physicsandmathstutor.com

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4736:** Decision Mathematics 1

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2011

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

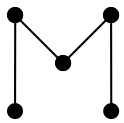
Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1	**(i)**	$y \geq x$ $x + y \leq 8$ $y \leq 2x$	M1 A1	**[2]**	Any two boundary lines correct (any form) All three inequalities correct (any form)	Allow = or wrong inequality sign for method mark Ignore $x \geq 0$, $y \geq 0$, if given
	(ii)	$(2\frac{2}{3}, 5\frac{1}{3})$ $P_2 = 13\frac{1}{3}$	B1 B1	**[2]**	Coordinates of point *A* seen, cao, fractions or 2.6 to 2.7 and 5.3 to 5.4 cao, fractions or 13.3 to 13.4	Must be written down, allow $x = 2\frac{2}{3}$, $y = 5\frac{1}{3}$ or as decimals 2.6 to 2.7, 5.3 to 5.4 BOD if not identified as *A* or as optimal
	(iii) a **b**	$m \geq 1$ $m \leq 1$	B1 B1	**[2]**	Condone > 1 Condone < 1	Do not accept 1, 2, 3 ... (integer valued) Do not accept 0, -1, ... (integer valued) $m = 1$ is not enough for either mark

physicsandmathstutor.com

2	**(i)**	<table><tr><th>*R*</th><th>*S*</th></tr><tr><td>8</td><td>5</td></tr><tr><td>5</td><td>4.1</td></tr><tr><td>4.1</td><td>4.0012 (4.00)</td></tr><tr><td>4.0012 (4.00)</td><td>3.9988 (4.00)</td></tr></table> Output 4.00	M1 A1	**[2]**	(N = 16) 8, 5 and 4.1 seen Final value (of R) 4.00 (accept 4 or anything that rounds to 4.00)	Need not be in table form
	(ii)	<table><tr><th>*R*</th><th>*S*</th></tr><tr><td>1</td><td>1.5</td></tr><tr><td>1.5</td><td>1.4167 (1.42)</td></tr><tr><td>1.4167 (1.42)</td><td>1.4142 (1.41)</td></tr><tr><td>1.4142 (1.41)</td><td>1.4142 (1.41)</td></tr></table> Output 1.41	M1 A1	**[2]**	(N = 2) 1, 1.5 and 1.4167 seen to at least 2 dp accuracy (ie accept 1, 1.5, 1.42) Final value (of R) 1.41 (or anything that rounds to 1.41) after 4 iterations	Need not be in table form 1.4166… = $\frac{17}{12}$ as a fraction Watch out for truncation on S = 1.4167
	(iii)	Output is the square root of input (to 2 dp)	B1	**[1]**	Square root, √, root	
	(iv)	<table><tr><th>*R*</th><th>*S*</th></tr><tr><td>-2</td><td>0</td></tr><tr><td>0</td><td>undefined, error or ?</td></tr></table>	B1	**[1]**	(N = -4) Stops when R = 0 (without wrong working) in second iteration (division by 0) Do not allow continuing on to further R values	‘Division by 0’ ⇒ B1 (without wrong working) Do not give mark for talking about square root of a negative number, unless evidence of division by 0 is also seen in working
	(v)	<table><tr><th>*R*</th><th>*S*</th></tr><tr><td>-1</td><td>0.5</td></tr><tr><td>0.5</td><td>-1.75</td></tr><tr><td>-1.75</td><td>-0.3036</td></tr><tr><td>-0.3036</td><td>3.1423</td></tr></table> The algorithm does not terminate (chaotic) Build in a stopping condition, such as counting how many times step 3 has been carried out	B1 B1 B1	**[2]**	(N = -2) Values not necessarily shown, but if comments are made then they must be true for the <u>correct</u> values The values never agree (to 2 dp) Values keep changing/do not settle down Needs a stopping condition, so that algorithm works for $N \geq 0$ and stops if $N < 0$	Allow correct listed values (ignoring comments) -1, 0.5, -1.75, -0.30 ($-\frac{17}{56}$) (or better) Allow ‘oscillate’ Not ‘use modulus of N’ Not suggesting changes to ‘S =’ line Not ‘if $N < 0$ then …’

physicsandmathstutor.com

3	**(i)**	Must have an even number of odd vertices Total order must be even	B1	**[1]**	(Cannot have) three odd vertices (It would have) three odd nodes 15÷2 = 7.5 and cannot have half an arc	Note: graph need not be simple or connected Do not consider descriptions of specific cases
	(ii)	The vertex of order 5 would either connect to at least one of the other vertices twice or connect to itself Graph is not simple	B1 B1	**[2]**	Identifying that vertex of order 5 causes problem, with a valid reason ('order 5 and repeated arcs' or 'order 5 and loop') Stating 'simple' (simple cannot be achieved)	Not a description of a specific case 'simple graph with 5 vertices means no vertex can have order greater than 4' ⇒ B1 B1 But **not** if also claim graph is not connected
	(iii)	5 arcs A tree with five vertices only has 4 arcs (must be using number of arcs)	B1 B1	**[2]**	10 ÷ 2 = 5 Tree has 4 arcs, or sum of vertex orders = 8 Explaining why 5 arcs means this is not a tree A tree has fewer arcs than vertices	 5 > *n*-1 (but *n*-1 on its own is not enough) Not a description of a specific case
	(iv)	 A tree must have at least two nodes of order 1, an Eulerian graph has no odd nodes so has no nodes of order 1	B1 B1	**[2]**	Any tree with five vertices that is a 'string' Any of: Eulerian so all even orders Eulerian so no nodes of order 1 A cycle cannot be a tree Two (or more) regions so not a tree	 Any of: A tree cannot contain a cycle A tree must have at least two nodes of order 1
	(v)	Would need either an Eulerian or a semi-Eulerian graph ⇒ 0 or 2 odd nodes, but the graph has 6 odd nodes Need to repeat 2 arcs	B1 B1	**[2]**	Accept 'needs 0 odd nodes' or 'needs 2 odd nodes' or both Cannot have more than 2 odd nodes 2 (allow two arcs written, using four nodes, eg *AB* and *CD*)	Condone '6 odd nodes' Condone 'all odd' [Note: route need not be a closed cycle]

physicsandmathstutor.com

4 (i)

P	*w*	*x*	*y*	*z*	*s*	*t*	*u*	RHS
1	3	-5	7	-2	0	0	0	0
0	1	2	-2	-1	1	0	0	10
0	2	0	3	-4	0	1	0	12
0	4	5	1	0	0	0	1	30

Pivot must be chosen from a column that has a negative entry in the objective row, so pivot cannot be chosen from the *w* column or the *y* column (both)

Pivot entry cannot be negative or zero, there are no positive entries in the *z* column so the pivot cannot be chosen from the *z* column

B1 Objective row correct (3, -5, 7, -2)
Tableau may be credited if seen in part (ii), but only if not attempted in part (i)

B1 Constraint rows correct, with three slack variable columns
Rows and columns may be in any order
Condone *P* column missing here

B1 *x* has a negative value (in the objective row)
Identifying the value -5
BOD: *x* most negative (in objective row)
Only follow through their initial tableau if it leads to *x* as a valid pivot column
But not just: *x* is the minimum value in the objective row (unless also say negative)

B1 Using signs of entries in columns to show that pivot cannot come from *z* column
Identifying the values -1 and -4
Allow numbers in *z* column are 'negative'
Only follow through their initial tableau if it leads to *x* as the only valid pivot column, but values on objective row led to two (or more) possible pivot columns

x is the only column with a negative in objective row and positives in column
⇒ B1, B1
x column has a negative value in objective row and positives in column ⇒ B1, B0

[4]

(ii)

P	*w*	*x*	*y*	*z*	*s*	*t*	*u*	RHS
1	5.5	0	2	-4.5	2.5	0	0	25
0	0.5	1	-1	-0.5	0.5	0	0	5
0	2	0	3	-4	0	1	0	12
0	1.5	0	6	2.5	-2.5	0	1	5

Decimals or fractions

New row 2 = (row 2) ÷ 2
New row 1 = row 1 + 5(new row 2)
New row 3 = row 3
New row 4 = row 4 - 5(new row 2)
(or equivalent, eg row 1 + 5/2×row 2)

w = 0, *x* = 5, *y* = 0, *z* = 0 and *P* = 25

M1 An augmented tableau with four basis columns, non-negative values in final column and value of objective having not decreased
P column required in part (ii)
A column of 0's ⇒ M0

A1 Correct tableau after one iteration (cao)
Rows and columns may be in any order
Ignore any working lines (or interim tableau)

B1 ft Method seen and correct – usually in margin and abbreviated eg ①+5⑥, R1 + 5×pivot, ①+5② (BOD new ②)
but not just +5⑥ or + 5×pivot
(condone row 3 and/or pivot row not explained, but do not ignore if explained and wrong)
Follow through their pivot choice, even if negative (but not if pivot value = 0)
This B mark may be credited if seen in part (i), but only if not attempted in part (ii)

B1 Condone 0 values missing
Ignore values for slack variables, if given
x = 5, *P* = 25 (and others not written), not ft

[4]

physicsandmathstutor.com

(iii)

P	*w*	*x*	*y*	*z*	*s*	*t*	*u*	RHS
1	8.2	0	12.8	0	-2	0	1.8	34
0	0.8	1	0.2	0	0	0	0.2	6
0	4.4	0	12.6	0	-4	1	1.6	20
0	0.6	0	2.4	1	-1	0	0.4	2

Decimals or fractions

M1

A1

$w = 0$, $x = 6$, $y = 0$, $z = 2$ and $P = 34$

B1

There is still a negative entry in the objective row but there is no further valid pivot choice

B1

The coefficients of z are never positive, so z can be increased without limit, and hence P can also be increased without limit

B1

[5]

(answer to (ii) below on scan)
An augmented tableau with four basis columns, non-negative values in final column and value of objective having not decreased

Correct tableau after two iterations

Condone 0 values missing
Ignore values for slack variables, if given

Have not reached optimum and cannot pivot further

z can be arbitrarily large

P column required in part (iii)

A column of 0's ⇒ M0

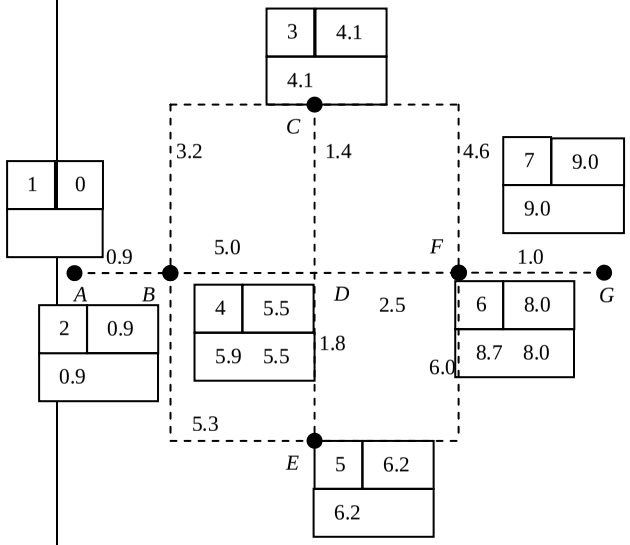
Rows and columns may be in any order
May be credited if seen in part (ii), but only if not attempted in part (iii), (unless crossed out)
Ignore any working lines (or interim tableau)

$x = 6$, $z = 2$, $P = 34$ (and others not written), not ft

From the final tableau
Explanation needs to be convincing

From the original formulation
Explanation needs to be convincing

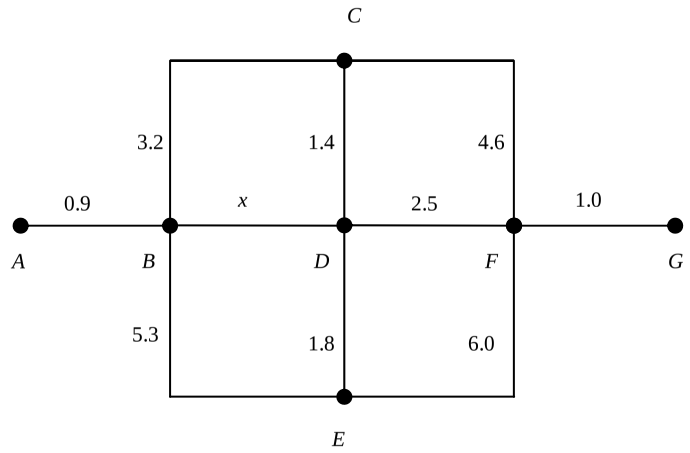
physicsandmathstutor.com

5	**(i)**	 *A B C D F G* 9.0 km	M1 A1 M1 A1 B1 B1	**[6]**	At least four sets of temporary labels (excluding *A*) correct with no extras All temporary labels correct with no extras At least four permanent labels correct, (excluding *A*) All permanent labels correct (condone blank at *A*) and order of labelling correct Not reversed 9	Temporary labels correct at (at least four of) *B, C, D, E, F, G* cao May be seen as permanent labels Permanent labels correct at (at least four of) *B, C, D, E, F, G* cao cao not ft, units not necessary
	(ii)	$x = 3.2+1.4 = 4.6$ $0.9+x+2.5+1.0 = 4.4+x$	B1 B1	**[2]**	4.6 seen (ignore what is said about it) $4.4+x$	cao cao, not a specific case Expression must be simplified
	(iii)(a)	$x+1.8 < 5.3 \Rightarrow x < 3.5$ Temporary labels are 6.2 and $2.7+x$	B1 B1	**[2]**	Condone 3.5 identified Condone $2.7+x$ without 6.2 as well	3.5, even with wrong inequality Expressions must be simplified
	(b)	$x+1.4 < 3.2 \Rightarrow x < 1.8$ Temporary labels are 4.1 and $2.3+x$	B1 B1	**[2]**	Condone 1.8 identified Condone $2.3+x$ without 4.1 as well	1.8, even with wrong inequality Expressions must be simplified
	(iv)	$20\times\left(\frac{100}{50}\right)^2 = 80$ seconds	M1 A1	**[2]**	Correct calculation 80 (or 1 minute 20 seconds)	Or equivalent correct method eg $100^2 \times 0.008$ Units not necessary

physicsandmathstutor.com

6	(i)	Need *A* and *G* odd so must pair *C* with *E*	B1		Recognising that <u>only</u> *C* and *E* need to be paired	May be implied from *CE* (*CDE*) seen in subsequent working (without *AG*) but not implied from 3.2 or from a route written
		CE = 1.4 + 1.8 = 3.2	M1		3.2 seen, without having to look for it in a list of additions	Allow 1.4 + 1.8 seen (may be implied from 29.9)
		Shortest route has length 29.9 + *x*	A1	**[3]**	29.9+*x*	cao

For reference:

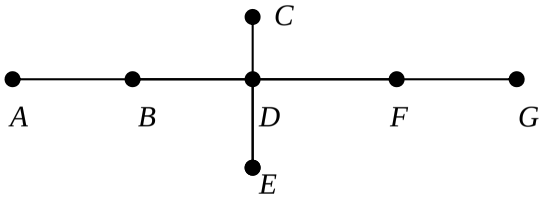
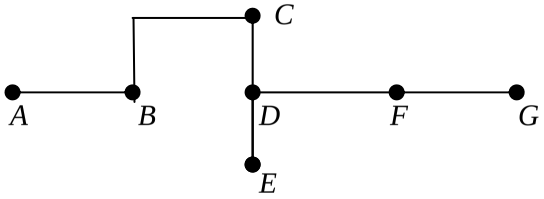


Error on question paper. If you see any evidence that a candidate has lost out through this (needs to be a substantial attempt at this part) please check with your team leader. May need to mark crossed out work, even if it has been replaced. Credit work for (a) even when labelled as $x \geq 1.8$, and similarly for (b)

physicsandmathstutor.com

	(ii)	If $x \leq 1.8$ $BC = 1.4+x$ or $AC = 2.3+x$ $EF = 4.3$ or $EG = 5.3$ $BE = 1.8+x$ or $AE = 2.7+x$ $CF = 3.9$ or $CG = 4.9$ $BF = 2.5+x$, $AG = 4.4+x$, $AF = 3.4+x$ or $BG = 3.5+x$ $CE = 3.2$ $\Rightarrow$ $5.7+x$ or $7.6+x$ or $6.6+x$ or $6.7+x$ Hence if $x \leq 1.8$, shortest route has length $1.9 + (26.7+x) + (5.7+x) = 1.9 + 32.4 + 2x$ km	M1 M1 M1 A1	**[4]**	See or imply any one of CE (CDE) = 3.2 EF (EDF) = 4.3 CF (CDF) = 3.9 EG ($EDFG$) = 5.3 CG ($CDFG$) = 4.9 See or imply any one of $AC = 2.3 + x$ $BC = 1.4 + x$ $AE = 2.7 + x$ $BE = 1.8 + x$ $AF = 3.4 + x$ $BF = 2.5 + x$ $AG = 4.4 + x$ $BG = 3.5 + x$ $5.7 + x$ or $7.6+x$ or $6.6+x$ or $6.7+x$ seen Correctly achieving $32.4 + 2x$ or $34.3 + 2x$ (from a valid method) **$34.3 + 2x$ seen** $\Rightarrow$ SC 4 (but NOT $34.2 + 2x$)	Just writing down a route and totalling, or using a specific value for x (including 1.8) $\Rightarrow$ 0 marks Note: expression $32.4 + 2x$ is given in the question Note: 'see or imply' may be seen in amongst additions, but must have vertices obvious' 'seen' must be explicit, not in amongst additions Must be simplified. The previous two M marks may be implied from this mark gained In any form, eg still involving additions BOD $34(+2x)$ or $34.2(+2x)$ (with or without $+2x$) for A mark, provided M marks were gained
		Identifying odd nodes: A or B, C, E, F or G (A,C,E,G or B,C,E,F or A,C,E,G or B,C,E,G) If x > 1.8 $BC = 3.2$ and $EF = 4.3$ $\Rightarrow$ 7.5 or $BC = 3.2$ and $EG = 5.3$ $\Rightarrow$ 8.5 or $AC = 4.1$ and $EF = 4.3$ $\Rightarrow$ 8.4 or $AC = 4.1$ and $EG = 5.3$ $\Rightarrow$ 9.4 Any other possibility is longer Hence, if x > 1.8, shortest route has length 1.9+ (26.7+x) + 7.5 = 1.9 + 34.2 + x km	B1 M1 M1 A1	**[4]**	Any one of these sets seen or implied anywhere in answer to part (ii) $BC = 3.2$ or $AC = 4.1$ seen or implied Any one of $BC + EF = 7.5$, $BC + EG = 8.5$, $AC + EF = 8.4$ or $AC + EG = 9.4$ Correctly achieving $34.2 + x$ or $36.1 + x$ (from a valid method) **$36.1 + x$ seen** $\Rightarrow$ SC3 (for M1, M1, A1)	Just writing down a route and totalling, or using a specific value for x (including 1.8) $\Rightarrow$ 0 marks Note: expression $34.2 + x$ is given in the question The values must be linked to correct pairs of vertices (accept BC, EF = 3.2+4.3 etc) Previous M mark may be implied from this mark gained In any form, eg still involving additions (with or without $+x$) for A mark, provided M marks were gained. Accept correct additions even when followed by an incorrect total. BOD $36(+x)$

physicsandmathstutor.com

(iii)(a)		B1		Tree may be difficult to see on printed diagram, give BOD if necessary cao	If drawn underneath assume labelling is as in question May list arcs instead
	Total weight = 7.6 + x km	B1		7.6 + x	Not ft
	This is the shortest way to connect the vertices, but Shauna needs the shortest cycle so there must be at least one extra arc, and hence her solution must be longer than this	B1	**[3]**	Recognition that Shauna needs a cycle, needs to return to A, or similar Need to travel (some) arcs more than once	Explaining why her solution must be longer than weight of tree Must repeat AB (or FG)
(b)	$A - B - D - C - F - G$	M1		cao (as far as G but not continuing past G)	Must be written here, not just shown on diagram
	Misses out E	A1	**[2]**	Saying that E is not visited	'Stalls' on its own is not enough 'need to repeat B (or F)' is not enough
(iv)(a)		B1		Tree may be difficult to see on printed diagram, give BOD if necessary cao	If drawn underneath assume labelling is as in question May list arcs instead
	Total weight = 10.8 km	B1	**[2]**	10.8	Not ft
(b)	$A - B - C - D - E - F - G$ Total weight = 14.3 km	B1 B1		$ABCDEFG$ or AB, BC, CD, DE, EF, FG 14.3	Must be written here, not just shown on diagram
	Use shortcut $E - D - F$ instead of EF to get a total weight of 12.6 km	B1		Showing a valid shortcut ('shortcut EF' is not enough)	Need not give new weight, but it needs to be a shortcut ie E-D-F
	Upper bound = 17.0 + x	B1	**[4]**	17 + x	Accept 21.6 or 25.2 or 28.6 as alternative upper bounds, but nothing else

physicsandmathstutor.com

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

© OCR 2011