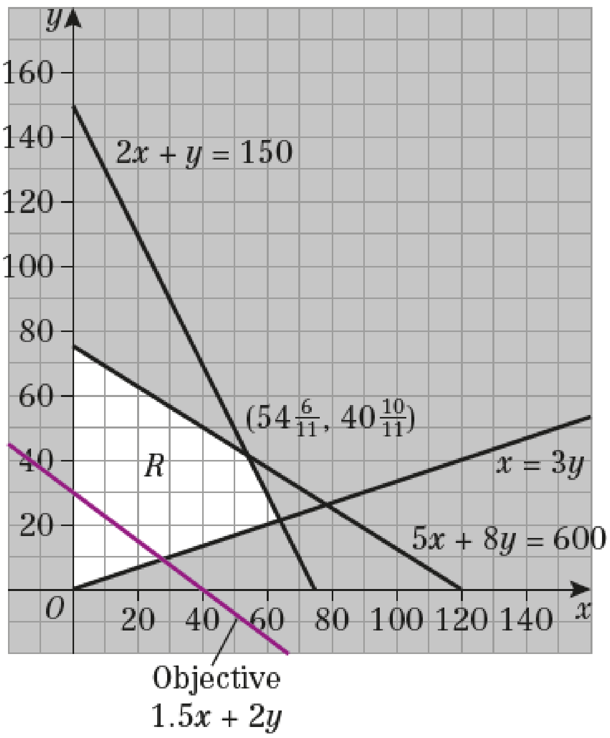
First pieces machine operates max 10h $\Rightarrow \frac{5}{60}x + \frac{8}{60}y \leqslant 10 \Rightarrow 5x + 8y \leqslant 600$

Second pieces machine operates max 10h $\Rightarrow \frac{8}{60}x + \frac{4}{60}y \leqslant 10 \Rightarrow 2x + y \leqslant 150$

At most 3 times as many Cludopoly as Trivago $\Rightarrow 3y \geqslant x$

Non-negativity $\Rightarrow x, y \geqslant 0$

5 b



c We wish to maximise profit $P = 1.5x + 2y$.

By objective line method we find the optimal non-integer point as an intersection

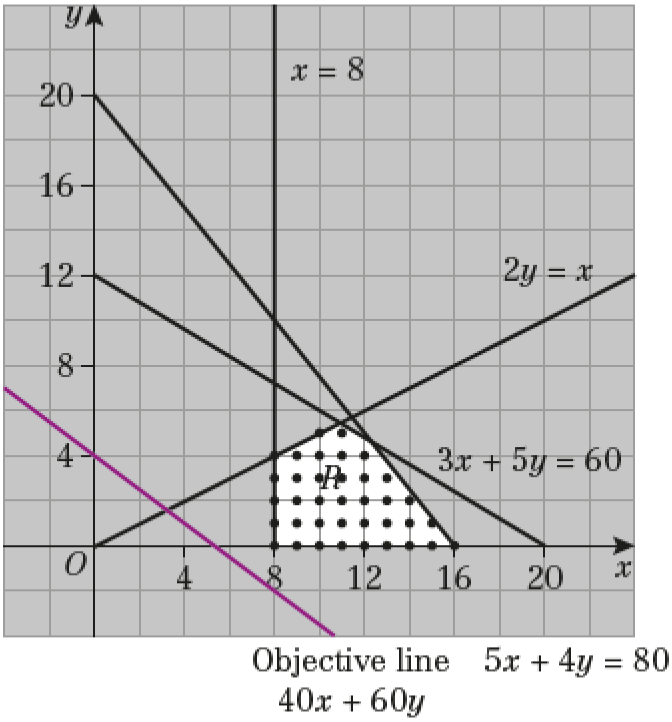
$5x+8y=600, 2x+y=150 \Rightarrow x=54\frac{6}{11}, y=40\frac{10}{11}$

Investigate neighbouring integer points

Points	$5x+8y \leqslant 600$?	$2x+y \leqslant 150$?	$1.5x+2y$
(54,50)	$590 \leqslant 600$	$148 \leqslant 150$	161
(54,41)	$598 \leqslant 600$	$149 \leqslant 150$	163
(55,40)	$599 \leqslant 600$	$150 \leqslant 150$	162.5
(55,41)	603 > 600	–	–

Maximal value of profit we have found is \$163 achieved by producing 54 games of Cludopoly and 41 games of Trivscrab.

6 Represent all inequalities on a diagram and find feasible region R.



Integer points are fairly sparse, so we can use Method 1 from Example 11 (p. 153), i.e. identify the optimal point directly by the ruler method.
Optimal integer solution is (11,5) and at this point $S = 740$.
Thus, the maximal amount of shelving is 740m and to achieve that the librarian should buy 11 bookcases of type 1 and 5 bookcases of type 2.

Using Method 1 from Example 13 shows you that the optimal integer solution is (11,5) giving 740 m of shelving.
Using Method 2 gives you the following solution:
Intersection of $3x + 5y = 60$ and $5x + 4y = 80$ giving $\left(12\frac{4}{13}, 4\frac{8}{13}\right)$

Points	$3x+5y \leqslant 60$	$5x+4y \leqslant 80$	$40x + 60y$
(12,4)	✓	✓	720
(12,5)	X	✓	
(13,4)	✓	X	
(13,5)	X	X	

Maximum value is 720 at (12,4).
In this instance, the solution produced by Method 2 is actually incorrect, but it requires a very particular set of circumstances to create this discrepancy. It is generally safe to assume that a solution found using Method 2 will be correct, but do check your graph to see whether there could be an alternative optimal integer solution.