

**Mark Scheme 4736  
June 2007**

SOLUTIONS

4736

D1

June 2007

FINAL

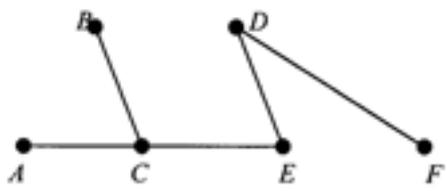
1	(i)	Example: $N-P-Q-T-S-R-N$ or: $P-Q-S-P$	B1	1	Any valid cycle (closed and does not repeat vertices, need not be a Hamiltonian cycle)
	(ii)	It passes through $Y$ twice	B1	1	Or, it includes a cycle (accept 'loop')
	(iii)	5	B1	1	
	(iv)	$A$ : neither $B$ : semi-Eulerian	B1	2	If graphs are not specified, assume $A$ is first
	(v)	$A$ : 2 $B$ : 1	B1	2	If graphs are not specified, assume $A$ is first
	(vi)	There are 4 odd nodes ( $N, P, S$ and $Z$ ) To connect these we must add 2 arcs	M1	2	$A$ : 1, $B$ : 0 $\Rightarrow$ B1 only Seen or implied
			A1	9	For 2

2	(i)	$d+f+g=120$	B1	1	For this equality. Condone an inequality
	(ii)	"(Area of) grass is not more than 4 times (area of) decking"	B1	1	Identifying the constraint in words (not just 'grass is less than or equal to 4 times decking' though)
	(iii)	$d \leq f$	B1	1	Do not accept $d < f$
	(iv)	$g \geq 40$ $\min d = 10$ $\min f = 20$	B1	3	Do not accept $g > 40$ $d \geq 10$ $f \geq 20$
	(v)	$5g + 10d + 20f$ or $g + 2d + 4f$	B1	1	Or any positive multiple of this
	(vi)	Minimise $g + 2d + 4f$ Subject to $d + f + g = 120$ $g - 4d + s = 0$ $d - f + t = 0$ $g \geq 40$ , and $d \geq 10, f \geq 20, s \geq 0, t \geq 0$	M1	3	For a reasonable attempt at setting up the minimisation problem using their expressions
			B1	10	For dealing with this slack variable correctly (variables on LHS and constant on RHS)
			A1	3	For a completely correct formulation (accept $d$ and $f \geq 0$ , or their min values for $d, f$ )

3	(i)	<table border="0"> <tr> <td></td> <td>8 6 9 7 5</td> <td>Comps</td> <td>Swaps</td> <td></td> <td></td> </tr> <tr> <td>After 1st pass:</td> <td>6 8 9 7 5</td> <td>1</td> <td>1</td> <td>M1</td> <td>Bubble sort or decreasing order loses first 4 marks</td> </tr> <tr> <td>After 2nd pass:</td> <td>6 8 9 7 5</td> <td>1</td> <td>0</td> <td>M1</td> <td>1st pass correct</td> </tr> <tr> <td>After 3rd pass:</td> <td>6 7 8 9 5</td> <td>3</td> <td>2</td> <td>M1</td> <td>2nd pass correct, follow through from 1st pass</td> </tr> <tr> <td>After 4th pass:</td> <td>5 6 7 8 9</td> <td>4</td> <td>4</td> <td>A1</td> <td>3rd pass correct, follow through from 2nd pass</td> </tr> </table> <p>Comparisons must be 1, 2, 3 or 4 with total <math>\leq 10</math> Swaps must be 0, 1, 2, 3 or 4 and no more than corresponding number of comparisons</p>		8 6 9 7 5	Comps	Swaps			After 1st pass:	6 8 9 7 5	1	1	M1	Bubble sort or decreasing order loses first 4 marks	After 2nd pass:	6 8 9 7 5	1	0	M1	1st pass correct	After 3rd pass:	6 7 8 9 5	3	2	M1	2nd pass correct, follow through from 1st pass	After 4th pass:	5 6 7 8 9	4	4	A1	3rd pass correct, follow through from 2nd pass	B1	6	4th pass correct Counting comparisons for at least three passes Counting swaps for at least three passes
		8 6 9 7 5	Comps	Swaps																															
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After 4th pass:	5 6 7 8 9	4	4	A1	3rd pass correct, follow through from 2nd pass																														
(ii)	<p>Step 1 A = 8 6 9 7 5</p> <p>Step 2 A = 6 9 7 5 X = 8</p> <p>Step 3 A = 9 7 5 B = 6</p> <p>Step 4 A = 7 5 C = 9</p> <p>Step 4 A = 5 B = 6 7</p> <p>Step 4 A is empty B = 6 7 5</p> <p>Step 6 N = 3</p> <p>Step 7 A = 6 7 5 8 9</p> <p>Step 8 Display 6 7 5 8 9</p>	M1	5	For identifying that $6 \rightarrow B$ or the sublist {6}																															
			M1	11	For identifying that $9 \rightarrow C$ or the sublist {9}																														
			M1		For identifying that $7 \rightarrow B$																														
			M1		For identifying that $5 \rightarrow B$																														
			A1	5	For the final A list or the display correct																														

<b>4</b>	(i)	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>u</i></th> <th></th> </tr> <tr> <td>1</td> <td>-3</td> <td>5</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>5</td> <td>1</td> <td>0</td> <td>0</td> <td>12</td> </tr> <tr> <td>0</td> <td>1</td> <td>-5</td> <td>0</td> <td>1</td> <td>0</td> <td>10</td> </tr> <tr> <td>0</td> <td>3</td> <td>10</td> <td>0</td> <td>0</td> <td>1</td> <td>45</td> </tr> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>		1	-3	5	0	0	0	0	0	1	5	1	0	0	12	0	1	-5	0	1	0	10	0	3	10	0	0	1	45	<p>B1 For correct use of three slack variable columns</p> <p>B1 For <math>\pm (-3 \ 5)</math> in objective row</p> <p>B1 For 1 5 12, 1 -5 10 and 3 10 45 in constraint rows</p> <p style="text-align: right;"><b>3</b></p>
	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>																																
	1	-3	5	0	0	0	0																															
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(ii)	<p>Pivot on second 1 in <i>x</i> column  <i>x</i> column has a negative entry in objective row  <math>12 \div 1 = 12</math>, <math>10 \div 1 = 10</math>, <math>45 \div 3 = 15</math>                      Least non-negative ratio is 10 so pivot on the second 1</p>	<p>B1 For correct pivot choice (cao)</p> <p>B1 For 'negative in top row for <i>x</i>', or equivalent, and a correct explanation of choice of row 'least ratio <math>10 \div 1</math>' (ft their pivot column)</p> <p style="text-align: right;"><b>2</b></p>																																				
(iii)	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>z</i></th> <th><i>s</i></th> <th><i>t</i></th> <th></th> </tr> <tr> <td>1</td> <td>0</td> <td>-10</td> <td>0</td> <td>3</td> <td>0</td> <td>30</td> </tr> <tr> <td>0</td> <td>0</td> <td>10</td> <td>1</td> <td>-1</td> <td>0</td> <td>2</td> </tr> <tr> <td>0</td> <td>1</td> <td>-5</td> <td>0</td> <td>1</td> <td>0</td> <td>10</td> </tr> <tr> <td>0</td> <td>0</td> <td>25</td> <td>0</td> <td>-3</td> <td>1</td> <td>15</td> </tr> </table> <p><math>x = 10, y = 0</math>  <math>P = 30</math></p>	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>		1	0	-10	0	3	0	30	0	0	10	1	-1	0	2	0	1	-5	0	1	0	10	0	0	25	0	-3	1	15	<p>ft their tableau if possible for method marks</p> <p>M1 For correct method evident for objective row</p> <p>M1 For a correct method evident for pivot row</p> <p>M1 For a correct method evident for other rows</p> <p>A1 For correct tableau CAO</p> <p>B1 For correct values from their tableau</p> <p>B1 <b>6</b> For correct value from their tableau</p>	
<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>s</i>	<i>t</i>																																	
1	0	-10	0	3	0	30																																
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0	1	-5	0	1	0	10																																
0	0	25	0	-3	1	15																																
(iv)	<p><math>11 + 5(0.2) = 12</math> or <math>s = 0</math>  <math>11 - 5(0.2) = 10</math> or <math>t = 0</math>  <math>3(11) + 10(0.2) = 35</math> or <math>u = 10</math>                      so all the constraints are satisfied</p> <p><math>P = 3(11) - 5(0.2) = 32</math>                      which is bigger than 30 from (iii)</p>	<p>B1 For showing (not just stating) that constraints are satisfied</p> <p>B1 <b>2</b> For calculating 32, or equivalent (eg 3<i>x</i> has increased by 3 but -5<i>y</i> has only decreased by 1)</p> <p style="text-align: right;"><b>13</b></p>																																				

5	(i)	<p>Shortest path from J to B: J G H E B Length of path: 125 metres</p>	<p>M1 M1 A1 B1 B1 B1 B1 B1</p>	<p>ANSWERED ON INSERT</p> <p>For correct initial temporary labels at F, G, I</p> <p>For correctly updating F and label at H</p> <p>For all temporary labels correct (including A) (allow extra 100 at C, 105 at D, 75 at H only)</p> <p>For order of becoming permanent correct</p> <p>For all permanent labels correct (A need not have a permanent label)</p> <p>For correct route (condone omission of J or B)</p> <p>For 125</p>
	(ii)	<p>Odd nodes: B C E J</p> <p><math>BC = 60</math>   <math>BE = 35</math>   <math>BJ = 125</math>  <math>EJ = 90</math>   <math>CJ = 95</math>   <math>CE = 70</math>                150       130       195</p> <p>Repeat BE and CJ (or BE, JI, IC)</p> <p>130 + 765 Shortest route: 895 metres</p>	<p>B1 B1 B1 M1 A1 M1 A1</p>	<p>For identifying or using B C E J or implied</p> <p>For any three of these weights correct, or implied or ft from their (i)</p> <p>For identifying the pairing BE, CJ to repeat or 130 (not ft)</p> <p>For 765 + their 130 (a valid pairs total)</p> <p>For 895 (cao)</p>
	(iii)	<p>Travelling salesperson problem</p>	<p>B1 M1 A1 B1</p>	<p>For graph structure correct</p> <p>For a reasonable attempt at arc weights (at least 9 correct, including the three given)</p> <p>For all arc weights correct</p> <p>For identifying TSP by name</p>

<b>6</b>	(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">1</td> <td style="text-align: center;">5</td> <td style="text-align: center;">2</td> <td style="text-align: center;">4</td> <td style="text-align: center;">3</td> <td style="text-align: center;">6</td> </tr> <tr> <td></td> <td style="text-align: center;"><i>A</i></td> <td style="text-align: center;"><i>B</i></td> <td style="text-align: center;"><i>C</i></td> <td style="text-align: center;"><i>D</i></td> <td style="text-align: center;"><i>E</i></td> <td style="text-align: center;"><i>F</i></td> </tr> <tr> <td style="text-align: center;"><i>A</i></td> <td style="text-align: center;">-</td> <td style="text-align: center;">6</td> <td style="text-align: center;">3</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> <tr> <td style="text-align: center;"><i>B</i></td> <td style="text-align: center;">6</td> <td style="text-align: center;">-</td> <td style="text-align: center;">5</td> <td style="text-align: center;">6</td> <td style="text-align: center;">-</td> <td style="text-align: center;">14</td> </tr> <tr> <td style="text-align: center;"><i>C</i></td> <td style="text-align: center;">3</td> <td style="text-align: center;">5</td> <td style="text-align: center;">-</td> <td style="text-align: center;">8</td> <td style="text-align: center;">4</td> <td style="text-align: center;">10</td> </tr> <tr> <td style="text-align: center;"><i>D</i></td> <td style="text-align: center;">-</td> <td style="text-align: center;">6</td> <td style="text-align: center;">8</td> <td style="text-align: center;">-</td> <td style="text-align: center;">3</td> <td style="text-align: center;">8</td> </tr> <tr> <td style="text-align: center;"><i>E</i></td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">4</td> <td style="text-align: center;">3</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> <tr> <td style="text-align: center;"><i>F</i></td> <td style="text-align: center;">-</td> <td style="text-align: center;">14</td> <td style="text-align: center;">10</td> <td style="text-align: center;">8</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> </tr> </table>		1	5	2	4	3	6		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>A</i>	-	6	3	-	-	-	<i>B</i>	6	-	5	6	-	14	<i>C</i>	3	5	-	8	4	10	<i>D</i>	-	6	8	-	3	8	<i>E</i>	-	-	4	3	-	-	<i>F</i>	-	14	10	8	-	-		ANSWERED ON INSERT
			1	5	2	4	3	6																																																				
			<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>																																																				
		<i>A</i>	-	6	3	-	-	-																																																				
		<i>B</i>	6	-	5	6	-	14																																																				
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		<i>F</i>	-	14	10	8	-	-																																																				
			Order: <i>A C E D B F</i>		M1	For choosing row <i>C</i> in column <i>A</i>																																																						
	Minimum spanning tree:		M1 dep	For choosing more than one entry from column <i>C</i>																																																								
			A1	For correct entries chosen																																																								
	Total weight: 23 miles		B1	For correct order, listed or marked on arrows or table, or arcs listed <i>AC CE ED CB DF</i>																																																								
			B1	For tree (correct or follow through from table, provided solution forms a spanning tree)																																																								
			B1	For 23 (or follow through from table or diagram, provided solution forms a spanning tree)																																																								
		<b>6</b>																																																										
	(ii)	MST for reduced network = 18 Two shortest arcs from <i>B</i> = 5 + 6 = 11 Lower bound = 29 miles	M1	For their 18 seen or implied																																																								
			M1	For 11 seen or implied																																																								
			A1	<b>3</b> For 29 (cao)																																																								
	(iii)	$F-D-E-C-A-B-F$  $8 + 3 + 4 + 3 + 6 + 14$ = 38 miles	M1	For $F-D-E-C-A-B$																																																								
			A1	For correct tour																																																								
			M1	For a substantially correct attempt at sum																																																								
			A1	<b>13</b> For 38 (cao)																																																								

