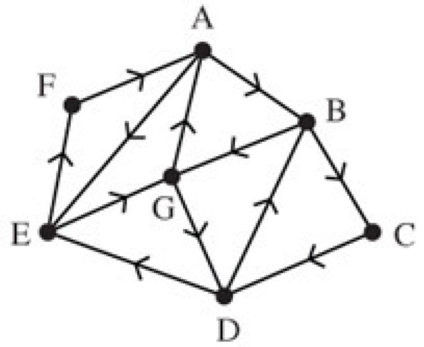
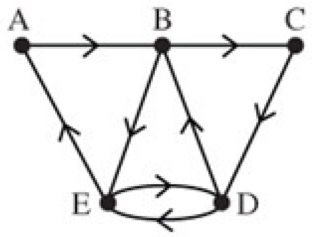
Exercise 4B

1 **a** All valencies are even, so the network is traversable and can return to its start.



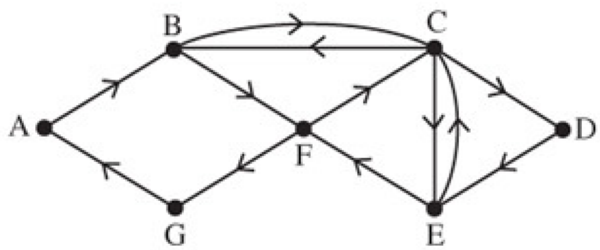
A possible route is: *ABCDBGDEGAEFA*
length of route = weight of network
= 285

b The valencies of *D* and *E* are odd, the rest are even.
We must repeat the shortest path between *D* and *E*, which is the direct path *DE*.
We add this extra arc to the diagram.



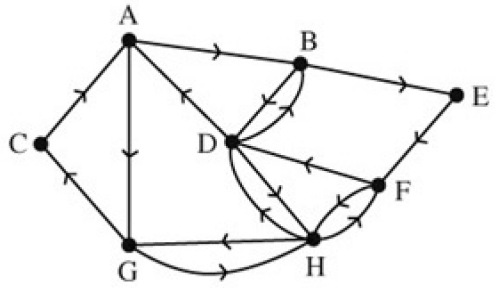
A possible route is: *ABCDEDBEA*
length of route = weight of network + arc *DE*
= 61 + 11
= 72

c The degrees of *B* and *E* are odd, the rest are even. We must repeat the shortest path from *B* to *E*.
By inspection this is *BCE*, length 260. We add these extra arcs to the diagram.



A possible route is: *ABCDECBFCEFGA*
length of route = weight of network + *BCE*
= 1055 + 260
= 1315

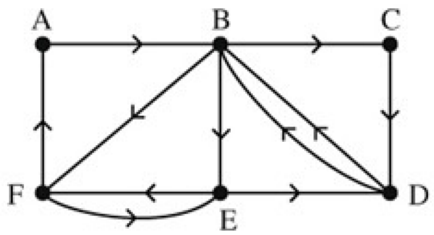
1 d The order of B and G are odd, the rest are even. We must repeat the shortest path from B to G.
By inspection this is $BDHG$, length 183. We add these arcs to the diagram.



A possible route is: *ABEFDBDAGHFHDHGCA*

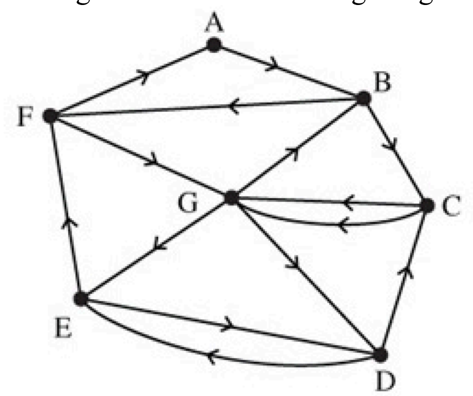
length of route = weight of network + $BDHG$
= 995 + 183
= 1178

2 a Odd valencies at B, D, E and F.
Considering all possible pairings and their weights.
$BD + EF = 130 + 85 = 215$ ← least weight
$BE + DF = 110 + 178 = 288$
$BF + DE = 125 + 93 = 218$
We need to repeat arcs BD and EF.
The length of the shortest route = weight of network + 215
= 908 + 215
= 1123
Adding BD and EF to the diagram gives.



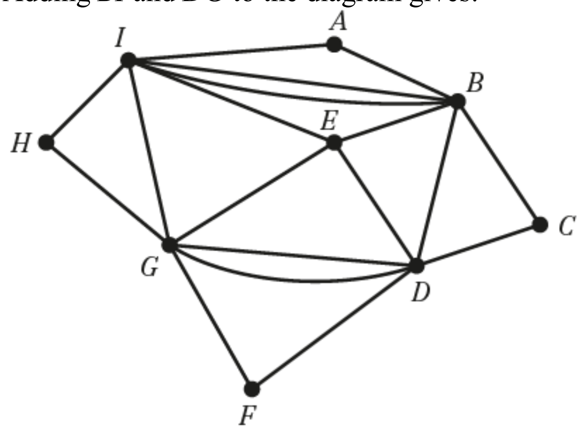
A possible route is: *ABCDBEDBFEFA*

2 b Odd valencies at C, D, E and G
Considering all possible pairings and their weights
$CD + EG = 130 + 75 = 205$
$CE + DG = 157 + 92 = 249$
$CG = DE = 82 + 120 = 202$ ← least weight
We need to repeat arcs CG and DE
The length of the shortest route = weight of network + 202
= 938 + 202
= 1140
Adding CG and DE to the diagram gives



A possible route is: *ABCGDCGEDEFGBFA*

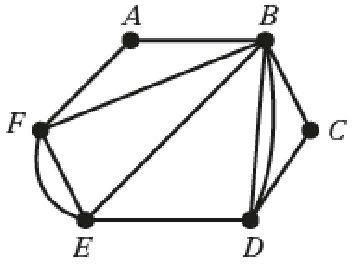
3 a The odd nodes are B, D, G and I.
The minimum path lengths for each pairing are:
$BD + GI = 22 + 30 = 52$
$BG + DI = 37 + 42 = 79$
$BI + DG = 32 + 18 = 50$
The arcs to be traversed twice are BI and DG.
Adding BI and DG to the diagram gives:



A possible route is: *ABCDBIEBIGDFGEDGHIA*.

b $BI + DG = 50$. So the minimum time = 326 + 50 = 376 ms

4 a Odd degrees are B, D, E and F.
Considering all possible pairings and their weight
$BD + EF = 250 + 200 = 450$ ← least weight
$BE + DF = 350 + 380 = 730$
$BF + DE = 300 + 180 = 480$
We need to repeat arcs BD and EF.
Adding these to the diagram gives



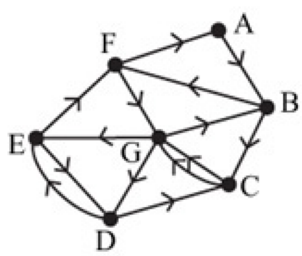
A possible route is: *ABCDBDEFBEFA*
Length = 1910 + 450 = 2360 m

b We will still have two odd valencies.
We need to select the pair that gives the least path.
From part **a** our six choices are
BD (250), EF (200), BE (350), DF (380), BF (300) and DE (180).
The shortest is DE (180) so we choose to repeat this.
It is the other two vertices (B and F) that will be our start and finish.
For example, start at B, finish at F.
Length = 1910 + 180 = 2090 m

5 a Each arc must be traversed twice, whereas in the standard problem each arc need only be visited once.
This has the same effect as doubling up all the edges.
The length of the route = 2×weight of network
= 2×89 = 178 km

b Odd nodes C, D, E, G.
Consider all possible pairings.
$CD + EG = 13 + 5 = 18$
$CE + DG = 15 + 3 = 18$
$CG + DE = 10 + 7 = 17$ ← least weight
We need to repeat arcs CG and DE.
Adding these to the network.

Shortest routes
CD is CGD
CE is CGE

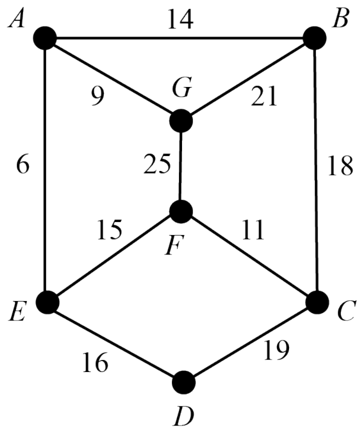


A possible route is: *ABCGDCGEDEFGBFA*
Length = 89 + 17 = 106 km

c If EG is omitted E and G become even and the only odd valencies are at C and D.
We must repeat the shortest path between C and D, which is CGD.
We no longer need to travel along EG so we can subtract 5 from the weight of the network.
The new length = (89 – 5) + 13 = 97 km.

Challenge

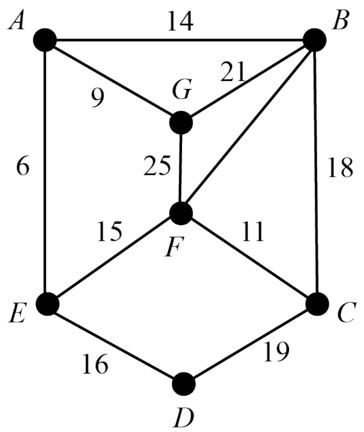
a



The total weight of the network is 154.
The odd nodes are *A*, *B*, *C*, *E*, *F*, *G*.
Starting at A and finish at C, so nodes left to consider, *B, E, F, G*. These can be paired in the following three ways:
BE, FG, weight 20 + 25 = 45
BF, *EG*, weight 29 + 15 = 44
EF ,*BG,* weight 21 + 15 = 36. This is the smallest option, so repeat these edges.
Giving a route of weight 154 + 36 = 190.

b Many possible solutions are available, such as
ABGFEDCFEAGBC.

c



New path is *ABGFBCFEAGEDC*.
Let the weight of *BF* be x
Odd nodes are *A, C, E, G.*
Starting and finishing at *A* and *C*, so edge *EG* needs to be repeated.
Minimum time to traverse all paths: $154 + 15 + x = 169 + x$.
The minimum time to traverse all paths, has been reduced by $2x$, so is now $190 - 2x$
So $190 - 2x = 169 + x$
$\Rightarrow x = 7$