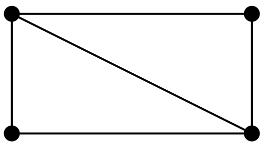
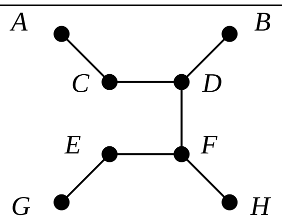
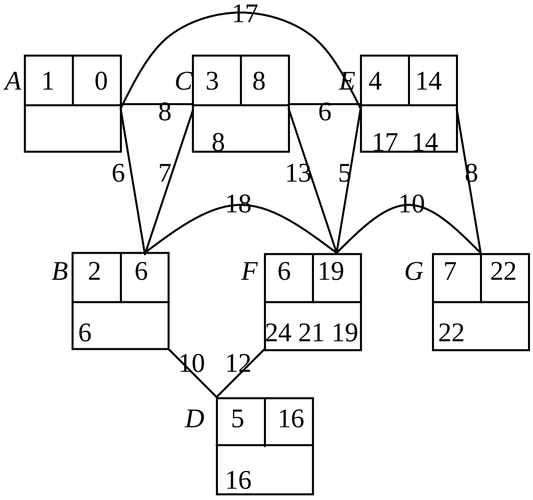
Mark Scheme 4736
June 2005

1	**(a) (i)**	8 7 5 4 3 3 3 3 2 2 First bag 8 2 Second bag 7 3 Third bag 5 4 Fourth bag 3 3 3 Fifth bag 2	M1 M1 A1	For sorting the list into decreasing order For trying to apply first-fit to their list For a completely correct solution
	(ii)	A packing that uses fewer bags could be First bag 8 2 Second bag 7 3 Third bag 5 3 2 Fourth bag 4 3 3	B1	For any valid packing into four bags (may be as an incorrect answer to using algorithm, need not be packed in this order)
	(b)	$\left(\frac{500}{100}\right)^3 \times 4$ or 125000000 × 0.000004 = 500	M1 A1 **6**	For scaling 4 seconds by 5^3 or for an equivalent valid and complete method. Condone minor errors with the number of zeros. For 500 or 500 seconds or 500 s. Accept 8 minutes 20 seconds or 8.3 minutes
2	**(i)**	eg 	B1	For any <u>simple</u> graph with 4 vertices and 5 arcs Vertices need not be labelled Need not be planar
	(ii)	The sum of the orders of the vertices is twice the number of arcs, and hence is even. Hence the sum of the odd orders must be even and so there must be an even number of odd vertices.	M1 A1	<u>Or</u> start from a null graph and successively add in arcs. Each time an arc is added the number of odd vertices is either unchanged or it increases or decreases by 2. So the number of odd nodes is always even
	(iii)	5 arcs ⇒ sum of orders of vertices = 10 Simple graph connecting vertices so each vertex has order 1, 2 or 3 1 + 3 + 3 + 3 = 10 or 2 + 2 + 3 + 3 = 10 But 1 + 3 + 3 + 3 is not possible since if three vertices have order 3 they are all connected to the fourth vertex so it also has order 3. With 2 + 2 + 3 + 3 the two vertices of order 2 cannot be adjacent, since otherwise two arcs connect the other two vertices so not simple. Hence only one possible graph.	M1 A1 A1 **6**	 Explaining why 1 + 3 + 3 + 3 is not possible. Explaining why there is only one graph with nodes of orders 2, 2, 3, 3.
3	**(i)**	 Kruskal: *DF*, *CD*, *BD* and *EF*, *FH*, *AC*, *EG* 40	M1 A1 B1	For a correct tree (labels not required) For a valid order (using Prim or Kruskal) For length = 40
	(ii)	*A C D F E G H B A*	M1	At getting at least as far as *A C D F E* (or shown on a diagram)

	A1	For a correct cycle, ending back at *A* (if shown on a diagram, needs direction shown)
(iii) **(A)** *ACEG* and *ABGH*	B1	For both, vertices in any order
(B) 5	B1	For 5
(C) *ABCD*	B1	For *ABCD*, vertices in any order
	8	
4 **(i)** 		Answer should be on insert sheet
	M1	For using Dijkstra's algorithm – updating at *E* and *F* (even if incomplete)
	A1	For all permanent labels correct
	B1	For valid order of assigning permanent labels
Vertex *B* *C* *D* *E* *F* *G* Length 6 8 16 14 19 22	B1	For copying their permanent labels, or correct values
A – *C* – *E* – *G*	B1	Correct answer only
(ii) The only odd nodes are *A* and *F* Shortest path from *A* to *F* has length 19 km	M1	For identifying *A* and *F* or value 19 or their 19
120 + 19	M1	For 120 + their 19
= 139 km	A1	For 139 (cao)
(iii) Need *A* and *G* odd and all other nodes even so need to connect *F* to *G* = 10 km	M1	For identifying *F* and *G* or value 10 as only extra
120 + 10 = 130 km	A1	For 130 (cao)
	10	

Question	Part	Answer	Marks	Guidance
5	(i)	X N T S 0 0 0 3 1 3 9 6 2 9 45 5 3 14 70 7 4 21 119 3 5 24 128	 M1 M1 A1	 For initial pass through step 3 correct For updating each of N, T and S correctly For final values of N, T and S correct
		$M = 4.8$ $D = 1.6$	B1 B1	For 4.8 (ft their $T \div N$) For 1.6 (ft their $\sqrt{\{(S \div N) - (M^2)\}}$
	(ii)	15 additions and 5 multiplications 20 + 5 = 25	B1 B1	 For 'their 20' + 5
	(iii)	$3n + n + 5$ $= 4n + 5$	M1 A1	For any function of n that gives their answer from (ii) when $n = 5$ For any expression that simplifies to $4n + 5$
	(iv)	(5000 ÷ 1000) × 2 = 10 seconds	B1 **10**	Or 2 ÷ 4005 × 20005 = 9.99 ~ 10 seconds

6 (i)

P	x	y	z	s	t	u	
1	-15	4	4	0	0	0	0
0	10	-4	8	1	0	0	40
0	10	6	9	0	1	0	72
0	-6	4	3	0	0	1	48

M1 For overall structure correct, including three slack variables

A1 For a correct initial tableau, with no extra constraints added. Accept equivalent forms.

(ii) Pivot on 10 in x column 40 row

1	0	-2	16	1.5	0	0	60
0	1	-0.4	0.8	0.1	0	0	4
0	0	10	1	-1	1	0	32
0	0	1.6	7.8	0.6	0	1	72

$x = 4$, $y = 0$, $z = 0$

$P = 60$

M1 For the correct pivot choice for their tableau

A1 For dealing with the pivot row correctly

M1 For dealing with the other rows correctly

A1 For a correct tableau

B1 For reading off x, y and z from their tableau

B1 For reading off P from their tableau

(iii) Pivot on 10 in y column

1	0	0	16.2	1.3	0.2	0	66.4
0	1	0	0.84	0.06	0.04	0	5.28
0	0	1	0.1	-0.1	0.1	0	3.2
0	0	0	7.64	0.76	-0.16	1	66.88

$x = 5.28$, $y = 3.2$, $z = 0$

$P = 66.4$

M1 For the correct pivot choice for their tableau

A1 For dealing with the pivot row correctly

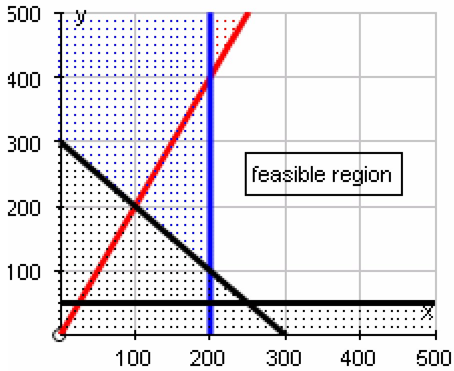
M1 For dealing with the other rows correctly

A1 For a correct tableau

B1 For the correct values of x, y and z at optimum

B1 For the correct value of P at optimum

14

7	(i)	Minimise 70*x* + 80*y* + 50*z*	B1	For 'minimise' a (non-zero) multiple of 7*x*+8*y*+5*z*
		'No more than twice as many packs of type *Y* as packs of type *X*'	B1	For identifying this constraint from the list, or equivalent
		Other constraints		Ignore extra 'constraints' unless contradictions
		$x \geq 200$, $0 \leq z \leq 50$	B1	For boundary constraints on *x* and *z*
		$y \geq z$	B1	For this, or an equivalent correct answer
		$x + z \geq 220$	B1	For this, or an equivalent correct answer
		$x + y \geq 300$	B1	For this, or an equivalent correct answer Use of strict inequalities – penalise first time only
	(ii) (a)	Minimise 70*x* + 80*y* (+ 2500) (or scaled through) Subject to	M1	For replacing *z* by 50
		$y \leq 2x$ $x \geq 200$ $y \geq 50$ $x + y \geq 300$	A1	For their $y \geq 50$
			M1	For at least two appropriate lines drawn on a graph with plausibly scaled axes.
			M1	For boundary lines drawn correctly (follow through their equations provided there are at least two horizontal or vertical lines and at least two lines that 'slope')
			A1	Feasible region correctly identified (correct answer only, not follow through)
	(b)	(200, 400), (200, 100), (250, 50)	M1	For reading off or calculating at least one of their vertices
			A1	For getting these three vertices correct with no extras
		(200, 100) gives 70*x* + 80*y* = 22000 (£245) (250, 50) gives 70*x* + 80*y* = 21500 (£240)	M1	For calculating their cost at one of their vertices or using an appropriate line of constant cost
		Cost is minimised when *x* = 250, *y* = 50	A1	For identifying vertex (250, 50)
		Cost = £240	B1	For £240 or 24000 p (with units)
	(iii)	eg *x* = 300, *y* = 0 , *z* = 0 only costs £210	M1	For finding a feasible point with $z < 50$ Or a written explanation
			A1	For finding such a feasible point with a lower cost than that in (ii)(b) and showing that cost is lower.
			18	