OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4736

Decision Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- Insert for Questions 1 and 6 (inserted)
- List of Formulae (MF1)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 3 6 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Questions **1** and **6**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

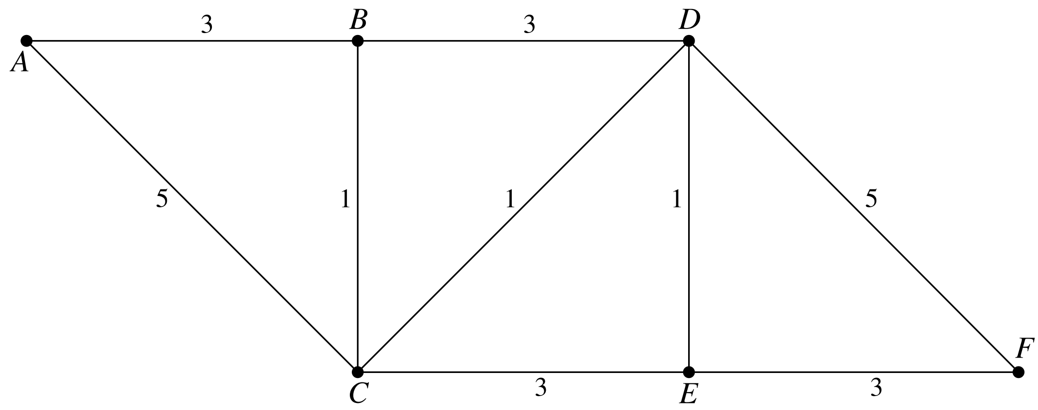
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

2R–9E02

Turn over

1 Answer this question on the insert provided.



(i) Apply Dijkstra's algorithm to the copy of this network in the insert to find the least weight path from A to F. State the route of the path and give its weight. **[5]**

(ii) Apply the route inspection algorithm, showing all your working, to find the weight of the least weight closed route that uses every arc at least once. Write down a closed route that has this least weight. **[4]**

An extra arc is added, joining B to E, with weight 2.

(iii) Write down the new least weight path from A to F. Explain why the new least weight closed route, that uses every arc at least once, has no repeated arcs. **[2]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) Explain why there is no simply connected graph with exactly five vertices each of which is connected to exactly three others. **[1]**

(ii) A simply connected graph has five vertices A, B, C, D and E, in which A has order 4, B has order 2, C has order 3, D has order 3 and E has order 2. Explain how you know that the graph is semi-Eulerian and write down a semi-Eulerian trail on this graph. **[2]**

A network is formed from the graph in part **(ii)** by weighting the arcs as given in the table below.

	A	B	C	D	E
A	–	5	3	8	2
B	5	–	6	–	–
C	3	6	–	7	–
D	8	–	7	–	9
E	2	–	–	9	–

(iii) Apply Prim's algorithm to the network, showing all your working, starting at vertex A. Draw the resulting tree and state its total weight. **[3]**

A sixth vertex, F, is added to the network using arcs CF and DF, each of which has weight 6.

(iv) Use your answer to part **(iii)** to write down a lower bound for the length of the minimum tour that visits every vertex of the extended network, finishing where it starts. Apply the nearest neighbour method, starting from vertex A, to find an upper bound for the length of this tour. Explain why the nearest neighbour method fails if it is started from vertex F. **[4]**

3 Maggie is a personal trainer. She has twelve clients who want to lose weight. She decides to put some of her clients on weight loss programme X, some on programme Y and the rest on programme Z. Each programme involves a strict diet; in addition programmes X and Y involve regular exercise at Maggie's home gym. The programmes each last for one month.

In addition to the diet, clients on programme X spend 30 minutes each day on the spin cycle, 10 minutes each day on the rower and 20 minutes each day on free weights. At the end of one month they can each expect to have lost 9 kg more than a client on just the diet.

In addition to the diet, clients on programme Y spend 10 minutes each day on the spin cycle and 30 minutes each day on free weights; they do not use the rower. At the end of one month they can each expect to have lost 6 kg more than a client on just the diet.

Because of other clients who use Maggie's home gym, the spin cycle is available for the weight loss clients for 180 minutes each day, the rower for 40 minutes each day and the free weights for 300 minutes each day. Only one client can use each piece of apparatus at any one time.

Maggie wants to decide how many clients to put on each programme to maximise the total expected weight loss at the end of the month. She models the objective as follows.

$$\text{Maximise } P = 9x + 6y$$

(i) What do the variables x and y represent? **[1]**

(ii) Write down and simplify the constraints on the values of x and y from the availability of each of the pieces of apparatus. **[3]**

(iii) What other constraints and restrictions apply to the values of x and y? **[1]**

(iv) Use a graphical method to represent the feasible region for Maggie's problem. You should use graph paper and choose scales so that the feasible region can be clearly seen. Hence determine how many clients should be put on each programme. **[6]**

4 Jack and Jill are packing food parcels. The boxes for the food parcels can each carry up to 5000 g in weight and can each hold up to 30 000 cm^3 in volume.

The number of each item to be packed, their dimensions and weights are given in the table below.

Item type	*A*	*B*	*C*	*D*
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm^3)	1000	24 000	10 000	4000
Weight (g)	1000	250	300	400

Jill tries to pack the items by weight using the first-fit decreasing method.

(i) List the 30 items in order of decreasing weight and hence show Jill's packing. Explain why Jill's packing is not possible. **[5]**

Jack tries to pack the items by volume using the first-fit decreasing method.

(ii) List the 30 items in order of decreasing volume and hence show Jack's packing. Explain why Jack's packing is not possible. **[5]**

(iii) Give another reason why a packing may not be possible. **[1]**

5 Consider the following LP problem.

$$\begin{aligned} \text{Minimise} \quad & 2a - 3b + c + 18, \\ \text{subject to} \quad & a + b - c \geqslant 14, \\ & -2a + 3c \leqslant 50, \\ & 10 + 4a \geqslant 5b, \\ \text{and} \quad & a \leqslant 20,\ b \leqslant 10,\ c \leqslant 8. \end{aligned}$$

(i) By replacing a by $20 - x$, b by $10 - y$ and c by $8 - z$, show that the problem can be expressed as follows.

$$\begin{aligned} \text{Maximise} \quad & 2x - 3y + z, \\ \text{subject to} \quad & x + y - z \leqslant 8, \\ & 2x - 3z \leqslant 66, \\ & 4x - 5y \leqslant 40, \\ \text{and} \quad & x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{aligned}$$

[3]

(ii) Represent the problem as an initial Simplex tableau. Perform **one** iteration of the Simplex algorithm. Explain how the choice of pivot was made and show how each row was obtained. Write down the values of x, y and z at this stage. Hence write down the corresponding values of a, b and c. **[11]**

(iii) If, additionally, the variables a, b and c are non-negative, what additional constraints are there on the values of x, y and z? **[2]**

6 Answer this question on the insert provided.

In this question you will need the result: $1 + 2 + \ldots + k = \frac{1}{2}k(k+1)$.

Dominic is writing a computer program to carry out Kruskal's algorithm. He starts by writing a procedure that enables him to input arcs and their weights, for example $\boxed{A \mid 8 \mid B}$ would represent an arc joining A to B of weight 8.

(i) If Dominic uses a network with five vertices, what is the greatest number of arcs that he needs to input? What is the greatest number of arcs for a network with n vertices? **[2]**

He then uses shuttle sort to sort the inputs into order of increasing weight.

(ii) (a) For a network with five vertices, write down

- the maximum number of passes
- the maximum number of comparisons in the first, second and third passes
- the maximum total number of comparisons. **[3]**

(b) Show that the maximum total number of comparisons for a network with n vertices is $\frac{1}{4}n(n-1)\left(\frac{1}{2}n(n-1)-1\right)$. **[2]**

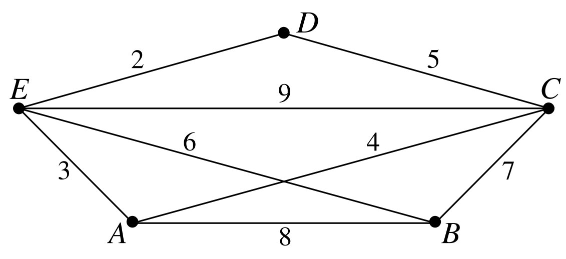
Dominic then sets up four memory areas. *M1* is for the vertices that are in the tree, *M2* is for the arcs that are in the tree and *M3* is for the vertices that are not in the tree. Initially, *M1*and *M2* are empty and *M3* contains a list of all the vertices. Dominic stores the sorted list of arcs and their weights in *M4*.

The first arc on the sorted list is added to the tree, the vertices at its ends are transferred from *M3* to *M1* and the arc is transferred from *M4* to *M2*.

The arc that is now first in *M4* is considered. Each of the two vertices that define the arc is compared with every entry in *M3*. If either of the vertices appears in *M3*, the arc is added to the tree by transferring the vertices at its ends from *M3* to *M1* and transferring the arc from *M4* to *M2*. If neither of the vertices appears in *M3*, the arc is just deleted from *M4*.

This is continued until *M4* is empty.

(iii) The insert shows the start of Dominic's program for the network shown below.



Complete the working on the table in the insert. **[4]**

(iv) Dominic's program has quartic order (order n^4). Dominic's program takes 30 seconds to process an input from a network with 100 vertices. Approximately how long would it take to process an input from a network with 500 vertices? **[2]**

BLANK PAGE

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4736

Decision Mathematics 1

INSERT for Questions 1 and 6

Monday 25 January 2010

Morning

Duration: 1 hour 30 minutes

Candidate Forename						Candidate Surname				
Centre Number						Candidate Number				

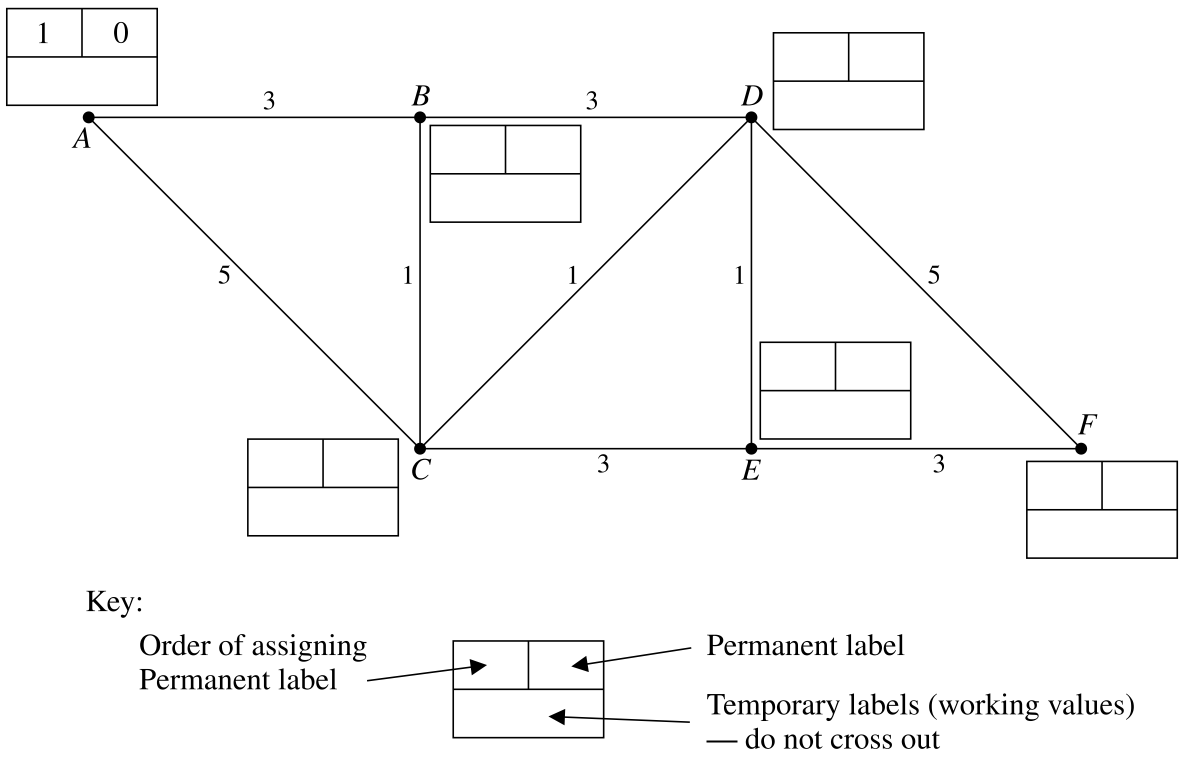
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Questions **1** and **6**.
- Write your answers to Questions **1** and **6** in the spaces provided in this insert, and attach it to your Answer Booklet.

INFORMATION FOR CANDIDATES

- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)**



....

....

Path ....

Weight ....

(ii) ....

....

Weight ....

Route ....

(iii) ....

....

....

6 **(i)** Greatest number of arcs

for a network with five vertices = ...

for a network with n vertices = ...

(ii) **(a)** For a network with five vertices

maximum number of passes = ...

maximum number of comparisons

in the first pass = ...

in the second pass = ...

in the third pass = ...

maximum total number of comparisons = ...

(b) For a network with n vertices

maximum total number of comparisons = ...

....

....

(iii)

M1 Vertices in tree	***M2*** Arcs in tree	***M3*** Vertices not in tree
		A B C D E
D E	*D* 2 *E*	*A B C*
	D 2 *E* ☐ ☐ ☐	
	D 2 *E* ☐ ☐ ☐ ☐ ☐ ☐	
	D 2 *E* ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐	

M4 Sorted list
~~*D* 2 *E*~~
A 3 *E*
A 4 *C*
C 5 *D*
B 6 *E*
B 7 *C*
A 8 *B*
C 9 *E*

(iv) ....

....