Exercise 4A

1 a

vertex	*A*	*B*	*C*	*D*	*E*	*F*
valency	2	3	2	3	3	3

There are 4 nodes with odd valency so the graph is *neither* Eulerian nor Semi-Eulerian.

b

vertex	*G*	*H*	*I*	*J*	*K*
valency	3	4	3	2	4

There are precisely 2 nodes of odd degree (*G* and *I*) so the graph is *semi-Eulerian.*
A possible route starting at *G* and finishing at *I* is: *GHKIJKGHI*

c

vertex	*L*	*M*	*N*	*P*	*Q*	*R*
valency	2	4	2	4	2	4

All vertices have even valency, so the graph is *Eulerian.*
A possible route starting and finishing at *L* is: *LMNPMRPQRL*

2 a i

vertex	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
valency	4	2	4	2	2	4	2	2

ii

vertex	*A*	*B*	*C*	*D*	*E*	*F*	*G*
valency	4	4	2	4	2	4	4

b i A possible route is: *ABCAFCEGHFDA*

ii A possible route is: *ACFABEGBDGFDA*

3 a i

vertex	*R*	*S*	*T*	*U*	*V*	*W*
valency	2	2	3	3	2	2

Precisely 2 vertices of odd valency (*T* and *U*) so semi-Eulerian.

ii

vertex	*H*	*I*	*J*	*K*	*L*	*M*	*N*
valency	2	4	3	2	3	4	4

Precisely 2 nodes of odd degree (*J* and *L*) so semi-Eulerian.

b i A possible route starting of *T* and finishing at *U* is: *TRSUWVTU*

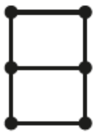
ii A possible route starts at *J* and finishes at *L*: *JKLMJIMNIHNL*

4 a The number of odd nodes of any graph must be even so this is not possible as there are 3 odd nodes.

4 b i $2x + 1 + 2x + 4x - 1 + 4x + 6x = 2E = 18$
$\Rightarrow 18x = 18$
$\Rightarrow x = 1$

ii Semi-Eulerian since there are two odd nodes.

c Numerous possible answers e.g.:

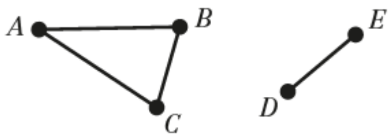


or

5 a Not connected. There are no connections from *A*, *B* or *C* to *D* or *E*.

b Neither. To be Eulerian or semi-Eulerian the graph must be connected.

c



6 Adding up the numbers in each row, the orders of *A*, *B*, *C*, *D*, *E* are 2, 2, 2, 4, 4 respectively. Since they are all even the graph must be Eulerian.

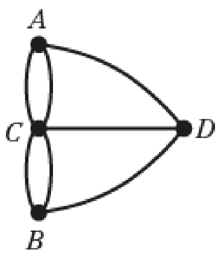
7 a *n* must be odd so that each vertex will have degree $n - 1$ which is even.

b 

8 The example given in the question **1a** is a counterexample. *ABEFCDA* is a Hamiltonian cycle, but the graph is not Eulerian.

Challenge

a

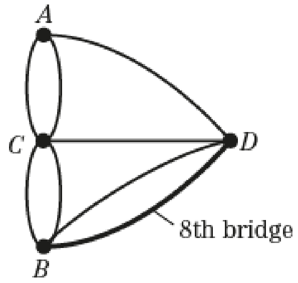


vertex	*A*	*B*	*C*	*D*
valency	3	3	5	3

There are more than two odd nodes, so the graph is *not* traversable.

Challenge

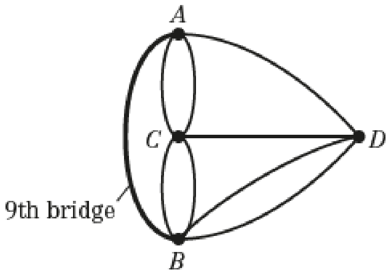
b



We will start at *A* and finish at *C* so these still need to have odd valency. We can only have two odd valencies so *B* and *D* must have even valencies (see table).
We need to change the valency of *B* and of *D*. So we build a bridge from *B* to *D*.

vertex	*A*	*B*	*C*	*D*
valency with 7 bridges	odd	*odd*	odd	*odd*
valency wanted	odd	*even*	odd	*even*

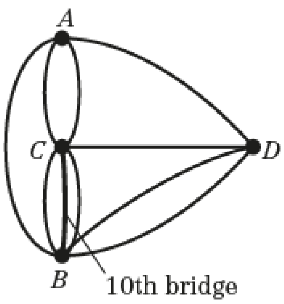
c



We will start at *B* and finish at *C* so these vertices need to be the two vertices with odd valency. We need *A* and *D* to have even valency (see table). We need to change the valency of node *A* and of node *B*. So we build a bridge from *A* to *B*.

vertex	*A*	*B*	*C*	*D*
valency with 8 bridges	*odd*	*even*	odd	even
valency wanted	*even*	*odd*	odd	even

d



All vertices now need to have even valency. This means we need to change the valencies of nodes *B* and *C*. So the 10th bridge needs to be built from *B* to *C*.

vertex	*A*	*B*	*C*	*D*
valency with 9 bridges	even	*odd*	*odd*	even
valency wanted	even	*even*	*even*	even