

## Exercise 5D

1 a  $D_7B_{12}C_8E_{14}A_{19}D = 60$

b  $E_8C_{11}A_{13}B_7D_{14}E = 53$

or

$E_8C_{11}D_7B_{13}A_{14}E = 53$

c The better upper bound is 53 since this is lower.

2 a  $Z_{10}X_{15}Y_{40}V_{55}W_{30}S_{70}Z = 220$

b  $X_{10}Z_{15}V_{40}Y_{45}W_{30}S_{55}X = 195$

$V_{15}Z_{10}X_{15}Y_{45}W_{30}S_{75}V = 190$

c The better upper bound is 190 because it is lower.

3 a  $R_{150}S_{210}T_{120}U_{180}V_{300}W_{240}R = 1200$  minutes

b  $U_{120}S_{150}R_{120}V_{150}T_{180}W_{270}U = 990$

and

$U_{120}T_{150}V_{120}R_{150}S_{240}W_{270}U = 1050$

c  $V_{120}R_{150}S_{120}U_{120}T_{180}W_{300}V = 990$

and

$V_{120}R_{150}U_{120}S_{210}T_{180}W_{300}V = 1080$

and

$V_{120}R_{150}U_{120}T_{180}W_{240}S_{210}V = 1020$

d The better upper bound is 990 because it is lower.

4 a The nearest neighbour route from  $B$  is either  $BEGADAFCB$  of total weight  $648 + x$  or  $BEGDFACB$  of total weight 779. It depends on the value of  $x$ , so that the second one is a nearest neighbour route only if  $x \geq 131$

Similarly, the nearest neighbour route from  $A$  is either  $ADEGCBFA$  of total weight  $639 + x$  or  $AEGDBCFA$  of total weight 836. The second one is a nearest neighbour route only if  $x \geq 197$

By considering all combinations, one of the following is true

$$1419 = 648 + x + 639 + x = 1287 + 2x \quad (1)$$

$$1419 = 779 + 639 + x = 1418 + x \quad (2)$$

$$1419 = 648 + x + 836 = 1484 + x \quad (3)$$

$$1419 = 779 + 836 = 1615 \quad (4)$$

(2) implies  $x = 1$ , but we need to remember that (2) is applicable only for  $x \geq 131$

(3) implies  $x = -65$  which is not physical

(4) is clearly false

Hence (1) must be true and so

$$2x = 1419 - 1287 \Rightarrow 2x = 132 \Rightarrow x = 66.$$

b 705 miles is an upper bound. In the previous part we have found upper bound  $648 + x = 714$  and  $639 + x = 705$ . The second one is better.