OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

Decision Mathematics 1

4771

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- Printed Answer Book (inserted)
- * MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the printed Answer Book.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- This document consists of **8** pages. Any blank pages are indicated.

Answer all questions in the printed answer book provided.

Section A (24 marks)

1 Alfred, Ben, Charles and David meet, and some handshaking takes place.

- Alfred shakes hands with David.
- Ben shakes hands with Charles and David.
- Charles shakes hands with Ben and David.

(i) Complete the bipartite graph in your answer book showing A (Alfred), B (Ben), C (Charles) and D (David), and the number of people each shakes hands with. **[4]**

(ii) Explain why, whatever handshaking takes place, the resulting bipartite graph cannot contain both an arc terminating at 0 and another arc terminating at 3. **[2]**

(iii) Explain why, whatever number of people meet, and whatever handshaking takes place, there must always be two people who shake hands with the same number of people. **[2]**

2 The following algorithm computes the number of comparisons made when Prim's algorithm is applied to a complete network on n vertices ($n > 2$).

Step 1	Input the value of n
Step 2	Let $i = 1$
Step 3	Let $j = n - 2$
Step 4	Let $k = j$
Step 5	Let $i = i + 1$
Step 6	Let $j = j - 1$
Step 7	Let $k = k + (i \times j) + (i - 1)$
Step 8	If $j > 0$ then go to Step 5
Step 9	Print k
Step 10	Stop

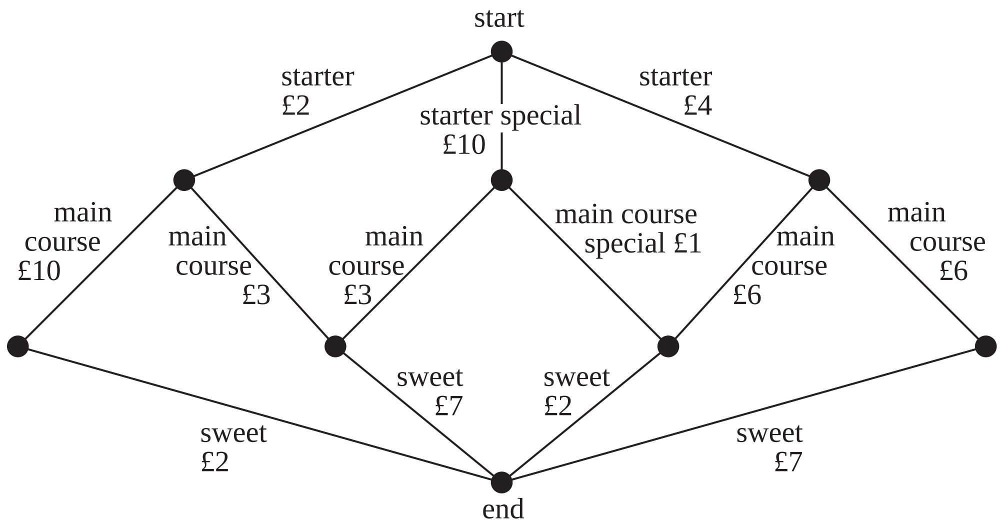
(i) Apply the algorithm when $n = 5$, showing the intermediate values of i, j and k. **[5]**

The function $f(n) = \frac{1}{6}n^3 - \frac{7}{6}n + 1$ gives the same output as the algorithm.

(ii) Showing your working, check that f(5) is the same value as your answer to part **(i)**. **[2]**

(iii) What does the structure of f(n) tell you about Prim's algorithm? **[1]**

3 Whilst waiting for her meal to be served, Alice tries to construct a network to represent the meals offered in the restaurant.



(i) Use Dijkstra's algorithm to find the cheapest route through the undirected network from "start" to "end". Give the cost and describe the route. Use the lettering given on the network in your answer book. **[6]**

(ii) Criticise the model and suggest how it might be improved. **[2]**

Section B (48 marks)

4 A ski-lift gondola can carry 4 people. The weight restriction sign in the gondola says "4 people – 325 kg".

The table models the distribution of weights of people using the gondola.

	Men	Women	Children
Weight (kg)	90	80	40
Probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

(i) Give an efficient rule for using 2-digit random numbers to simulate the weight of a person entering the gondola. **[5]**

(ii) Give a reason for using 2-digit rather than 1-digit random numbers in these circumstances. **[1]**

(iii) Using the random numbers given in your answer book, simulate the weights of four people entering the gondola, and hence give its simulated load. **[3]**

(iv) Using the random numbers given in your answer book, repeat your simulation 9 further times. Hence estimate the probability of the load of a fully-laden gondola exceeding 325 kg. **[6]**

(v) What in reality might affect the pattern of loading of a gondola which is not modelled by your simulation? **[1]**

5 The tasks involved in turning around an "AirGB" aircraft for its return flight are listed in the table. The table gives the durations of the tasks and their immediate predecessors.

	Activity	Duration (mins)	Immediate Predecessors
A	Refuel	30	–
B	Clean cabin	25	–
C	Pre-flight technical check	15	A
D	Load luggage	20	–
E	Load passengers	25	A, B
F	Safety demonstration	5	E
G	Obtain air traffic clearance	10	C
H	Taxi to runway	5	G, D

(i) Draw an activity on arc network for these activities. **[4]**

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. **[6]**

Because of delays on the outbound flight the aircraft has to be turned around within 50 minutes, so as not to lose its air traffic slot for the return journey. There are four tasks on which time can be saved. These, together with associated costs, are listed below.

Task	A	B	D	E
New time (mins)	20	20	15	15
Extra cost	250	50	50	100

(iii) List the activities which need to be speeded up in order to turn the aircraft around within 50 minutes at minimum extra cost. Give the extra cost and the new set of critical activities. **[6]**

6 A company is planning its production of "MPowder" for the next three months.

- Over the next 3 months 20 tonnes must be produced.
- Production quantities must not be decreasing. The amount produced in month 2 cannot be less than the amount produced in month 1, and the amount produced in month 3 cannot be less than the amount produced in month 2.
- No more than 12 tonnes can be produced in total in months 1 and 2.
- Production costs are £2000 per tonne in month 1, £2200 per tonne in month 2 and £2500 per tonne in month 3.

The company planner starts to formulate an LP to find a production plan which minimises the cost of production:

Minimise $2000x_1 + 2200x_2 + 2500x_3$
subject to $x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$
$x_1 + x_2 + x_3 = 20$
$x_1 \leq x_2$
• • •

(i) Explain what the variables x_1, x_2 and x_3 represent, and write down two more constraints to complete the formulation. **[4]**

(ii) Explain how the LP can be reformulated to:

Maximise $500x_1 + 300x_2$
subject to $x_1 \geq 0 \quad x_2 \geq 0$
$x_1 \leq x_2$
$x_1 + 2x_2 \leq 20$
$x_1 + x_2 \leq 12$ **[3]**

(iii) Use a graphical approach to solve the LP in part **(ii)**. Interpret your solution in terms of the company's production plan, and give the minimum cost. **[9]**