OCR
RECOGNISING ACHIEVEMENT

Friday 25 January 2013 – Afternoon

AS GCE MATHEMATICS

4736/01 Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4736
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(i)** Use shuttle sort to put this list of values into decreasing order (from largest to smallest).

18 7 9 20 15 21 6 10 22

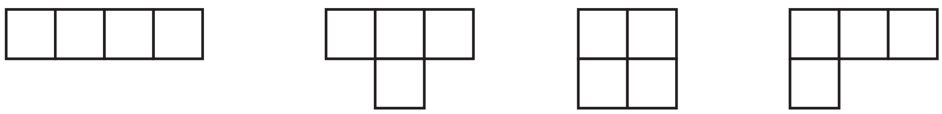
Show the result at the end of each pass through the algorithm and write down the number of comparisons and the number of swaps used in each pass. **[5]**

(ii) The values give the weights, in kg, of sacks of grain. The sacks are to be loaded into boxes, each of which can hold at most 30 kg. Use the first-fit decreasing method to show which sacks should be put into which box. **[2]**

(iii) Suppose that some stronger boxes are available, each of which can hold at most W kg. Find the least value of W for which only four boxes are needed. Show a packing using four of these stronger boxes. **[2]**

2 A tetromino is a two-dimensional shape made by joining four squares edge-to-edge. Joins are along complete edges.

(i) Represent each of the tetrominoes below by a graph in which the nodes represent the squares and two nodes are joined by an arc if the squares share a common edge. **[2]**



(A) (B) (C) (D)

(ii) Six simply connected graphs with four nodes are shown below. For each graph, either draw a tetromino that can be represented by the graph, as in part **(i)**, or explain why this is not possible. **[3]**

(1) (2) (3) (4) (5) (6)

Two tetrominoes are regarded as being the same if one can be rotated or reflected to form the other. Derek claims that each tetromino corresponds to a unique tree with four nodes, and each tree with four nodes corresponds to a unique tetromino. Derek's claim is wrong.

(iii) From the diagrams above, find:

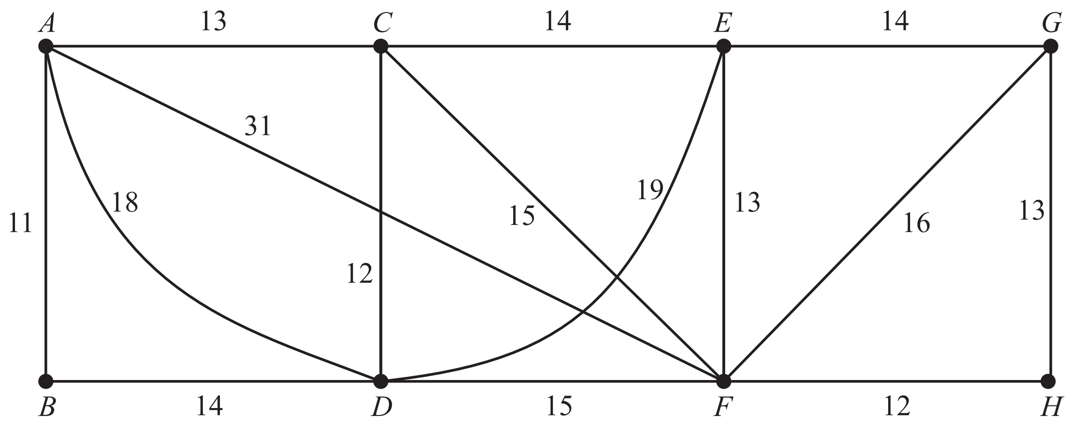
(a) a tetromino whose graph does not correspond to a tree; **[1]**

(b) two different tetrominoes whose graphs correspond to the same tree. **[1]**

A pentomino is a two-dimensional shape made by joining five squares edge-to-edge. Joins are along complete edges. Two pentominoes are regarded as being the same if one can be rotated or reflected to form the other. There are twelve distinct pentominoes.

(iv) When the pentominoes are represented by graphs, as in part **(i)**, there are only four distinct graphs. Draw these four graphs. **[3]**

3 The total weight of the arcs in the network below is 230.



(i) Apply Dijkstra's algorithm to the copy of the network in the answer book to find the least weight path from A to H. Give the path and its weight. **[6]**

In the remainder of this question, any least weight paths required may be found without using a formal algorithm.

(ii) The arc AD is removed. Apply the route inspection algorithm, showing your working, to find the weight of the least weight **closed** route that uses every arc (except AD) at least once. **[4]**

(iii) Suppose, instead, that the arc AD is available, but arcs AC and CD are both removed. Apply the route inspection algorithm, showing your working, to find the weight of the least weight **closed** route that uses every arc (except AC and CD) at least once. **[4]**

4 Pam has seven employees. When it snows they all need to be contacted by telephone.

The table shows the expected time, in minutes, that it will take Pam and her employees to contact each other.

	Pam	Alan	Bob	Caz	Dan	Ella	Fred	Gita
Pam	–	10	4	8	18	12	12	9
Alan	10	–	6	10	18	12	11	9
Bob	4	6	–	9	17	10	11	10
Caz	8	10	9	–	15	13	10	7
Dan	18	18	17	15	–	16	19	20
Ella	12	12	10	13	16	–	13	14
Fred	12	11	11	10	19	13	–	18
Gita	9	9	10	7	20	14	18	–

(i) Use the nearest neighbour method, starting from Pam, to find a cycle through all the employees and Pam. If there is a choice of names choose the one that occurs first alphabetically. Calculate the total weight of this cycle. **[5]**

(ii) Apply Prim's algorithm to the copy of the table in the answer book, starting by crossing out the row for Pam and looking down the column for Pam. List the **arcs** in the order in which they were chosen. Draw the resulting minimum spanning tree and calculate its total weight. **[6]**

(iii) Find a lower bound for the minimum weight cycle through Pam and her seven employees by initially removing Gita from the minimum spanning tree. **[3]**

Pam realises that it takes less time if she splits the employees into teams.

(iv) Use the minimum spanning tree to suggest how to split the employees into two teams, so that Pam contacts the two team leaders and they each contact the members of their team. Using this solution, find the minimum elapsed time by which all the employees can be contacted. **[3]**

5 Roland Neede, the baker, is making cupcakes. He makes three sizes of cupcake: miniature, small and standard. Miniature cupcakes are sold in boxes of 24 and each cupcake uses 3 units of topping and 2 decorations. Small cupcakes are sold in boxes of 20 and each cupcake uses 5 units of topping and 3 decorations. Standard cupcakes are sold in boxes of 12 and each cupcake uses 7 units of topping and 4 decorations.

Roland has no restriction on the amount of cake mix that he uses but he only has 5000 units of topping and 3000 decorations available. Cupcakes are only sold in complete boxes, and Roland assumes that he can sell all the boxes of cupcakes that he makes. Irrespective of size, each box of cupcakes sold will give Roland a profit of £1. Roland wants to maximise his total profit.

Let x denote the number of boxes of miniature cupcakes, y denote the number of boxes of small cupcakes and z denote the number of boxes of standard cupcakes that Roland makes.

(i) Construct an objective function, P, to be maximised. **[1]**

(ii) By considering the number of units of topping used, show that $18x + 25y + 21z \leqslant 1250$. **[2]**

(iii) Construct a similar constraint by considering the number of decorations used, simplifying the coefficients so that they are integers with no common factor. **[2]**

(iv) Set up an initial Simplex tableau to represent Roland's problem. **[3]**

(v) Perform **one** iteration of the Simplex algorithm, choosing a pivot from the x column. Explain how the choice of pivot row was made and show how each row was calculated. **[5]**

(vi) Write down the values of x, y and z from the first iteration of the Simplex algorithm. Hence find the maximum profit that Roland can make, remembering that cupcakes can only be sold in complete boxes. Calculate the number of units of topping and the number of decorations that are left over with this solution. **[4]**

(vii) The constraint from the number of units of topping can be rewritten as $18P + 7y + 3z \leqslant 1250$. Form a similar expression for the constraint from the number of decorations. Use this to find the number of boxes of small cupcakes which maximises the profit when there are no decorations left over. Find the solution which gives the maximum profit using all the topping and all the decorations, and find the values of x, y and z for this solution. **[5]**

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