## Exercise 3A

1 a *EF* (11) add to tree
  BD (12) add to tree
  CD (12) add to tree
  AH (14) add to tree
  DF (15) add to tree
  AC (16) add to tree
  BC (17) reject
  GH (18) add to tree
  BE (18)
  CH (20)
  CG (21) — reject all remaining arcs.
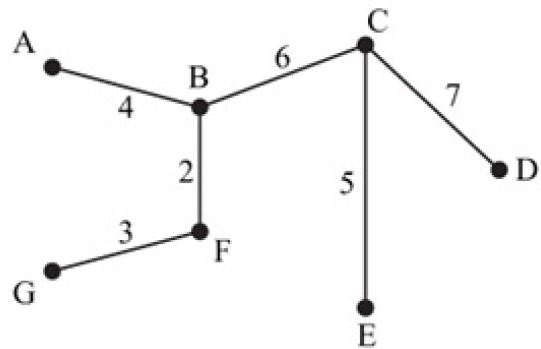 *FG* (24)
  AB (25)

b *BF* (2) add to tree
  FG (3) add to tree
  AB (4) add to tree
  BG (4) reject
  AG (5) reject
  CE (5) add to tree
  BC (6) add to tree
  EG (6) reject
  AC (7) reject
  CD (7) add to tree
  EF (7) — reject all remaining arcs.
  DE (8)

c *DE* (1.4) add to tree
  EF (1.7) add to tree
  CD (2.1) add to tree
  DF (2.1) reject
  CE (2.2) reject
  EG (2.3) add to tree
  GJ (3.1) add to tree
  FG (3.2) reject
  AB (3.8) add to tree
  GH (3.8) add to tree
  GH (4.1) reject
  HJ (4.1) reject
  BH (4.2) add to tree
  AH (4.3) reject
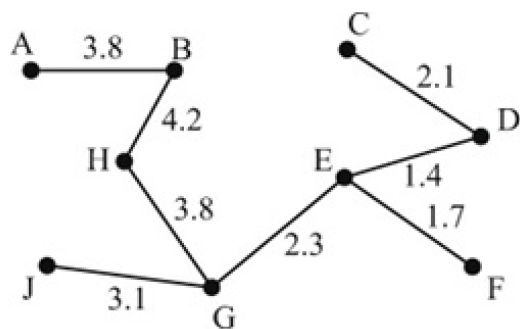
2 a i A tree is a connected graph with no cycles.

2 a ii A minimum spanning tree is a tree of minimum total weight that connects all of the nodes.
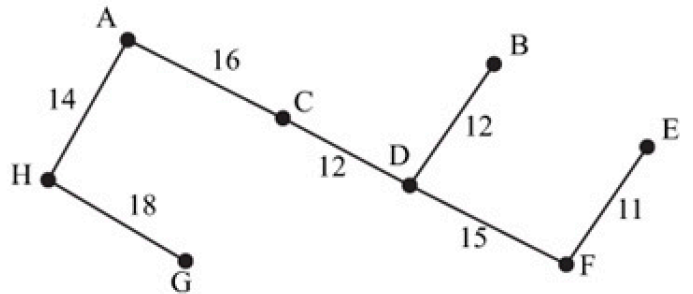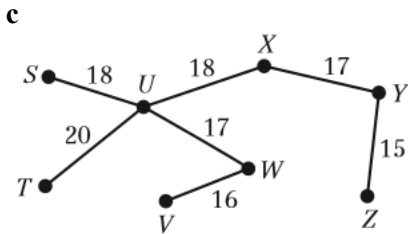

weight: 98

weight: 27

weight: 22.4

2 b By inspection the order of the arcs is
$YZ(15), VW(16), XY(17), UW(17), UX(18),$

$WX(18), SU(18), WZ(18), UV(19),$

$TU(20), ST(22), TV(23)$

Underlined arcs are in the minimum spanning tree. Total weight = 121
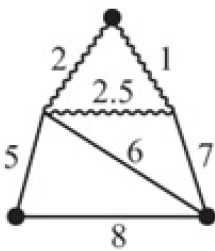
c

d This minimum spanning tree is not unique. For example, UX can be replaced with WX.
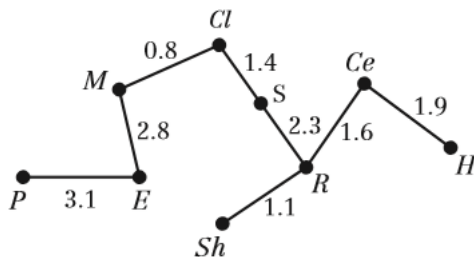
3 a For example;
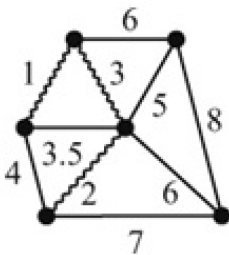
The three shortest edges form a cycle.

b For example;

The three shortest edges do not form a cycle.

4 a Add arcs in the order MCl (0.8), ShR (1.1), ClS(1.4), CeR(1.6), CeH(1.9), SR(2.3), ME(2.8), PE(3.1)

All vertices are connected so this is a minimum spanning tree.

4 b $0.8 + 1.1 + 1.4 + 1.6 + 1.9 + 2.3 + 2.8 + 3.1$
$= 15$ km