Exercise 7C

1 **a** Need intersection of $4x + y = 1400$ and $3x + 2y = 1200$
$(320,120)$ $M = 760$

Objective line passes through (200,0) and (0,400)

b $(0,400)$ $N = 1600$

Objective line passes through (400,0) and (100,0)

c Need intersection of $x + 3y = 1200$ and $3x + 2y = 1200$
$\left(171\frac{3}{7}, 342\frac{6}{7}\right)$ $P = 514\frac{2}{7}$

Objective line passes through (200,0) and (0,200)

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d $(350,0)$ $Q = 2100$

Objective line passes through (100,0) and (0,600)

2 **a** $(0,90)$ $E = 90$

b Need intersection of $6y = x$ and $3x + 7y = 420$
$(100.8,16.8)$ $F = 168$

c Need intersection of $9x + 10y = 900$
$3x + 7y = 420$
$\left(63\frac{7}{11}, 32\frac{8}{11}\right)$ $G = 321\frac{9}{11}$

Objective line passes through (80,0) and (0,60)

d Same intersection as in **b** $(100.8,16.8)$ $H = 201.6$

Objective line passes through (120,0) and (0,20)

3 **a** Need intersection of $3x + y = 60$ and $5y = 3x$
$\left(16\frac{2}{3}, 10\right)$ $J = 56\frac{2}{3}$

b Need intersection of $y = 4x$ and $9x + 5y = 450$
$\left(15\frac{15}{29}, 62\frac{2}{29}\right)$ $K = 77\frac{17}{29}$

c Need intersection of $3x + y = 60$ and $y = 4x$
$\left(8\frac{4}{7}, 34\frac{2}{7}\right)$ $L = 85\frac{5}{7}$

Objective line passes through (10,0) and (0,60)

d Need intersection of $9x + 5y = 450$ and $5y = 3x$
$(37.5,22.5)$ $M = 97.5$

Objective line passes through (40,0) and (0,80)

4 **a** *C*

b *A*

c *B*

d *D*

e *C*

f *A*

g *B*

h *D*

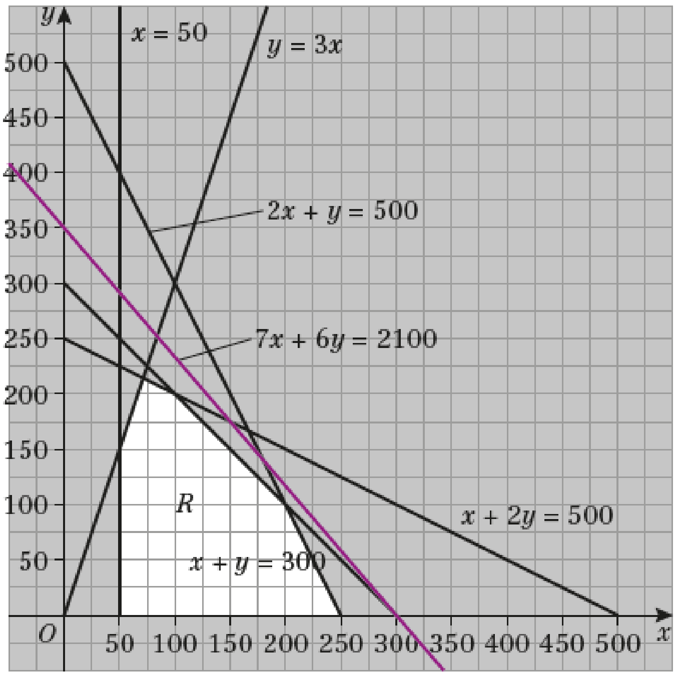
i *C*

j *D*

5 Let x be the mass of indoor feed and y be the mass of outdoor feed, in kilograms.
Recall that we want to maximise $P = 7x + 6y$, subject to

$$x + 2y \leqslant 500,$$
$$2x + y \leqslant 500,$$
$$x + y \leqslant 300,$$
$$y \leqslant 3x,$$
$$y \geqslant 0 \quad x \geqslant 50,$$

Draw the diagram including all constraints and mark the feasible region as R.
Objective line passes through (0,350) and (300,0).
Maximum point is (200,100). $P_{max} = 2000$

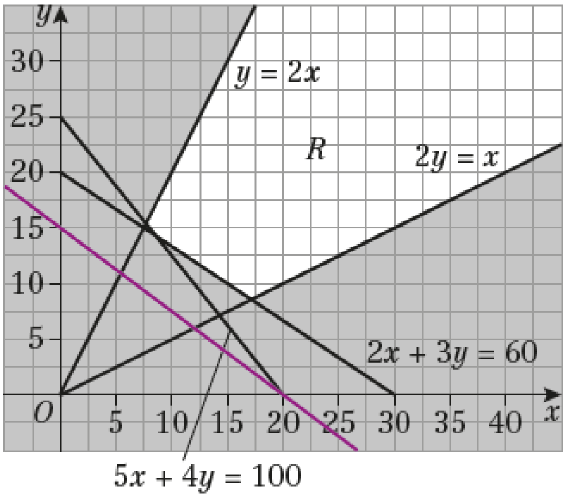


Using the ruler method, we identify that optimal point is (200,100) At this point $P = 2000$.
Hence, we conclude that in order to maximise its profit, the company should produce 200kg of indoor feed and 100kg of outdoor feed. The profit will be £2000.

6 Decision variables: x = hours of work for factory R, y = hours of work for factory S
Recall that we wish to minimise $C = 300x + 400y$ subject to:

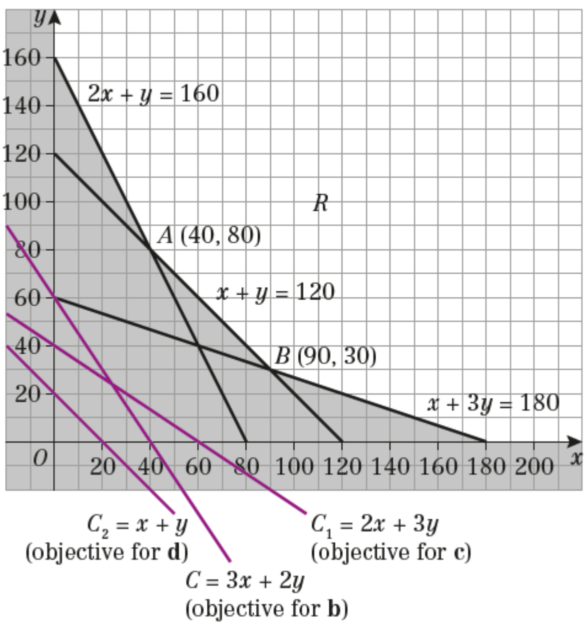
$$5x+4y\geqslant 100$$
$$2x+3y\geqslant 60$$
$$2x\geqslant y$$
$$2y\geqslant x$$
$$x,y\geqslant 0$$

Draw the diagram including all constraints and mark the feasible region as R.



We can use the ruler method; in the picture we have drawn an objective line passing through points (0,15) and (20,0).
This way, we identify the optimal point as the intersection of lines $5x + 4y = 100$ and $2x + 3y = 60$
By solving simultaneous equations, we find the optimal point $\left(8\frac{4}{7}, 14\frac{2}{7}\right)$ and optimal value $C = 8285\frac{5}{7}$
We conclude that in order to minimise operating cost, factory R should work for $8\frac{4}{7}$ h and factory S for $14\frac{2}{7}$

7 a



7 b

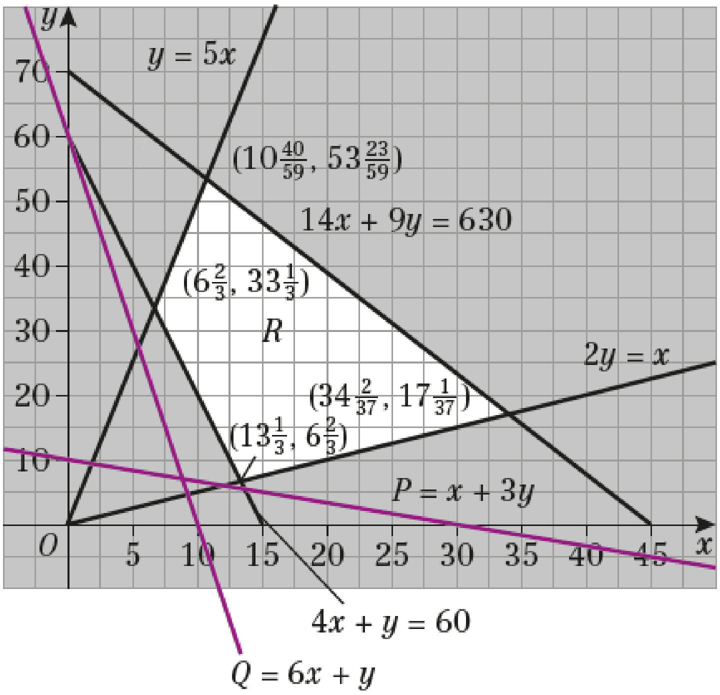
Vertices	$C = 3x + 2y$
(0,160)	320
(40,80)	280
(90,30)	330
(180,0)	540

so minimum is (40,80) value of C =280

c (90,30) $C_1 = 270$

d C_2 is parallel to $x + y = 120$ so all points from A to B are optimal points.

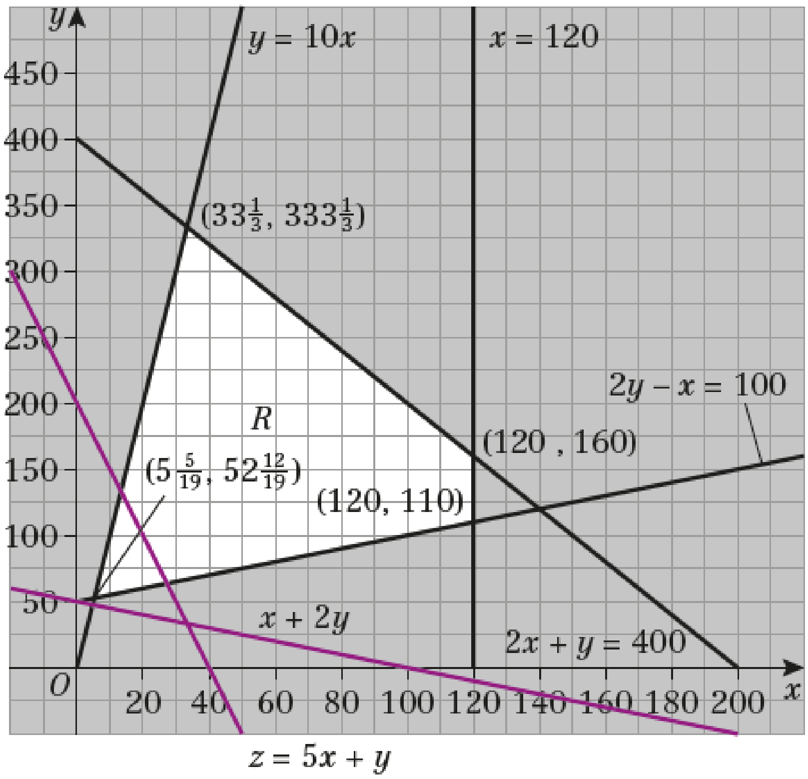
8 a



b i $\left(13\frac{1}{2}, 6\frac{2}{3}\right)$ $P = 33\frac{1}{3}$

ii $\left(34\frac{2}{37}, 17\frac{1}{37}\right)$ $Q = 221\frac{13}{37}$

9 a



b i We can identify 4 vertices of the feasible region as intersections of respective lines

$y = 10x, 2y - x = 100 \Rightarrow A = \left(5\tfrac{5}{19}, 52\tfrac{12}{19}\right)$

$y = 10x, 2x + y = 400 \Rightarrow B = \left(33\tfrac{1}{3}, 333\tfrac{1}{3}\right)$

$x = 120, 2x + y = 400 \Rightarrow C = (120,160)$

$x = 120, 2y - x = 100 \Rightarrow D = (120,110)$

Now we apply the vertex testing method.

Vertex	Coordinates	Value of $z = 5x + y$
A	$x = 5\frac{5}{19}, y = 52\frac{12}{19}$	$78\frac{18}{19}$
B	$x = 33\frac{1}{3}, y = 333\frac{1}{3}$	500
C	$x = 120, y = 160$	760
D	$x = 120, y = 110$	710

Maximal value of z in the feasible region is 760

ii Minimal value of z in the feasible region is $78\frac{18}{19}$

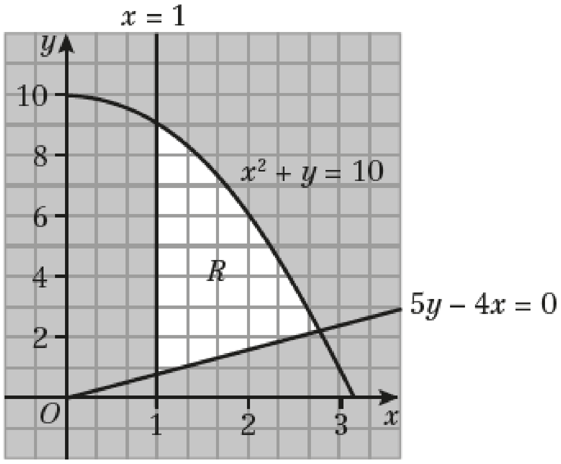
c We apply the vertex testing method to the points identified in part **b**.

Vertex	Coordinates	Value of $x + 2y$
A	$x = 5\frac{5}{19}, y = 52\frac{12}{19}$	$110\frac{10}{19}$
B	$x = 33\frac{1}{3}, y = 333\frac{1}{3}$	700
C	$x = 120, y = 160$	440
D	$x = 120, y = 110$	340

Maximal value of $x + 2y$ is 700

Challenge

a Sketch the feasible region by noting the boundary line corresponding to $x^2 + y \leqslant 10$ is a parabola.



b Objective function is $3x + y$, so gradient of an objective line is –3.
When we apply the ruler method we observe that at the optimal point parabola is tangent to the objective line.

For $x^2 + y = 10$ we have $\frac{dy}{dx} = -2x$

$-2x = -3 \Rightarrow x = 1.5, y = 10 - (1.5)^2 = 7.75.$

Maximal value of $P = 3\times(1.5) + 7.75 = 12.25$.