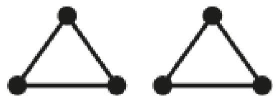
Chapter Review 4

1 a The graph is Eulerian as all vertices are even.

b The graph is neither as there are more than 2 odd nodes.

2 Any not connected graph with 6 even nodes, e.g.



If the graph is connected it will be Eulerian

3 a $3^{2x} - 700 + 3^{x+1} - 60 + 20 - x + x = 2 \times 35$

$\Rightarrow 3^{2x} + 3^{x+1} - 740 = 70$

$\Rightarrow \left(3^x\right)^3 + 3 \times 3^x - 810 = 0$

$\Rightarrow (3^x - 27)(3^x + 30) = 0$

$\Rightarrow 3^x = 27$

$\Rightarrow x = 3$

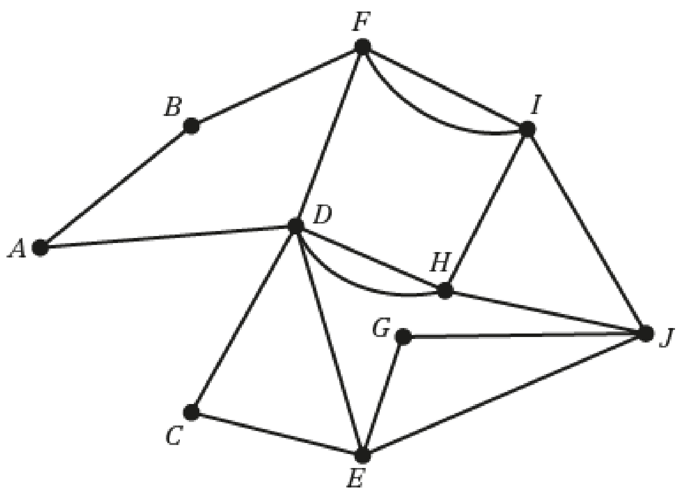
b The orders of the vertices are 29, 21, 17 and 3
The graph is neither Eulerian not semi-Eulerian since it has more than 2 odd vertices.

4 a

vertex	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
degree	2	2	2	5	4	3	2	3	3	4

b *DF* + *HI* = 19 + 36 =55
DH + *FI* = 22 + 27 = 49 ← least weight
DI + *HF* = 46 + 41 = 87
Repeat *DH* and *FI*
Add these to the network to get

Shortest routes
D to *I* is *DFI*
H to *F* is *HDF*



A possible route is *ABFIJGEJHDFIHDCEDA*

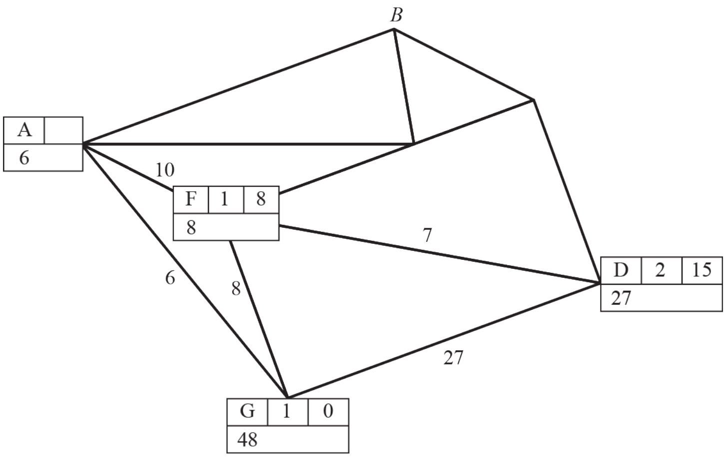
c length = 725 + 49 = 774

5 a The odd vertices are Q, R, T and V
$QR + TV = 104 + 189 = 293$
$QT + RV = 153 + 115 = 268$ ← Shortest route Q to T is QST
$QV + RT = 163 + 123 = 286$
The postman can repeat QT via S and RV so QS, ST and RV are repeated.

b The total length of the route is 1890 + 268 = 2176 m

c Only QV now needs to be repeated.
Total length = 1890 − 123 + 163 = 1930 m
The route is now 246 m shorter.

6 a Minimum weight of $A = 6$
Minimum weight of $F = 8$
Minimum weight of $E = 13$
So shortest route is $GFD = 15$



b The odd vertices are G, B, C and D
$GB + CD = 16 + 3 = 19$ ← least weight
$GC + BD = 18 + 10 = 28$
$GD + BC = 15 + 7 = 22$
GA, AB and CD should be traversed twice.
Total length = 118 + 19 = 137 m

c GB and CD will not need to be repeated as they are now even
BD with weight 10 will be repeated.

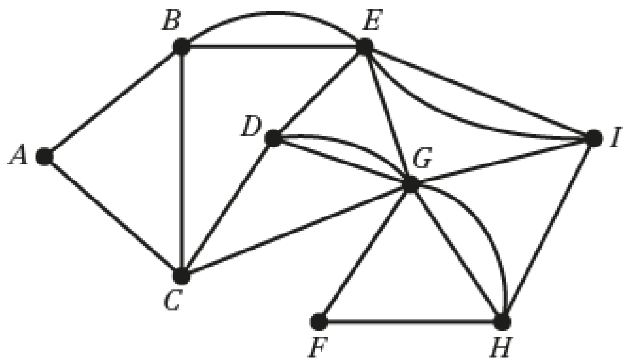
So $x + 10 = 10 \Rightarrow x = 9$

7 a

vertex	A	B	C	D	E	F	G	H	I
degree	2	3	4	3	4	2	6	3	3

Odd valencies at B, D, H and I

7 b Considering all possible complete pairings and their weight
$BD + HI = 7.2 + 3.4 = 10.6$
$BH + DI = 7.6 + 4 = 11.6$
$BI + DH = 5.6 + 4.3 = 9.9$ ← least weight
Repeat *BE*, *EI* and *DG*, *GH*.
Adding these arcs to the network gives



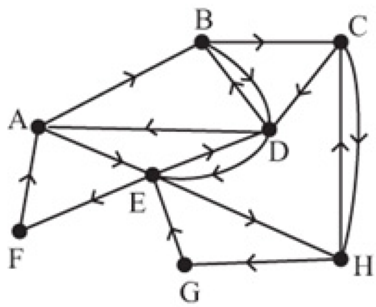
A possible route is: *ABEIHGIEBCDGDEGHFGCA*

c length = 51.4 + 9.9 = 61.3 km

d If *BD* is included *B* and *D* now have even valency.
Only *H* and *I* have odd valency.
So the shortest path from *H* to *I* needs to be repeated.
Length of new route = 51.4 + *BD* + path from *H* to *I*
= 51.4 + 6.4 + 3.4
= 61.2 km
This is (slightly) shorter than the previous route so choose to grit *BD* since it saves 0.1 km.

8 a Odd valencies *B*, *C*, *E*, *H*
Considering all possible complete pairings and their weight
$BC + EH = 68 + 150 = 218$
$BE + CH = 95 + 73 = 168$ ← least weight
$BH + CE = 141 + 85 = 226$
Repeat *BD*, *DE* and *CH*
Adding these arcs to the network gives

Shortest routes
BE is *BDE*
BH is *BCH*, *CE* is *CDE*



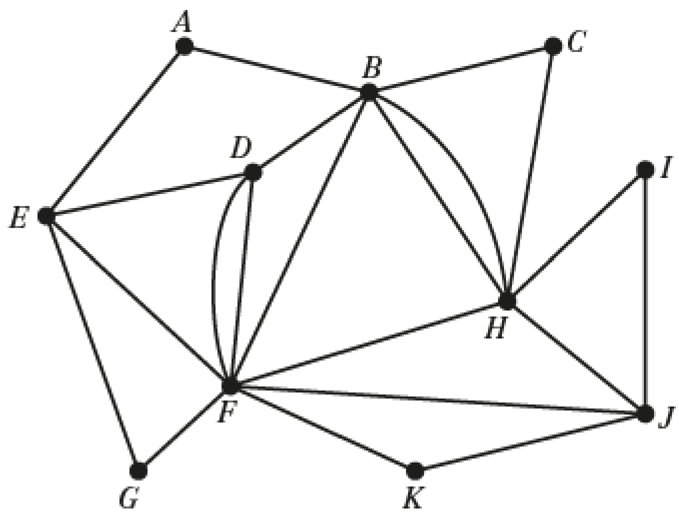
A possible route is: *ABDBCHCDEDAEHGEFA*
Length = 1011 + 168
= 1179 km

8 b This would make *B* the start and *C* the finish.
We would have to repeat the shortest path between *E* and *H* only.
New route = 1011 + 150 = 1161 m
1161 < 1179
So this would decrease the total distance by 18 m.

9 **a** The route inspection algorithm.

b Odd vertices B, D, F, H
Considering all complete pairings
$BD + FH = 14 + 5 = 29$
$BF + DH = 10 + 26 = 36$
$BH + DF = 12 + 16 = 28 \leftarrow$ least weight
Repeat BH and DF
Adding these arcs to the network gives

The shortest route DH is DBH.



A possible route is: *ABHCBHIJHFJKFBDEFGEA*

c length of route = 249 + 28 = 277

d **i** We will still have to repeat the shortest path between a pair of the odd nodes.
We will choose the pair that requires the shortest path.
The shortest path of the six is BF (10)
We will use D and H as the start and finish nodes.

ii 249 + 10 = 259

e Each edge, having two ends, contributes two to the sum of valencies for the network.
Therefore the sum = 2 × number of edges.
The sum is even so any odd valencies must occur in pairs.

Challenge

a Odd nodes are A, B, D, E, F and G
Starting at B so can leave as odd
Case (i): Land at D
$AE + FG = 19 + 10 = 29$
$AF + EG = 7 + 22 = 29$
$AG + EF = 6 + 12 = 18 \leftarrow$ least weight
Case (ii) Land at F
$AD + EG = 26 + 22 = 48$
$AE + DG = 19 + 20 = 39$
$AG + ED = 6 + 14 = 20$
Better to use landing strip at D

b 168 + 18 = 186 miles

Challenge

c Odd nodes unchanged.
Case (i): Land at D
$AE + FG = 40 + 13 = 53$
$AF + EG = 7 + 34 = 41$
$AG + EF = 6 + 47 = 53$
Case (ii) Land at F
$AD + EG = 26 + 34 = 60$
$AE + DG = 40 + 20 = 60$
$AG + ED = 6 + 14 = 20$ ← least weight
Now better to land at F
$168 - (10 + 25 + 12) + 20 = 141$ miles