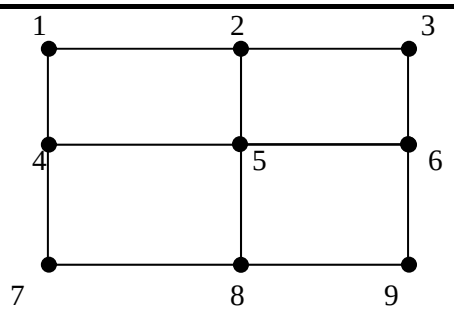
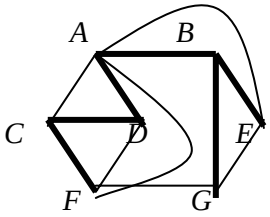
4736 Decision Mathematics 1

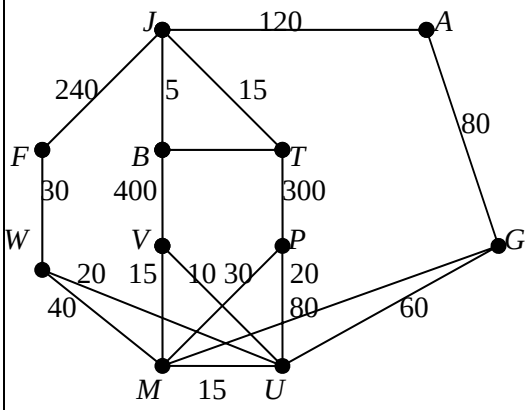
1	**(i)**	5 2 4 3 8 Bin 1: 5 2 3 Bin 2: 4 Bin 3: 8	M1 A1	First bin correct All correct in three bins	[2]
	(ii)	8 5 4 3 2 Bin 1: 8 2 Bin 2: 5 4 Bin 3: 3	M1 A1	First bin correct All correct in three bins	[2]
	(iii)	The heaviest box is originally at the bottom of the stack	B1	Referring to the physical act of sorting the weights into decreasing order	[1]
	(iv)	Bins in any order and boxes in any order Bin 1: 8 or 8 Bin 2: 5 3 5 2 Bin 3: 4 2 4 3	B1	Any valid packing into three bins of capacity 8 kg.	[1]
				Total =	**6**

2	**(i)**	 4 moves	M1 A1 B1	A connected graph with nine vertices labelled 1 to 9 Correct graph Stating 4	[3]
	(ii)	Neither It has four odd nodes The nodes 2, 4, 6, 8 each have three arcs joined to them whereas an Eulerian graph has no odd nodes and a semi-Eulerian graph has exactly two odd nodes	M1 A1	'Neither', together with an attempt at a reason A correct reference to the number of odd nodes for this graph. Be careful about whether 'odd' refers to the parity or the value. However, just defining Eulerian and semi-Eulerian, without reference to this graph, is not enough	[2]
				Total =	**5**

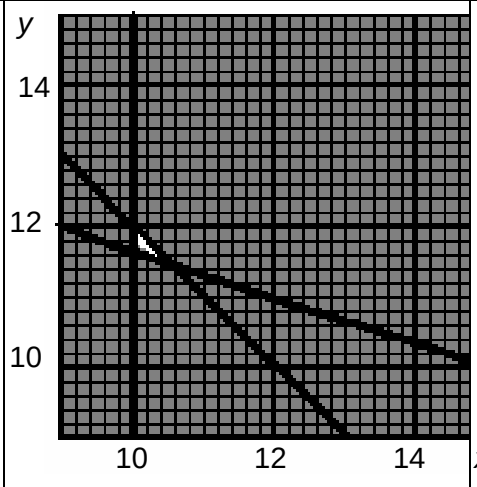
ANSWERED ON INSERT

3	**(i)**	*AD* = 16 *CD* = 18 *CF* = 21 ~~*AC* = 23~~ ~~*DF* = 34~~ *BE* = 35 *BG* = 46 *AB* = 50 ~~*EG* = 55~~ ~~*FG* = 58~~ ~~*AE* = 80~~ ~~*AF* = 100~~  Total weight = 186	M1 A1 M1 A1 B1	Using Kruskal: Not selecting *AC* and *DF* Selecting correct arcs in list, or implied (16+18+21+35+46+50, in this order with no others, can imply M1, A1) Drawing a spanning tree for these six vertices Correct (minimum) spanning tree drawn 186 (cao)	[5]
	(ii)	Delete *BG* from spanning tree 186 – 46 = 140 Two shortest arcs from *G* are *BG* and *EG* 140 + 46 + 55 = 241 Lower bound = 241	 B1 M1 A1	Correct working for wrong vertex deleted can score B1, M1, A0 Weight of MST on reduced network (ft from part (i) Adding two shortest arcs to MST 241 (cao)	[3]
	(iii)	*A–D–C–F–G–* … or 16+18+21+58+ … *A – D – C – F – G – B – E - A* Upper bound = 274	M1 A1 B1	Using nearest neighbour Correct closed tour listed, not just weights added 274 (cao)	[3]
				Total =	**11**

ANSWERED ON INSERT

4 (i)	 Strictly, these are directed arcs, but they are shown as undirected arcs	B1	Times for flying route, *JA* = 120 *AG* = 80 *GU* = 60 *UM* = 15 *GM* = 80	
		B1	Times for train route correct *JT* = 15 *JB* = 5 *BT* = 20 *TP* = 300 *PU* = 20 *PM* = 30	
		B1	Times for coach route and driving route correct *BV* = 400 *VU* = 10 *VM* = 15 *JF* = 240 *FW* = 30 *WU* = 20 *WM* = 40	**[3]**
			Follow through their arc weights if reasonable	
	J 1 | 0 *A* 4 | 120 (120) *F* 6 | 240 (240) *B* 2 | 5 (5) *T* 3 | 15 (15) *V* (405) *P* (315) *W* 8 | (270) *G* 5 | 200 (200) *M* 9 | 275 (280 275) *U* 7 | 260 (260)	M1	Permanent values correct at *A, F, B, T* *A* = 120, *F* = 240, *B* = 5, *T* = 15	
		M1 d	Both 280 and 275 seen at *M* (updating at *M*)	
		A1 ft	All temporary labels correct (or implied) and no extras	
		B1 ft	All permanent labels correct (or implied) (condone labelling past *M*)	
		B1 ft	Order of labelling correct (condone labelling past *M*)	
	Alternatively, if treating as undirected: *J, A, F, B* and *T* are unchanged, then Or *V* = 8th and *W* = 9th *V* 9 | 270 (405 270) *P* (315 280) *W* 8 | 270 (270) *G* 5 | 200 (200) *M* 10 | 275 (280 275) *U* 7 | 260 (260)		Marked as above	
	Route: *J - A - G - U - M*	B1	Correct answer only	**[6]**

(ii)	The quickest journey time from Jenny's house to the meeting venue	B1	Quickest journey / least travel time or equivalent	**[1]**
(iii)	Does not allow for waiting for connections There may be delays at the airport She may not want to fly because of the 'carbon footprint' She may want to choose the cheapest route rather than the quickest route She may not like flying She may want to see her friend She may want to break the journey overnight	B1 B1	Any reasonable suggestion for why she may not want to use the drive/fly/underground route or why she may want to use a different route Any second reasonable suggestion	**[2]**
				Total = 12

5	**(i)**	x = area of wall to be panelled (m^2) y = area to be painted z = area to be covered with pinboard	B1 B1	Reference to area or m^2 (at least once) Identifying x as panelling, y as paint and z as pinboard, in any way	**[2]**
	(ii)	Cost $\leq$ £150 $\Rightarrow 8x + 4y + 10z \leq 150$ $\Rightarrow 4x + 2y + 5z \leq 75$ (given)	B1 B1	Use of word 'cost' or equivalent $8x + 4y + 10z \leq 150$ seen or explicitly referred to	**[2]**
	(iii)	(Minimise P =) $15x + 30y + 20z$	B1 ft	Any positive multiple of this eg $3x + 6y + 4z$ or $\frac{1}{4}x + \frac{1}{2}y + \frac{1}{3}z$	**[1]**
	(iv)	(Minimise P = 480 +) $-5x + 10y$ Subject to $x + 3y \geq 45$ $x \geq 10$ $y \geq 0$ $x + y \leq 22$	B1 ft B1 B1	Any positive multiple of this, eg $2y–x(+c)$ - or maximise a negative multiple Any equivalent simplified form $x \geq 10$ may be implied $y \geq 0$ may be implied $x + y \leq 22$, any equivalent simplified form	**[3]**
	(v)		M1 M1 M1 A1	ANSWERED ON GRAPH PAPER x = 10 drawn accurately with a sensible scale $x + y$ = 22 drawn accurately with a sensible scale Their $x + 3y$ = 45 drawn accurately with a sensible scale Shading correct or identification of the feasible region (triangle with (10, $11\frac{2}{3}$), (10, 12) and ($10\frac{1}{2}$, $11\frac{1}{2}$) as vertices)	**[4]**
					Total = 12

6	**(i)**	<table><tr><td>*P*</td><td>*x*</td><td>*y*</td><td>*z*</td><td>*s*</td><td>*t*</td><td></td></tr><tr><td>1</td><td>-25</td><td>-14</td><td>32</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>6</td><td>-4</td><td>3</td><td>1</td><td>0</td><td>24</td></tr><tr><td>0</td><td>5</td><td>-3</td><td>10</td><td>0</td><td>1</td><td>15</td></tr></table>	B1 B1	Rows and columns may be in any order Objective row with -25, -14, 32 Constraint rows correct (condone omission of *P* column) **[2]**
	(ii)	*x* column has a negative value in objective row Cannot use *y* column since it has negative entries in all the other rows $24 \div 6 = 4$ $15 \div 5 = 3$ Least non-negative ratio is 3, so pivot on 5	B1 B1 B1	'negative in top row', '-25', or similar 'most negative in top row' $\Rightarrow$ bod B1 Correct reason for not choosing *y* column Both divisions seen and correct choice made (or both divisions seen and correct choice implied from pivoting) **[3]**
	(iii)	<table><tr><td>1</td><td>0</td><td>-29</td><td>82</td><td>0</td><td>5</td><td>75</td></tr><tr><td>0</td><td>0</td><td>-0.4</td><td>-9</td><td>1</td><td>-1.2</td><td>6</td></tr><tr><td>0</td><td>1</td><td>-0.6</td><td>2</td><td>0</td><td>0.2</td><td>3</td></tr></table> New row 3 = $\frac{1}{5}$ row 3 New row 1 = row 1 + 25×new row 3 oe New row 2 = row 2 - 6×new row 3 oe $x = 3$, $y = 0$, $z = 0$ $P = 75$	 M1 A1 B1 B1 B1 B1 ft B1 ft	Follow through their sensible tableau (with two slack variable columns) and pivot Pivot row correct (no numerical errors) **[2]** Other rows correct (no numerical errors) Calculation for pivot row Calculation for objective row **[3]** Calculation for other row *x*, *y* and *z* from their tableau *P* from their tableau, provided $P \geq 0$ **[2]**
	(iv)	Problem is unbounded No limit to how big *y* (and hence *P*) can be Only negative in objective row is *y* column, but all entries in this column are negative	B1	Any one of these, or equivalent. If described in terms of pivot choices, must be complete and convincing **[1]**
				Total = 13

		$F = N \div B$ $G = \text{INT}(F)$ $H = B \times G$ $C = N - H$ $N = G$						For reference only
7	**(i)**	*F*	*G*	*H*	*C*	*N*	M1	A reasonable attempt at first pass (presented in any form)
		2.5	2	4	1	2	A1	*F* = 2.5 and *G* = 2
							A1	*H* = 4 (or double their *G* value) and *C* = 5 – their *H*
		1	1	2	0	1	A1	*F*, *G*, *H*, *C* and *N* correct for second pass (ft their *N* value)
		0.5	0	0	1	0	A1	*F*, *G*, *H*, *C* and *N* correct for third pass (ft their *N* value) [5]
	(ii)	*F*	*G*	*H*	*C*	*N*	M1	A reasonable attempt
		-2.5	-3	-6	1	-3	M1 d	First pass correct (or implied)
		-1.5	-2	-4	1	-2		
		-1	-1	-2	0	-1		
		-0.5	-1	-2	1	-1	A1	Reaching two lines with the same value for *G*
		-0.5	-1	-2	1	-1		If described in words only, then M1 for a correct statement; M1 d for all correct statements (sufficient to guarantee result), and A1 for convincingly correct explanation of how they know these to be true and why the result follows
		Does not terminate					B1	Saying 'does not stop', or equivalent [4]
	(iii)	*F*	*G*	*H*	*C*	*N*		
		3.7	3	30	7	3	M1	First pass correct
		0.3	0	0	3	0	A1	All correct
		The first value is the units digit of *N*, the second value is the tens digit, the third value is the hundreds digit, and so on.					M1 A1	Outputs are digits of *N* In reverse order [4]
								Total = 13