OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **2**, **5** and **6**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 7 printed pages, 1 blank page and an insert.

Section A (24 marks)

1 Table 1 shows a precedence table for a project.

Activity	Immediate predecessors	Duration (days)
A	-	5
B	-	3
C	A	3
D	A, B	4
E	A, B	5

Table 1

(i) Draw an activity-on-arc network to represent the precedences. [3]

(ii) Find the early event time and late event time for each vertex of your network, and list the critical activities. [3]

(iii) Extra resources become available which enable the durations of three activities to be reduced, each by up to two days. Which three activities should have their durations reduced so as to minimise the completion time of the project? What will be the new minimum project completion time? [2]

2 Answer this question on the insert provided.

An algorithm is specified in Fig. 2. It operates on two lists of numbers, each sorted into ascending order, to create a third list.

Step 1	Let A equal the first number in List 1. Delete the first number in List 1. Let B equal the first number in List 2. Delete the first number in List 2.
Step 2	If A ⩽ B go to Step 3. Otherwise go to Step 4.
Step 3	Write A down at the end of List 3. If List 1 is not empty let A equal the first number in List 1, delete the first number in List 1 and go to Step 2. If List 1 is empty write down B at the end of List 3 and then copy the numbers in List 2 at the end of List 3. Then stop.
Step 4	Write B down at the end of List 3. If List 2 is not empty let B equal the first number in List 2, delete the first number in List 2 and go to Step 2. If List 2 is empty write down A at the end of List 3 and then copy the numbers in List 1 at the end of List 3. Then stop.

Fig. 2

(i) Complete the table in the insert showing the outcome of applying the algorithm to the following two lists:

List 1: 2, 34, 35, 56

List 2: 13, 22, 34, 81, 90, 92 [4]

(ii) What does the algorithm achieve? [1]

(iii) How many comparisons did you make in applying the algorithm? [1]

(iv) If the number of elements in List 1 is x, and the number of elements in List 2 is y, what is the maximum number of comparisons that will have to be made in applying the algorithm, and what is the minimum number? [2]

3 Fig. 3 shows a graph representing the seven bus journeys run each day between four rural towns. Each directed arc represents a single bus journey.

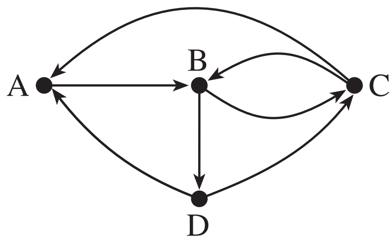

Fig. 3

(i) Show that if there is only one bus, which is in service at all times, then it must start at one town and end at a different town.

Give the start town and the end town. [3]

(ii) Show that there is only one Hamilton cycle in the graph.

Show that, if an extra journey is added from your end town to your start town, then there is still only one Hamilton cycle. [4]

(iii) A tourist is staying in town B. Give a route for her to visit every town by bus, visiting each town only once and returning to B. [1]

Section B (48 marks)

4 Table 4 shows the butter and sugar content in two recipes. The first recipe is for 1 kg of toffee and the second is for 1 kg of fudge.

	Toffee	Fudge
Butter	100 g	150 g
Sugar	800 g	700 g

Table 4

A confectioner has 1.5 kg of butter and 10 kg of sugar available. There are no constraints on the availability of other ingredients.

(i) What is the maximum amount of toffee which the confectioner could make? How much butter or sugar would be left over?

What is the maximum amount of fudge which the confectioner could make? How much butter or sugar would be left over? [4]

(ii) Formulate an LP to find the maximum total amount of toffee and fudge which the confectioner can make.

Solve your LP graphically. [8]

The confectioner charges £5.50 for 1 kg of toffee and £4.50 for 1 kg of fudge.

(iii) What quantities should he make to maximise his income? Justify your answer.

By how much would the price of toffee have to change for the maximum income solution to change? [4]

5 Answer this question on the insert provided.

Table 5 specifies a road network connecting 7 towns, A, B, … , G. The entries in Table 5 give the distances in miles between towns which are connected directly by roads.

	A	B	C	D	E	F	G
A	–	10	–	–	–	12	15
B	10	–	15	20	–	–	8
C	–	15	–	7	–	–	11
D	–	20	7	–	20	–	13
E	–	–	–	20	–	17	9
F	12	–	–	–	17	–	13
G	15	8	11	13	9	13	–

Table 5

(i) Using the copy of Table 5 in the insert, apply the tabular form of Prim's algorithm to the network, starting at vertex A. Show the order in which you connect the vertices.

Draw the resulting tree, give its total length and describe a practical application. [7]

(ii) The network in the insert shows the information in Table 5. Apply Dijkstra's algorithm to find the shortest route from A to E.

Give your route and its length. [6]

(iii) A tunnel is built through a hill between A and B, shortening the distance between A and B to 6 miles. How does this affect your answers to parts **(i)** and **(ii)**? [3]

6 Answer part (iv) of this question on the insert provided.

There are two types of customer who use the shop at a service station. 70% buy fuel, the other 30% do not. There is only one till in operation.

(i) Give an efficient rule for using one-digit random numbers to simulate the type of customer arriving at the service station. [1]

Table 6.1 shows the distribution of time taken at the till by customers who are buying fuel.

Time taken (mins)	1	1.5	2	2.5
Probability	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

Table 6.1

(ii) Specify an efficient rule for using one-digit random numbers to simulate the time taken at the till by customers purchasing fuel. [2]

Table 6.2 shows the distribution of time taken at the till by customers who are not buying fuel.

Time taken (mins)	1	1.5	2	2.5	3
Probability	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Table 6.2

(iii) Specify an efficient rule for using two-digit random numbers to simulate the time taken at the till by customers not buying fuel.

What is the advantage in using two-digit random numbers instead of one-digit random numbers in this part of the question? [4]

The table in the insert shows a partially completed simulation study of 10 customers arriving at the till.

(iv) Complete the table using the random numbers which are provided. [7]

(v) Calculate the mean total time spent queuing and paying. [2]