

Exercise 5A

1 a

1	A	B	C	D	E
A	-	7	10	9	5
B	7	-	3	11	12
C	10	3	-	8	12
D	9	11	8	-	4
E	5	12	12	4	-

AC – the shortest route is ABC length 10

AD – the shortest route is AED length 9

BD – the shortest route is BCD length 11

BE – the shortest route is BAE length 12

b

	A	B	C	D	E
A	-	5	12	7	9
B	5	-	7	2	4
C	12	7	-	5	7

ACD – the shortest route is $ABDC$ length 12

AE – the shortest route is $ABDE$ length 9

BC – the shortest route is BDC length 7

BE – the shortest route is BDE length 4

CE – the shortest route is CDE length 7

c

	A	B	C	D	E	F
A	-	10	18	13	15	18
B	10	-	8	3	5	8
C	18	8	-	5	3	10
D	13	3	5	-	2	5

ACE – the shortest route is $ABDEC$ length 18

AF – the shortest route is $ABDF$ length 18

BCF – the shortest route is $BDEC$ length 8

BE – the shortest route is BDE length 5

BF – the shortest route is BDF length 8

CD – the shortest route is CED length 5

CF – the shortest route is $CEDF$ length 10

EF – the shortest route is EDF length 7

1 d

	A	B	C	D	E	F	G
A	-	10	9	10	17	20	20
B	10	-	3	20	11	10	20
C	9	3	-	19	8	13	23
D	10	20	19	-	27	20	10
E	17	11	8	27	-	8	18
F	20	10	13	20	8	-	10
G	20	20	23	10	18	10	-

AF – shortest route is ABF length 20

AG – the shortest route is ADG length 20

BE – the shortest route is BCE length 11

CF – the shortest route is CBF length 13

CG – the shortest route is $CBFG$ length 23

DE – the shortest route is $DACE$ length 27

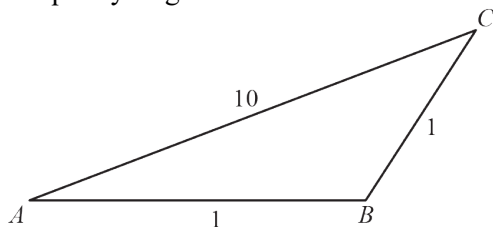
2 a In the classical problem, each vertex must be visited exactly once. In the practical problem, each vertex must be visited at least once.

b i It is helpful to find the shortest path from A to D , as this is the most distant node from A . If possible, we should also avoid edges of high weight (AE and CD). Keeping that in mind, by inspection we find that the solution is $ABECEDEBA$ (or $ABEDECEBA$). Weight of the tour is
 $2 + 1 + 3 + 3 + 4 + 4 + 1 + 2 = 20$

ii $ABCDEA$ and $AEDCBA$ are in fact the only tours starting at A which visit every vertex exactly once. Hence, either of them provides solution to the classical problem. Total weight is $2 + 3 + 9 + 4 + 8 = 26$.

3 a The solution must begin and end with edge PQ . By inspection we find $PQRSTUQP$ (or $PQUSTRQP$) as a solution. Observe that $QRSTUQ$ contains 5 edges of least weight in the graph. As the solution requires at least 7 edges (2 of them being PQ), we conclude it is optimal. Total weight is
 $7 + 3 + 5 + 6 + 8 + 3 + 7 = 39$

- 3 b Regardless of the starting vertex, we need to include edge PQ twice. The remaining problem is to find the solution to the practical problem for subgraph consisting of vertices Q, R, S, T, U and edges between them. It must contain at least 5 edges. As discussed in part a, optimal solution is provided by 5 edges of least weight. We can form a cycle starting at any vertex out of them, e.g. $QRTSUQ$ starting in Q or $RTSUQR$ starting in R . By adding PQ in appropriate position we can construct the solution or total weight 39 starting at any point, e.g. $QPQRTSUQ$ starting in Q or $RTSUQPQR$ starting in R .
- c As PQ is the only edge connecting P to the rest of the graph, we must visit Q at least twice.
- 4 The simplest example is a graph with 3 vertices where the weights of edges do not satisfy triangle inequality. E.g.



Challenge

In a complete graph any two vertices are connected. Hence, after fixing the start vertex, you can visit all other vertices in arbitrary order. The tour of that type which has minimum total weight provides the solution to the classic problem.

For example consider K_4 with vertices A, B, C, D and take A as a start vertex. Since there is an edge between any two nodes, $ABCDA, ABDCA, ACBDA, ACDBA, ADBCA$ and $ADCBA$ are all possible tours. The one of minimum weight provides a solution to classical problems. Similarly, for any other vertex.