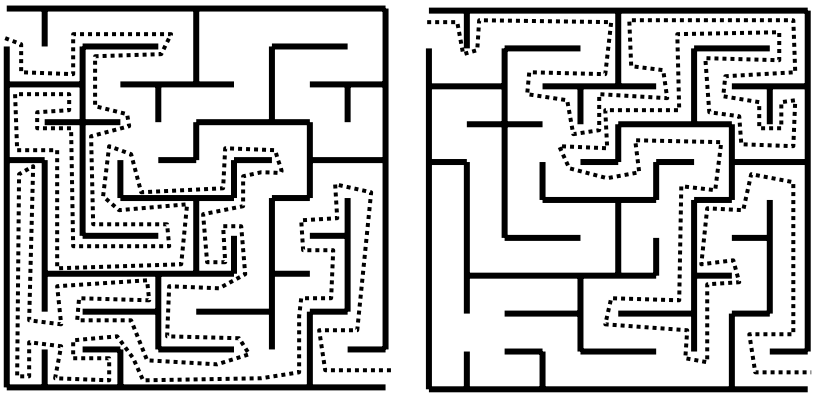
Mark Scheme 4771
June 2005

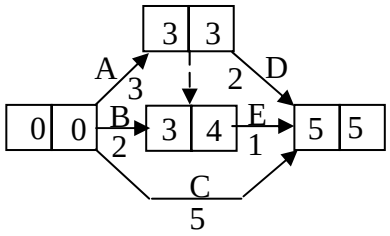
1.

(i)	Any connected tree.	M1 A1
	12 connections	B1
(ii)	14 connections	B1
(iii)	e.g. He might be able to save cable by using it. e.g. To avoid overloading.	B1
(iv)	Yes. A minimum connector is a tree. This gives the min number of arcs (n–1). This gives the minimum no of connections (2(n–1)).	B1 B1 B1

2.

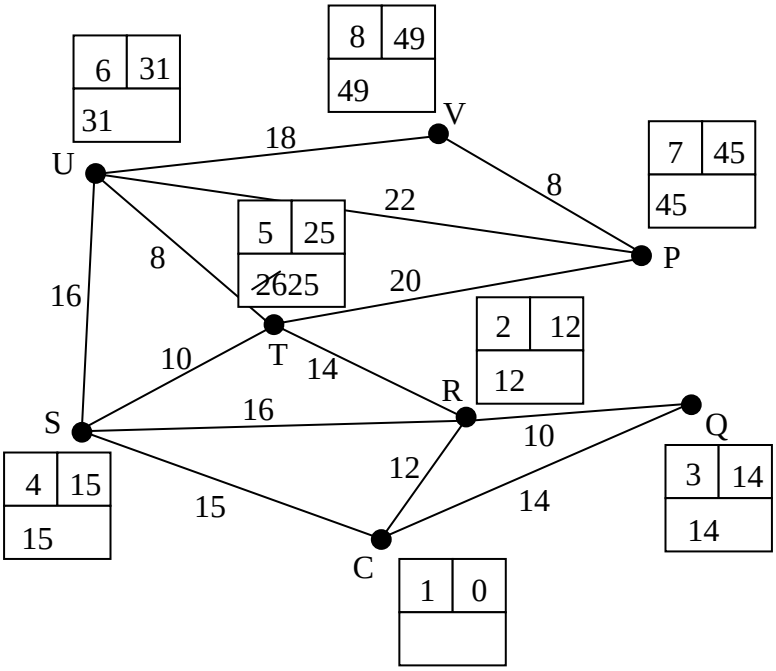
(i)	Janet John 	M1 A1 A1
(ii)	Yes Janet's route traces west and south walls plus "attachments". John's route traces north and east walls plus "attachments". − or equivalent (Any "islands" are irrelevant.)	M1 A1 B1
(iii)	Yes	B1
(iv)	Yes All avenues covered by forward and backward pass (i.e. by John's original route + Janet's route).	B1

3.

(i)		M1 A1 B1 B1
(ii)	Critical − A, D and C	B1
(iii)	Total float for B = 2 Independent float for B = 1 Total float for E = 1 Independent float for E = 0	B1 both total floats A1 B's independent A1 E's independent

4.

(i)



B1 starting at C

M1 Dijkstra
A1 labels
A1 order of labelling
A1 working values

P	Q	R	S	T	U	V
45	14	12	15	25	31	49

B1

P T S C
V U S C

B1
B1

(ii) PV ST CR RT UV Length = 80
TU QR

M1
A1 first 5
A1 last 2
B1 length

(iii) CP reduced to 26
CV reduced to 34

B1 (both and no more)

(iv) UV replaced by PQ New length = 74

B1

(v) Q
Semi-Eulerian. (Order of P changed from 3 to 4, but order of Q changed from 2 to 3 − so still 2 odd vertices.)
or Cross the bridge and proceed as before
or A valid route

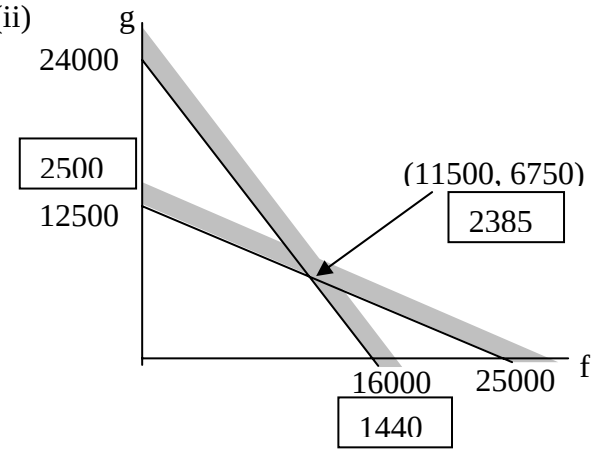
M1

A1

5.

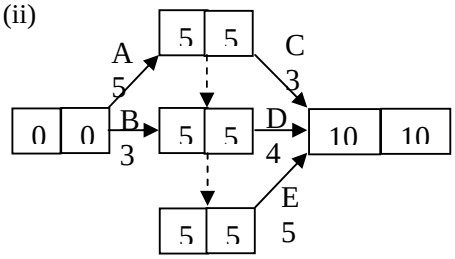
(i)	eg. 00–19 → 0 20–49 → 1 50–69 → 2 70–84 → 3 85–99 → 4	 M1 sca at proportions A1
(ii)	1, 0, 2, 3, 1, 3, 4, 3, 0, 0	M1 A1
(iii)	eg. 00–15 → 0 16–39 → 1 40–63 → 2 64–95 → 3 96–99 → ignore	 M1 missing some A1 times
(iv)	1, 0, 1, 0, 1, 1, 3, 3, 2, 2	B1 one ignored B1 rest
(v)	Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 0 0 0 0 2 4 Disptd 0 0 0 0 1 0 2 1 0 0 0	M1 A1 A1
(vi)	Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 1 0 0 1 3 5 Disptd 0 0 0 0 0 0 0 1 0 0 0	M1 using both ret dists A1 A1
	Only 1 disappointed under new policy against 4 under old policy. Not definitely, but pretty convincingly.	B1 B1

6.

(i)	Let f be the number of litres of Flowerbase produced Let g be the number of litres of Growmuch produced	B1
	Max 9f + 20g s.t. 0.75f + 0.5g ≤ 12000 f + 2g ≤ 25000	M1 A1 M1 A1 A1
(ii)		B1 labels + scales B1 B1 lines B1 shading
	Max profit = £2500 by producing 12500 litres of Growmuch	M1 A1
(iii)	No effect	B1
(iv)	No effect The profit on Flowerbase will be reduced by more than that suffered by Growmuch, since it uses more fibre. The objective gradient will thus increase from −9/20, making it even less attractive to produce any Flowerbase.	M1 A1
(v)	£3000	B1

Mark Scheme 4771
January 2006

1.

(i) & (ii)  	B1 C OK B1 D OK B1 E OK
Critical: A, E	M1 early and late A1 times B1 critical
(iii) A, E and D 6 days	B1 B1

2.

(i)

Step number	List 1	List 2	A	B	List 3
	2, 34, 35, 56	13, 22, 34, 81, 90, 92			
1	34, 35, 56	22, 34, 81, 90, 92	2	13	
3	35, 56	22, 34, 81, 90, 92	34	13	2
4	35, 56	34, 81, 90, 92	34	22	2, 13
4	35, 56	81, 90, 92	34	34	2, 13, 22
3	56	81, 90, 92	35	34	2, 13, 22, 34
4	56	90, 92	35	81	2, 13, 22, 34, 34
3		90, 92	56	81	2, 13, 22, 34, 34, 35
3		90, 92	56	81	2, 13, 22, 34, 34, 35, 56, 81, 90, 92

M1 sca
A1 to first step 3 inc.
A1 to second step 3
A1 rest

(ii) Merges ordered lists to give an ordered list — B1

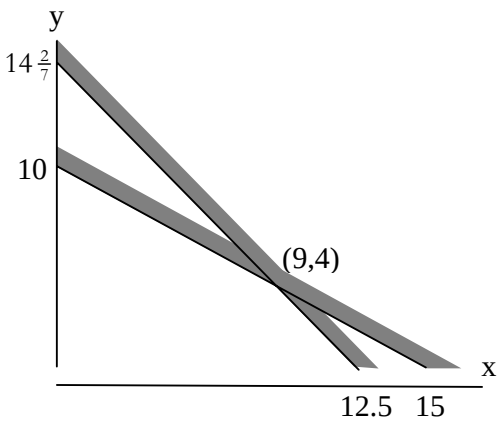
(iii) 7 — B1

(iv) Max = x + y – 1 — B1
Min = min (x, y) — B1

3.

(i)	Ins and outs One more out than in at D. Vice-versa at A. Start at D and end at A	M1 A1 B1
(ii)	Existence – A B D C A Uniqueness – Only alternative is A B C …!!! Extra arc – New possibility A D C B … !!!	B1 M1 A1 A1
(iii)	B D C A B	B1

4.

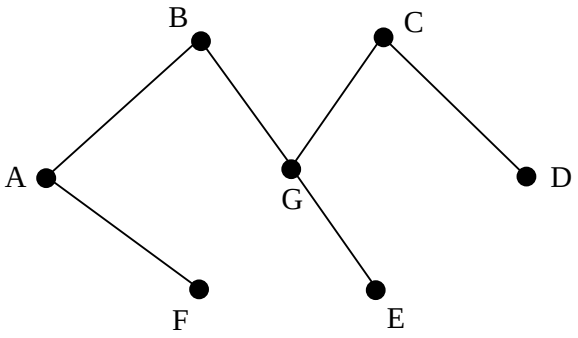
(i)	12.5 kg 250 g (of butter) 10 kg 3 kg (of sugar)	B1 B1 B1 B1
(ii)	Identification of variables e.g. Let x = kg of toffee made Let y = kg of fudge made	
	Max x + y st $100x + 150y \leq 1500$ $800x + 700y \leq 10000$	B1 B1 B1
		B1 axes labelled and scaled B1 butter line B1 sugar line B1 shading
	Make 9 kg toffee and 4 kg fudge	B1 max x+y + solution
(iii)	12.5 kg of toffee and no fudge – either by comparing 68.75 with 67.50 with 45, or by a gradient argument	M1 A1
	Toffee price must <u>decrease</u> by £0.36, or to £5.14.	B1 B1

5.

(i)

	1	2	5	6	4	7	3
	A	B	C	D	E	F	G
~~A~~	~~–~~	~~10~~	~~–~~	~~–~~	~~–~~	~~12~~	~~15~~
~~B~~	10	~~–~~	~~15~~	~~20~~	~~–~~	~~–~~	~~8~~
~~C~~	~~–~~	~~15~~	~~–~~	~~7~~	~~–~~	~~–~~	11
~~D~~	~~–~~	~~20~~	7	~~–~~	~~20~~	~~–~~	~~13~~
~~E~~	~~–~~	~~–~~	~~–~~	~~20~~	~~–~~	~~17~~	9
F	12	–	–	–	17	–	13
~~G~~	~~15~~	8	~~11~~	~~13~~	~~9~~	~~13~~	~~–~~

M1
A1 selections
A1 order of selecting
A1 deletions

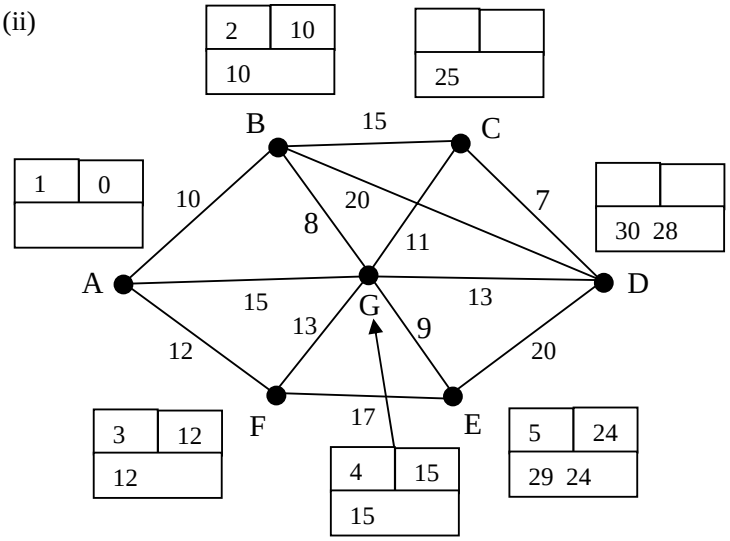


B1

Total length = 57 miles
Might be used to determine where to lay pipes or cables to connect the towns.

B1
B1

(ii)



M1 sca Dijkstra
A1 labels
A1 order of labelling
A1 working values

Shortest route: AGE
Length: 24

(iii) Shortens mst to 53 miles (√ by 4)
New shortest route ABGE – 23 miles (√ by 1)

B1
B1

B1
B1 B1

6.

(i) e.g. 0 – 6 petrol
7 – 9 other

B1

(ii) e.g. 0 – 2 1 min
3 – 6 1.5 mins
7 – 8 2 mins
9 2.5 mins

M1
A1

(iii) e.g. 00 – 13 1 min
14 – 41 1.5 mins
42 – 69 2 mins
70 – 83 2.5 mins
84 – 97 3 mins
98, 99 reject

M1 some rejected
A1 2 rejected
A1

Two digits – fewer rejects

B1

(iv)

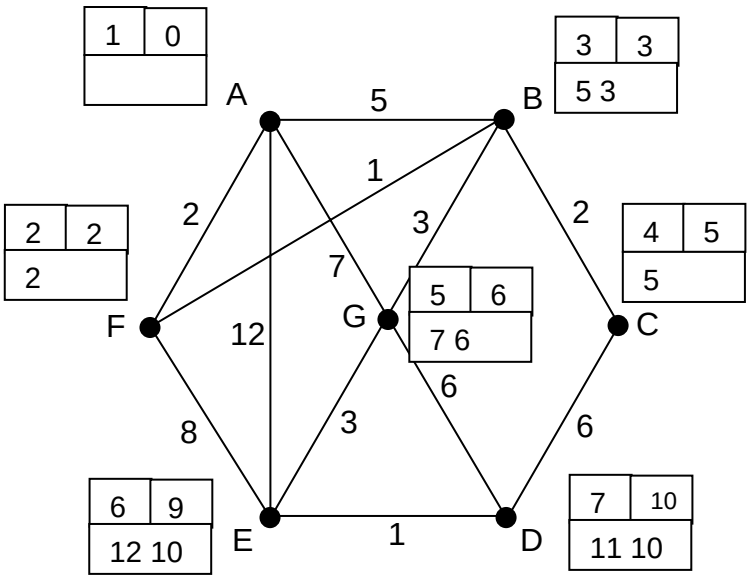
Customer number	Inter-arrival time	Arrival time	Type of customer	Arrival at till	Time at till	Departure time	Queuing + paying
1	1	1	F	1	1	2	1
2	0.5	1.5	N	2	2	4	2.5
3	3.5	5	N	5	1.5	6.5	1.5
4	3	8	F	8	1.5	9.5	1.5
5	1	9	F	9.5	1	10.5	1.5
6	0.5	9.5	F	10.5	1	11.5	2
7	1.5	11	F	11.5	2.5	14	3
8	2	13	N	14	2.5	16.5	3.5
9	2	15	F	16.5	2	18.5	3.5

B1 arrival times
M1 types
M1 service start
M1 service duration
M1 service end
M1 time in shop
A1

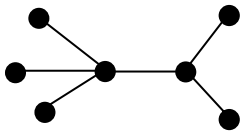
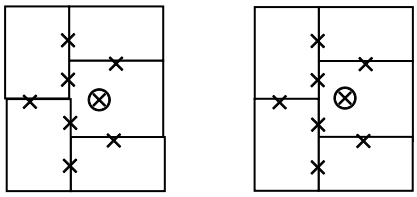
(v) 24.5/10 = 2.45 mins

M1 A1

Mark Scheme 4771
June 2006

(i)  Least weight route: A F B G E D Weight = 10 (ii) 11 From working value. Can't be bettered since new least weight must be bigger than 10.	M1 sca Dijkstra A1 labels A1 order of labelling A1 working values B1 B1 B1 B1

2.

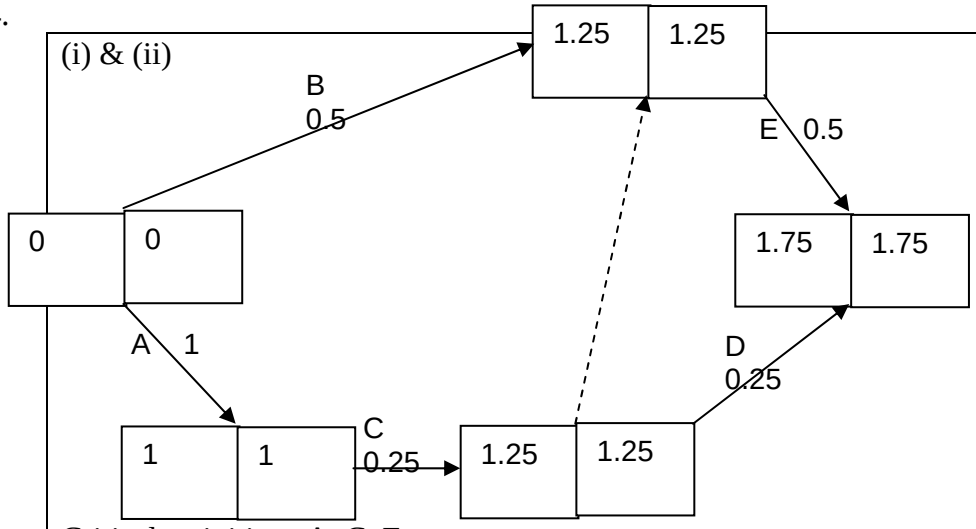
(i) e.g.  a tree (ii) 13 (iii) 14 (iv) e.g. 	M1 A1 B1 B1 B1 M1 A1 A1

3.

(i)	M = 1	B1
	f(M) = –1	B1
	L = 1	B1
	M = 1.5	B1
	f(M) = 0.25	B1
	R = 1.5	B1
(ii)	Solves equations (Allow "Finds root 2".)	B1
(iii)	A termination condition	B1

4.

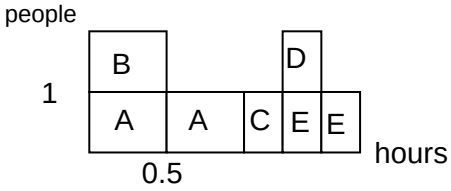
(i) & (ii)



Critical activities: A, C, E

M1 sca activity-on-arc
A1 A, B, C
A1 D
A1 E
B1 forward pass (1.25 at end of B/dummy)
B1 backward pass (1.25 at start of dummy/D)
B1

(iii)



M1
A1

M1
A1

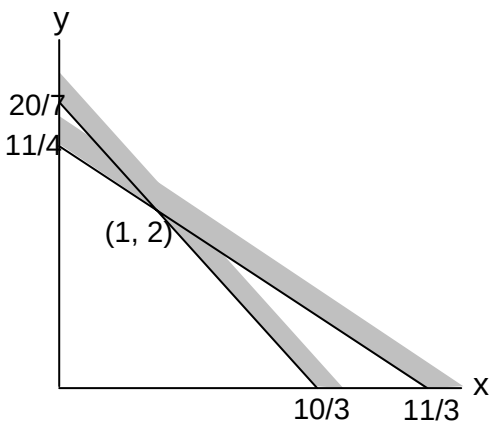
(iv) 2 hours (resource smoothing on A/B, but extra time needed for D/E).

(v)

P	–
Q	–
R	–
S	Q, R
T	Q, R
U	R
V	S, T, U
W	U

B1
B1
B1
B1
B1

5.

(i) Let x be the number of hours spent at badminton Let y be the number of hours spent at squash	B1
$3x + 4y \leq 11$ $1.5x + 1.75y \leq 5$	B1 B1
(ii) 	B1 axes labelled and scaled B1 line B1 line B1 shading B1 intercepts B1 (1, 2)
(iii) $x + 2y$	B1
(iv) $22/4 > 5 > 10/3$, so 5.5 at (0, 11/4)	M1 A1
(v) Squash courts sold in whole hours 1 hour badminton and 2 hours squash per week	B1 B1
(vi) 3 hours of badminton and no squash	B1 B1

6.

(i) year 1: 00 – 09 failure, otherwise no failure — M1 A1
year 2: 00 – 04
year 3: 00 – 01
year 4: 00 – 19 — A1
year 5: 00 – 19
year 6: 00 – 29

(ii)(A)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
year 1	√	√	√	√	x	√	√	x	√	√
year 2	√	√	√	√		√	√		√	√
year 3	√	√	√	√		√	√		√	√
year 4	√	√	√	x		√	√		x	√
		√	√			√	√			√

M1 ticks and crosses
A1 run 1
A1 runs 2–4
A1 runs 5–7
B1 runs 8–10

(B) 0.6 — B1

(iii)
(A) if no failure then continue after year 3 – but using rules for yrs 1 to 3 — B1 B1

(B)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
year 1	√	√	√	√	x	√	√	x	√	√
year 2	√	√	√	√		√	√		√	√
year 3	√	√	√	√		√	√		√	√
year 4	√	√	√	√		√	√		x	√
	√	√	√	√		√	√			√

M1
A1 runs 1–5
A1 runs 6–10

(C) 0.3 — B1

(iv) more repetitions — B1

D1 June '06

6(ii) (A)

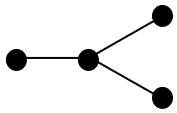
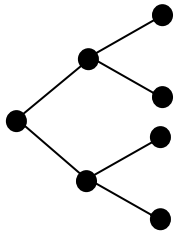
	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
Year 1	✓	✓	✓	✓	×	✓	✓	×	✓	✓
Year 2	✓	✓	✓	✓		✓	✓		✓	✓
Year 3	✓	✓	✓	✓		✓	✓		✓	✓
Year 4	✓	✓	✓	×		✓	✓		×	✓
Year 5	×	✓	✓			✓	✓			✓
Year 6		✓	×			✓	✓			✓

6(iii) (B)

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
Year 1	✓	✓	✓	✓	×	✓	✓	×	✓	✓
Year 2	✓	✓	✓	✓		✓	✓		✓	✓
Year 3	✓	✓	✓	✓		✓	✓		✓	✓
Year 4	✓	✓	✓	✓		✓	✓		×	✓
Year 5	✓	✓	✓	✓		✓	✓			✓
Year 6	✓	✓	✓	✓		✓	✓			✓

Mark Scheme 4771
January 2007

1.

(i) 	B1
(ii) Any two of 1 or 2 or 3 or 5 or 7	B1 B1
(iii) 	M1 branching tree A1
(iv) 	M1 branching tree A1
(v) A tree	B1

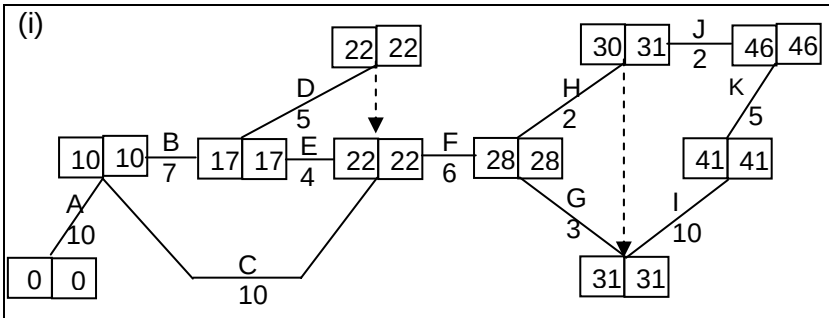
2.

(i) 109; 32; 3; 523; 58 32; 3; 109; 58; 523 4 comparisons and 3 swaps 3; 32; 58; 109; 523 3 and 2 3; 32; 58; 109; 523 2 and 0 3; 32; 58; 109; 523 1 and 0 10 and 5 in total	M1 A1 only if all iterations completed B1 B1
(ii) 523; 109; 58; 32; 3 10 swaps	B1 B1
(iii) 1.5×100^2 = 15000 seconds = 4 hrs 10 mins	M1 A1 hours and minutes

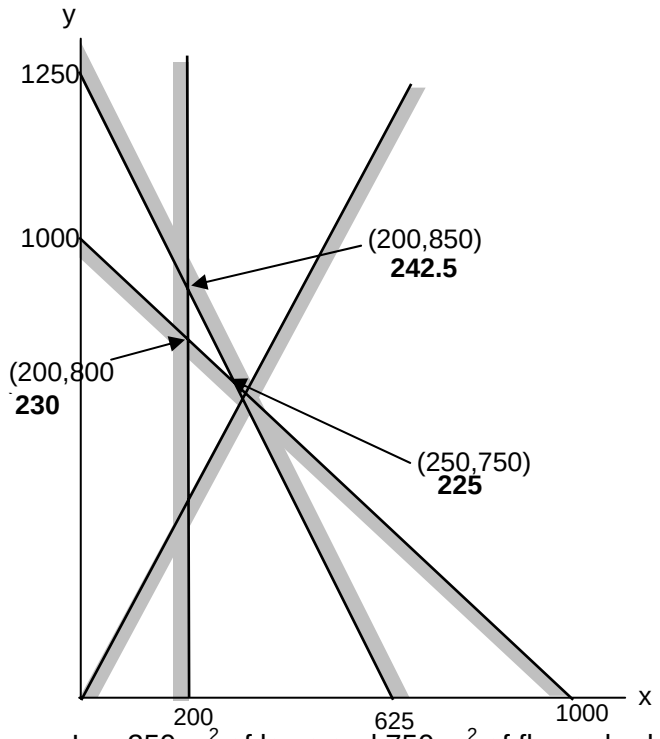
3.

(i) e.g. 0, 1 → A 2, 3 → B 4, 5 → C 6, 7 → D 8, 9 → E	M1 A1 proportions OK B1 efficient
(ii) e.g: 3, 4, 4, 4, 1	M1 A1
(iii) In the above simulation mean = 3.2 (Correct expectation is 2.5 – geometric rand variable)	M1 A1
(iv) More repetitions	B1

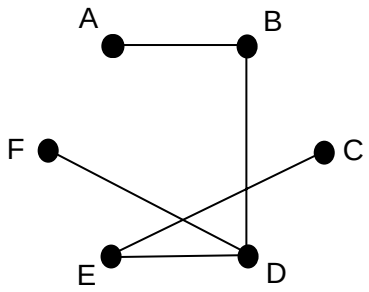
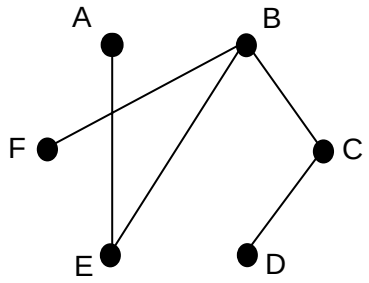
4.

(i)		M1 activity-on-arc A1 single start and end A1 dummy 1 A1 dummy 2 A1 rest
(ii)	See above Critical activities: A; B; D; F; G; I; K Duration = 46	M1 A1 forward pass M1 A1 backward pass B1 critical activities B1 duration
(iii)	E: total float = 1; independent float = 1 H: 1 and 0 J: 14 and 13 C: 2 and 2	B1 total floats B1 independent floats
(iv)	Tiler (I) – 2 days – £500 Electrician (D) – 1 day – £300 Bricklayer (B) – 1 day – £350	B1 tiler B1 electrician B1 bricklayer

5.

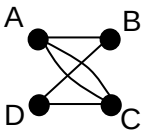
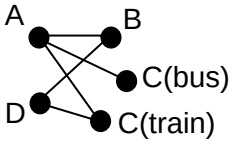
(i) Let x be the number of m^2 of lawn. Let y be the number of m^2 of flower beds.	B1
$x + y \geq 1000$	B1
$0.80x + 0.40y \leq 500$, i.e. $2x + y \leq 1250$	B1
$y \geq 2x$	B1
$x \geq 200$	B1
Minimise $0.15x + 0.25y$	B1 B1
(ii) & (iii) 	B1 axes labelled + scaled B4 lines B1 shading
Lay 250 m^2 of lawn and 750 m^2 of flower beds. Annual maintenance = £225.	M1 A1
(iv) Intersection of $y \geq 2x$ & area constraint is at (333.33,666.67) so max useful capital is £533.33. So £33.33.	B1 (allow £533.33)

6.

(i) DtoE; BtoD; CtoE; DtoF; AtoB	M1 A1 no BC nor BE A1
	B1
Total length = 20	B1
(iii) e.g.	
1 3 4 6 2 5 A B C D E F A – – – – 12 – B – – 5 – 6 6 C – 5 – 8 – 7 D – – 8 – – – E 12 6 – – – 7 F – 6 7 – 7 –	B1 reduced table M1 delete/select/delete A1 first 2 rows A1 rest of table A1 order
	B1
Total length = 37	B1
(iii) Lengths are 27 and 28. Shorter and more nearly equal.	B1 B1 B1 B1

Mark Scheme 4771
June 2007

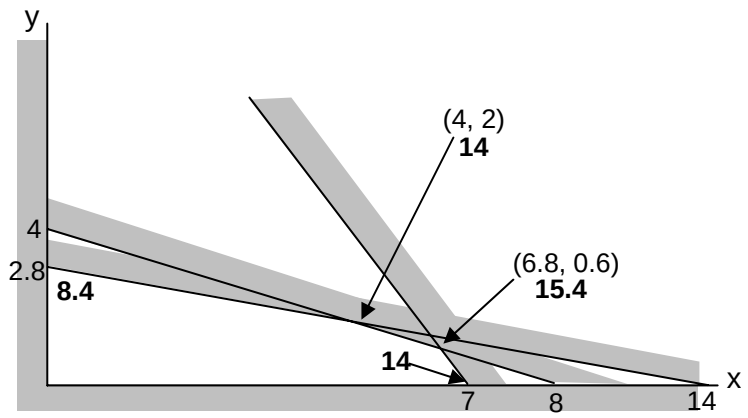
1.

(i)		M1 4 nodes and 5 arcs A1
(ii)	No. Two arcs AC.	M1 A1
(iii)		M1 5 nodes and 5 arcs A1
(iv)	No. ABDC(train)A is a cycle.	M1 A1

2.

(i)	Rucksack 1: 14; 6 Rucksack 2: 11; 9 final item will not fit.	M1 6 must be in R1 A1 B1
(ii)	Order: 14, 11, 9, 6, 6 Rucksack 1: 14; 11 Rucksack 2: 9; 6; 6	B1 ordering M1 11 in R1 A1
(iii)	Rucksack 1: 14; 9 Rucksack 2: 11; 6; 6 e.g. weights.	B1 B1

3.



Optimum of 15.4 at x = 6.8 and y = 0.6.

B1 axes scaled & used
M1 lines
A1
B1 shading
M1 two intersection
A1 points
M1 solution
A1
(or by using the objective gradient to identify the optimal point)

4.

(i)

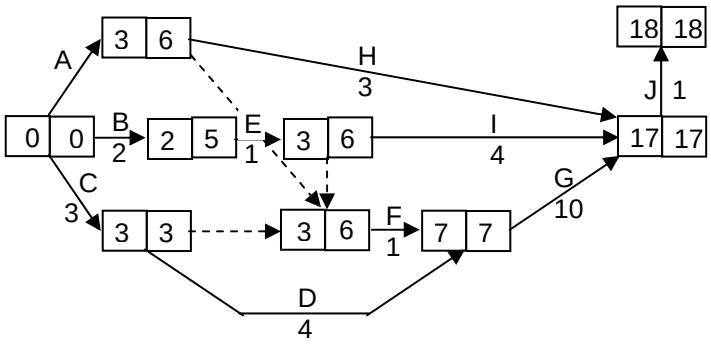
Activity		Duration (minutes)	Immediate predecessors
A	Rig foresail	3	–
B	Lower sprayhood	2	–
C	Start engine	3	–
D	Pump out bilges	4	C
E	Rig mainsail	1	B
F	Cast off mooring ropes	1	A, C, E
G	Motor out of harbour	10	D, F
H	Raise foresail	3	A
I	Raise mainsail	4	E
J	Stop engine and start sailing	1	G, H, I

B1 A, B, C, D, E, H & I

B1 F

B1 G and J

(ii)



M1 A1 forward pass

M1 A1 backward pass

Critical activities: C; D; G; J — B1

Project duration: 18 minutes — B1

(iii) H and I — B1

(iv) 25 mins — B1

Must do A, B, E, C, F, D (in appropriate order) then H and I with G, then J. — B1

(v) 18 mins — B1

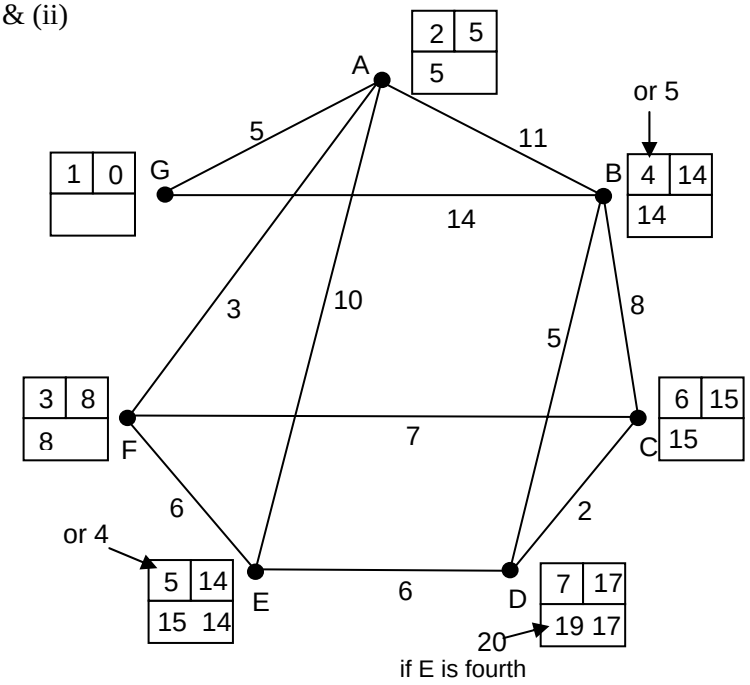
e.g. Colin does C, D — B1

Crew does A, B, E, F — B1

Thence G et al — B1

5.

(i) & (ii)



M1
A1 arcs
A1 arc weights

M1 Dijkstra
A1 labels
A1 order of labelling
A2 working values

Route: G A F C D Weight: 17

B1 B1

(iii) Route: G B C F E D or G B A E D Weight: 6

B1 B1

Any capacitated route application.

B1

(iv) Compute min(label, arc) and update working value if result is larger than current working value.

B1 B1

Label unlabelled vertex with largest working value.

B1

6.

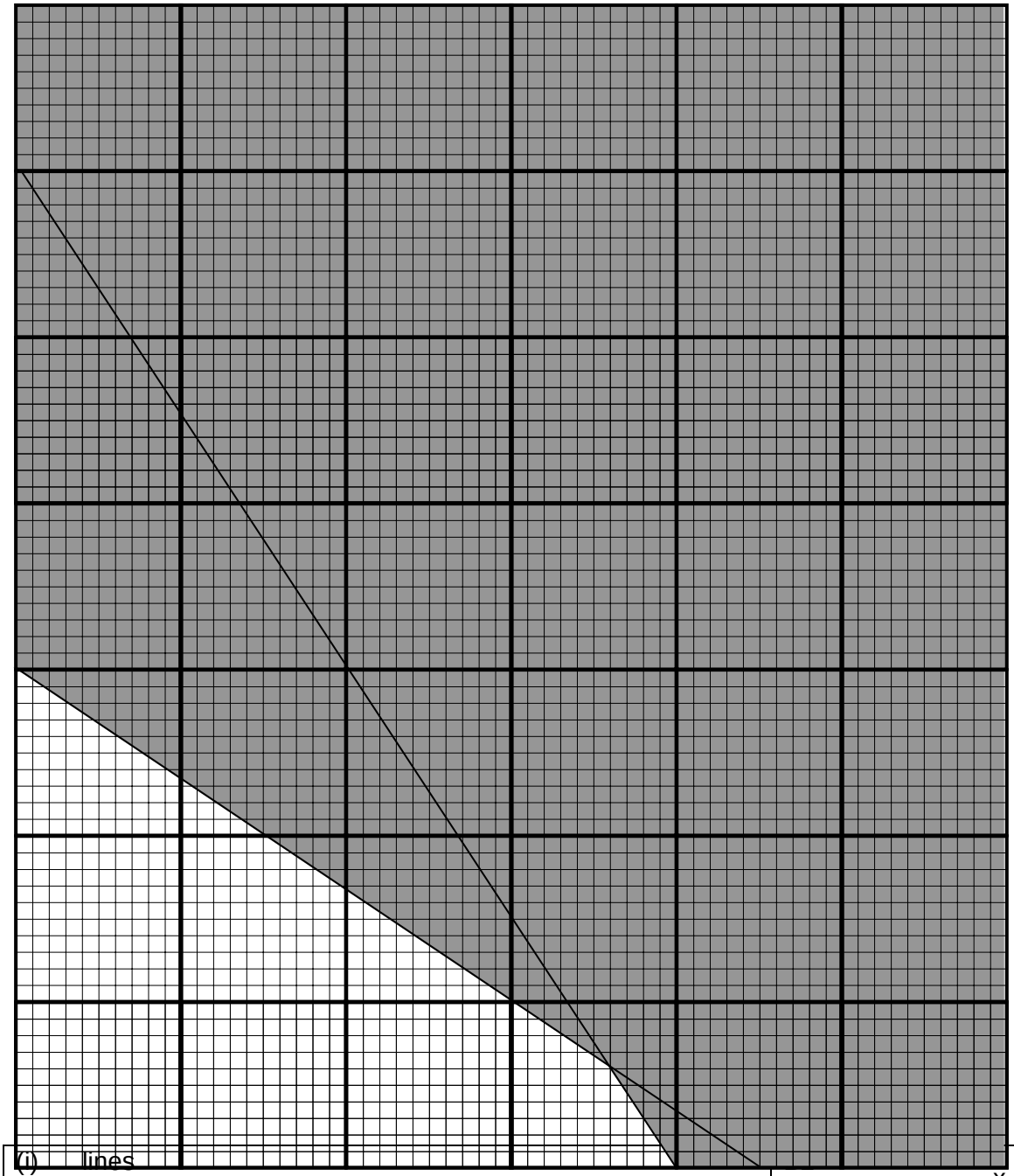
(i)(a) e.g. Dry: 00 – 39 Wet: 40 – 69 Snowy: 70 – 99	M1 A1	proportions efficient
(b) e.g. Dry: 00 – 19 Wet: 20 – 69 Snowy: 70 – 99	M1 A1	proportions efficient
(c) e.g. Dry: 00 – 27 Wet: 28 – 55 Snowy: 56 – 97 Reject: 98 & 99	M1 A1 A1	reject some proportions reject 2
(ii) D (today) → D → S → S → W → S → D → D	M1 A1 A1 A1	applying their rules sometimes dry rules wet rules snowy rules
(iii) 3/7 (or 4/8)	B1	
(iv) a (much) longer simulation run, with a "settling in" period ignored.	B1 B1	
(v) Defining days as dry, wet or snowy is problematical. Assuming that the transition probabilities remain constant. Weather depends on more than just previous day's weather	B1 B1	

4771 Decision Mathematics 1

1

(i)	6 routes M→A→I→T→Pi→C M→A→I→T→Pi→R→C M→A→I→T→Pi→H→R→C M→V→I→T→Pi→C M→V→I→T→Pi→R→C M→V→I→T→Pi→H→R→C	B1 B1
(ii)	6 routes M→A→I→Pa→Pi→C M→A→I→Pa→Pi→R→C M→A→I→Pa→Pi→H→R→C M→V→I→Pa→Pi→C M→V→I→Pa→Pi→R→C M→V→I→Pa→Pi→H→R→C	B1 B1
(iii)	M→V→I→Pa→Pi→H→R→Pi→C A R→H	B1
(iv)	e.g. P→T→I→V→M→A→I→Pa→P→H→R→C→P→R	M1 ends at R A2 (–1 each error/omission)

2. y



x

(i)	lines	
	shading	B1
	(3.6, 0.6)	B1 graph or sim. eqns
	25.8 at (3.6, 0.6) versus 21 and 24 (or profit line)	M1 A1
(ii)	25 at (3, 1)	B1 B1

3.

y = 2008 c = 2008/100 = 20 n = 2008 – 19 x (2008/19) = 2008 – 19 x (105) = 13	
k = 3/25 = 0	B1
i = 20 – 5 – 20 / 3 + 19 × 13 + 15 = 271	B1
i = 1	
i = 1 – 0 = 1	B1
j = 2008 + 502 + 1 + 2 – 20 + 5 = 2498	
j = 6	B1
p = – 5	B1
m = 3	B1
d = 23	B1
So 23rd March	B1

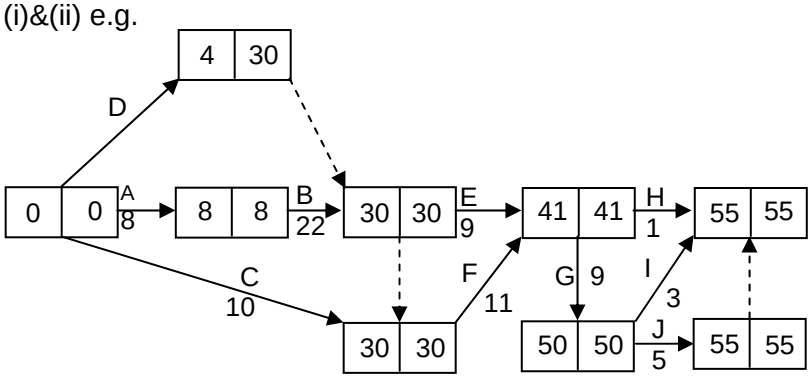
4.

(i) e.g. 0–3→brown 4–7→blue 8–9→green	M1 A1 proportions OK A1 efficient
(ii) e.g. 0–1→brown 2–5→blue 6–7→green 8–9→reject	M1 some rejected A2 proportions OK (–1 each error) A1 efficient
(iii) e.g.	
Eye colours	B1 br/br→br (4 times) B1 br/gr→bl B1 gr/gr→gr
	M1 br/bl rule A1 application A1 application
	B1 bl/bl application
	M1 gr/bl rule A1 application

Parent 1	brown	brown	brown	blue	
Parent 2	brown	blue	brown	blue	
Offspring	brown	brown	brown	brown	

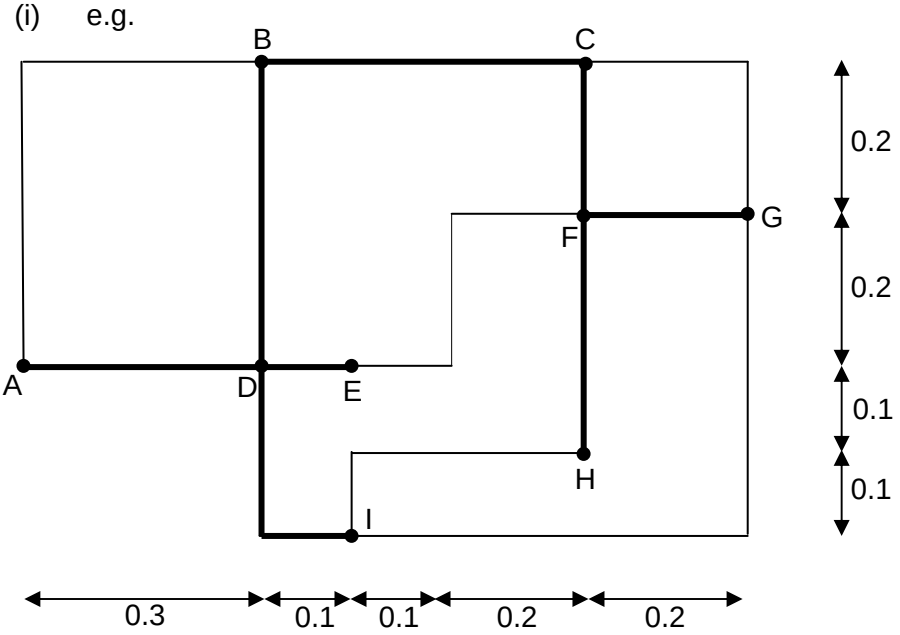
brown	green	blue	green	brown	brown
brown	blue	brown	green	brown	green
brown	blue	brown	green	brown	blue

5.

(i)&(ii) e.g.  	M1 sca (activity on arc) A1 dummy activities + E and F A1 A, B, C, D A1 G, H, I, J M1 forward pass A1 M1 backward pass A1
time – 55 weeks critical – A; B; F; G; J	B1 cao B1 cao
(iii) 50 weeks (49 weeks if G crashed rather than H)	B1
(iv) E – 1 week F – 3 weeks J – 2 weeks (G − 1 week, if crashed)	M1 A3
(v) £115000 (£121000)	A1

6.

(i) e.g.



Total length = 2.2 km

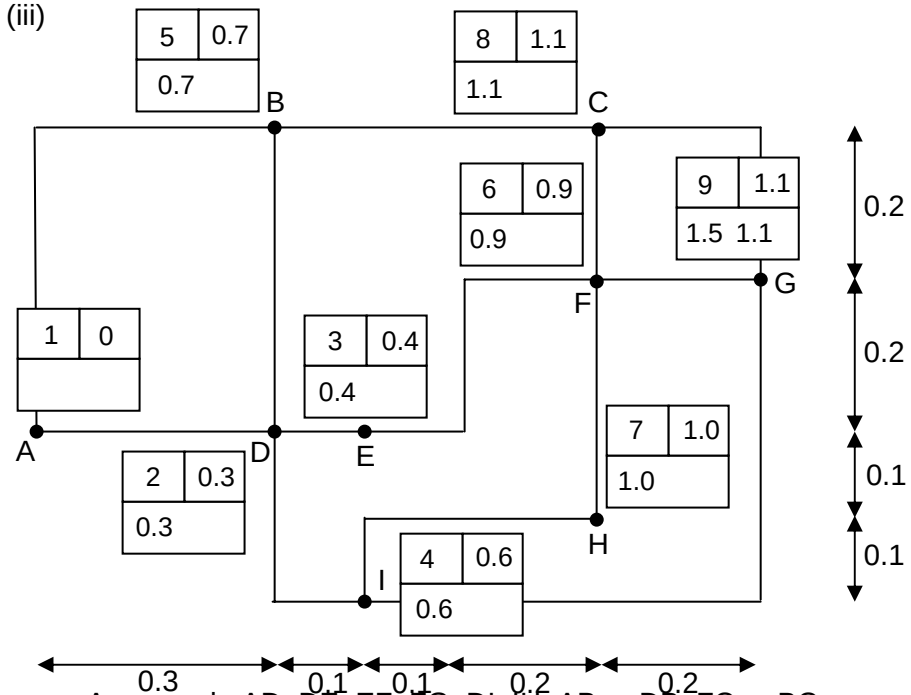
M1 A1

M1 connecting tree
A1 DE
A1 FC, FG
A1 AD, DI, FH
A1 2 of length 0.4

(ii) Prim: connect in nearest to connected set
Kruskal: Shortest arc s.t. no cycles

M1 name
A1 description

(iii)



Arcs used: AD, DE, EF, FG, DI, IH, AB or DB, FC or BC
Total length = 2.7 km (AB&FC) or 2.9 km (AB&BC) or 2.4 km (DB&FC) or 2.6 km (DB&BC)

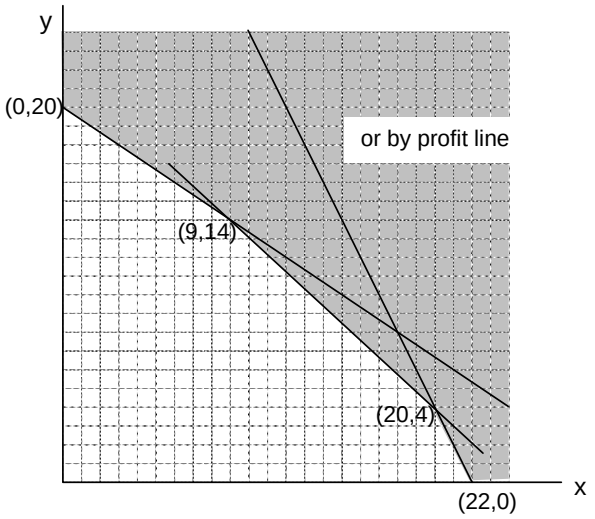
M1 Dijkstra
A1 working values (see vertex G)
A1 order of labelling
A1 labels

M1 arcs counted
A1 only once
A1

4771 Decision Mathematics 1

Solutions

1.



Objective has maximum value of 24 at (20,4)

M1 A1	third line
B1	shading
B1	(0,20) and (22,0)
B1	(9,14)
B1	(20,4)
M1 A1	solution

or M1 A1 B1,
B1 scale (implied OK),
B1 profit line, B1 (20,4)
M1 A1 (20,4) A1 (24)

2.

(i)

	X	Y
5, 14, 153, 6, 24, 2, 14, 15	5, 14, 153	5, 2
5, 14, 6, 24,14, 15	5, 14, 24	5
14, 6, 14, 15,	14, 15	14, 6
14, 14		

M1

Answer = 14 A1
Comparisons = 30 A1

(ii)

	X	Y
5, 14, 153, 6, 24, 2, 14	5, 14, 153	5, 2
5, 14, 6, 24,14	5, 14, 24	5
14, 6, 14	14	14, 6
14		

M1

Answer = 14 A1
Comparisons = 24 A1

(iii) Median B1

(iv) Time taken approximately proportional to square of length of list (or twice length takes four times the time, or equivalent). B1

3.

(i) $T_1 \rightarrow T_2$ $T_1 \rightarrow T_3 \rightarrow T_2$ M1
$T_1 \rightarrow T_3$ $T_1 \rightarrow T_2 \rightarrow T_3$ A1
$T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4$ $T_1 \rightarrow T_3 \rightarrow T_4$

(ii) $T_4 \rightarrow T_3 \rightarrow T_2 \rightarrow T_1$ $T_4 \rightarrow T_3 \rightarrow T_1$ M1
$T_4 \rightarrow T_3 \rightarrow T_1 \rightarrow T_2$ $T_4 \rightarrow T_3 \rightarrow T_2$ A1
$T_4 \rightarrow T_3$

(iii) 22 M1 allow for 23
A1

(iv) 11 M1 halving (not 11.5)
A1

4.

(i) e.g. 00–09→1
10–39→2
40–79→3
80–89→4
90–99→5

M1
A1 proportions OK
A1 efficient

(ii) e.g. 00–15→1
16–47→2
48–55→3
56–79→4
80–87→5
88–95→6
96, 97, 98, 99 reject

M1 some rejected
A2 proportions OK
(–1 each error)
A1 efficient

(iii) & (iv)

Sim. no.	Cars arriving after Joe – time interval | number of passengers										Time to 15 passengers (minutes)
1	3	2	2	1	1	2	2	2	3	1	6
2	3	1	2	2	1	4	1	2	5	1	6
3	5	1	2	2	2	1	3	4	2	2	12
4	4	6	3	2	4	1	1	2	2	3	4
5	5	1	4	1	3	2	5	4	2	2	17
6	4	4	4	2	5	3	1	4	1	4	8
7	4	1	4	2	3	1	5	4	1	3	16
8	2	2	2	2	2	4	3	5	1	2	6
9	1	1	1	1	1	1	1	1	1	2	5
10	2	4	3	2	2	6	2	5	2	1	5

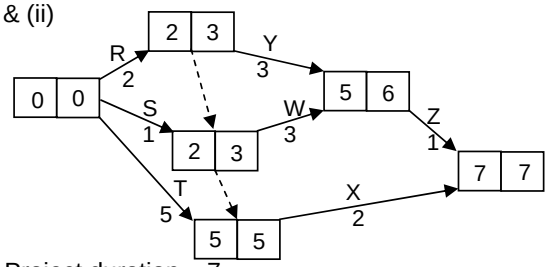
M1
A2 (–1 each error)

M1 simulation
A1 time intervals
A1 passengers
A1 time to wait

(v) 0.8
more runs

B1
B1

5.

(a)(i) Activity D. Depends on A and B in project 1, but on A, B and C in project 2.	M1 A1 A1
(ii) Project 1: Duration is 5 for x<3, thence x+2. Project 2: Duration is 5 for x<2, thence x+3	B1 "5" B1 B1 beyond 5
(b) (i) & (ii)  Project duration – 7 Critical activities – T, X	M1 activity-on-arc A1 single start and single end A2 precedences (–1 each error) M1 A1 forward pass M1 A1 backward pass B1 B1

6.

(i)

Order of inclusion	1	3	6	4	5	2
	A	B	C	D	E	F
A	~~–~~	~~10~~	~~7~~	~~–~~	~~9~~	~~5~~
B	~~10~~	~~–~~	~~–~~	~~1~~	~~–~~	4
C	7	–	–	–	3	–
D	~~–~~	1	~~–~~	~~–~~	~~2~~	~~–~~
E	~~9~~	~~–~~	~~3~~	2	~~–~~	~~–~~
F	5	~~4~~	~~–~~	~~–~~	~~–~~	~~–~~

M1
A1 select
A1 delete
A1 order

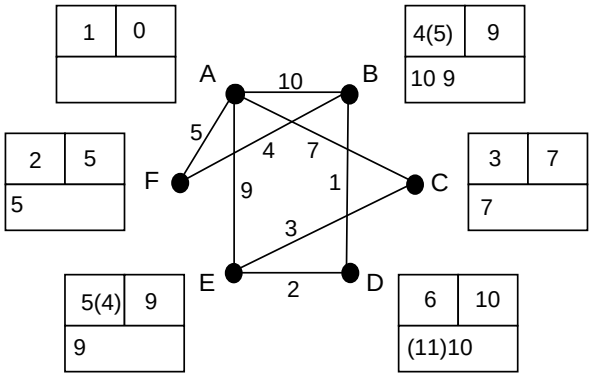
Arcs: AF, FB, BD, DE, EC

B1

Length: 15

B1

(ii) & (iii)



B1 arcs
B1 lengths

M1 Dijkstra
A1 working values
A1 order of labelling
A1 labels

Arcs: AF, FB, BD, AC, AE

M1
A1

Length: 26

(iv) Cubic

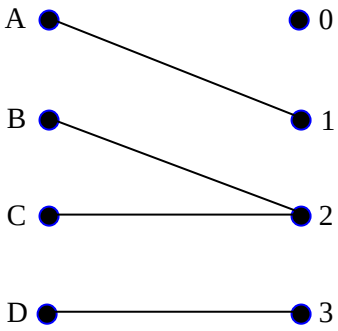
B1

n applications of Dijkstra, which is quadratic

B1

4771 Decision Mathematics 1

1.

(i)		M1 bipartite A1 one arc from each letter A1 David A1 rest
(ii)	Can't both have someone shaking hands with everyone and someone not shaking hands at all.	B1 $0 \Rightarrow \sim 3$ B1 $3 \Rightarrow \sim 0$
(iii)	n arcs leaving By (ii) only n–1 destinations	B1 B1

2.

(i)		

n	i	j	k
5	1	3	3
	2	2	8
	3	1	13
	4	0	16

B1
B1
B1
B1

k = 16

B1

(ii) $f(5) = 125/6 - 35/6 + 1 = 90/6 + 1 = 16$

(Need to see 125 or $20.8\dot{3}$ for A1)

M1 substituting

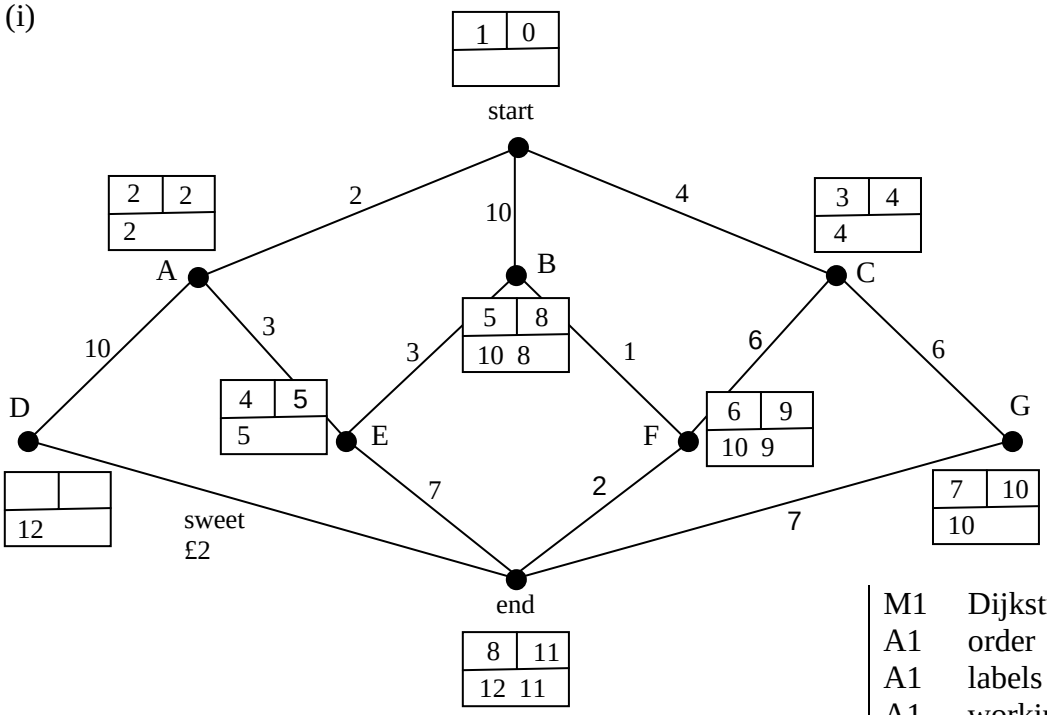
A1

(iii) cubic complexity

B1

3.

(i)



Cheapest: £11
[start (£2 starter)] → A (£3 main) → E (£3 main) → B (£1 main) → F (£2 sweet) → [end]

M1	Dijkstra
A1	order
A1	labels
A1	working values
B1	£11
B1	route

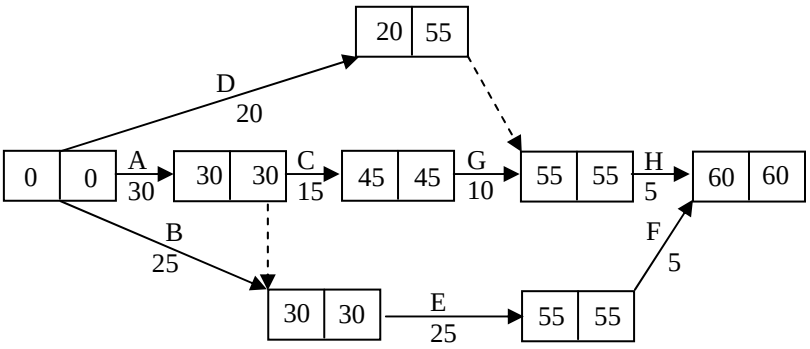
(ii) repeated mains ! B1
directed network B1

4.

(i) e.g.	00–47→90 48–79→80 80–95→40 96, 97,98, 99 ignore	M1 some rejected A3 correct proportions (–1 each error) A1 efficient
(ii)	smaller proportion rejected	B1
(iii) e.g.	90, 90, 90, 80 350	M1 A1 A1√
(iv) e.g.	90, 80, 90, 80 340 80, 90, 80, 80 330 90, 40, 80, 90 300 40, 90, 90, 90 310 90, 90, 90, 90 360 80, 80, 40, 90 290 80, 80, 80, 90 330 90, 80, 90, 90 350 90, 40, 40, 80 250	M1 A3 (–1 each error) √
	prob (load>325) = 0.6	M1 A1
(v)	e.g. family groups	B1

5.

(i)&(ii) e.g.



time – 60 minutes
critical – A; C; E; F; G; H

M1 sca (activity on arc)
A1 single start & end
A1 dummy
A1 rest

M1 forward pass
A1
M1 backward pass
A1

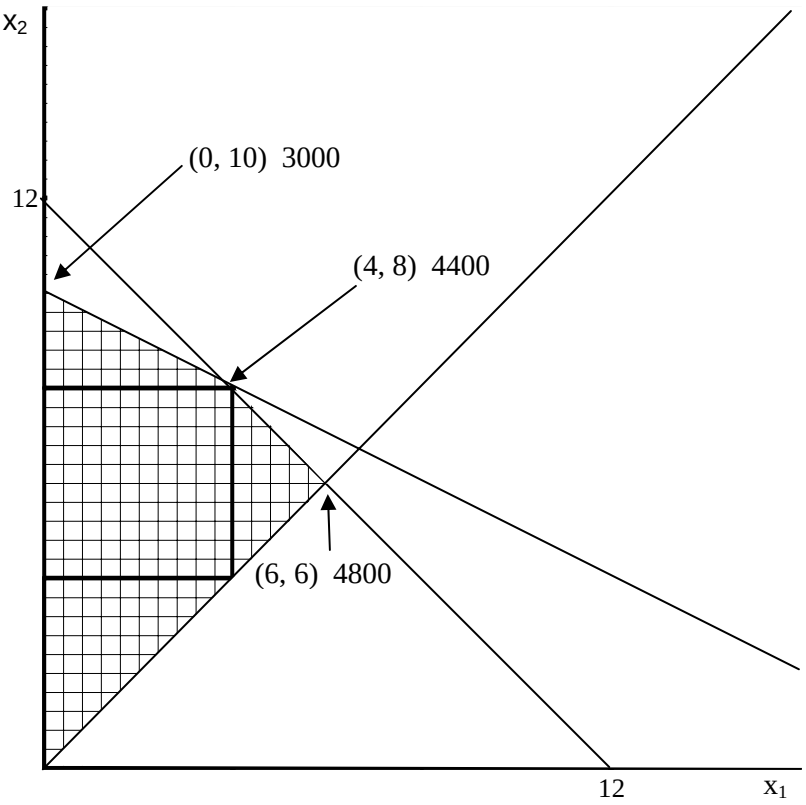
B1 √
B1 cao

(iii) A and B at £300

A; C; G; H
B; E; F

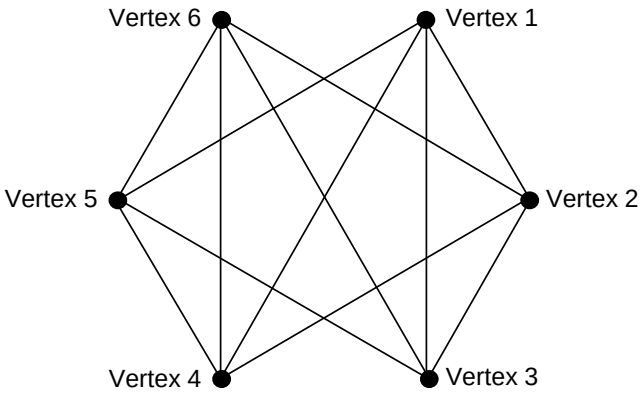
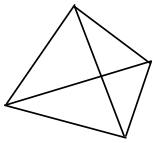
B1 2 out of A, B, E
B1 A
B1 B
B1 300 from A and B
B1
B1

6.

(i) x_i represents the number of tonnes produced in month i $x_2 \leq x_3$ $x_1 + x_2 \leq 12$	M1 quantities A1 tonnes B1 B1
(ii) Substitute $x_3 = 20 - x_1 - x_2$ $x_2 \leq x_3 \rightarrow x_1 + 2x_2 \leq 20$ Min $2000x_1 + 2200x_2 + 2500x_3 \rightarrow$ Max $500x_1 + 300x_2$	M1 A1 A1
(iii) 	M1 sca A3 lines A1 shading M1 >1 evaluated point or profit line A1 (6, 6) or 4800
Production plan: 6 tonnes in month 1 6 tonnes in month 2 8 tonnes in month 3	M1 √ all 3
Cost = £45200	A1 cao

4771 Decision Mathematics 1

Question 1

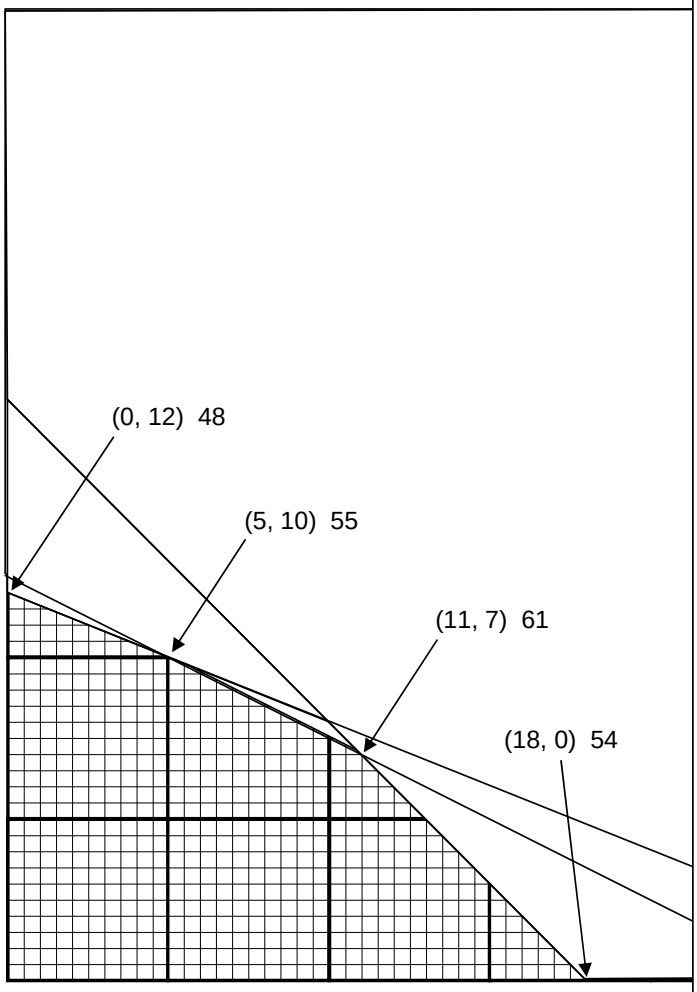
(i) 1 and 6, 2 and 5, 3 and 4	B1
(ii) 	M1 10 to 14 edges A4 (–1 each edge error)
(iii) 	B1 identification B1 sketch

Question 2.

(i)	A's c takes 2, leaving 3. You have to take 1. A's c takes one and you lose.	M1 A1 A1
(ii)	A's c takes 3 leaving 3. Then as above.	M1 A1
(iii)	A's c takes 3 leaving 4. You can then take 1, leading to a win.	M1 A1 A1

Question 3.

(i)



B3 lines
B1 shading

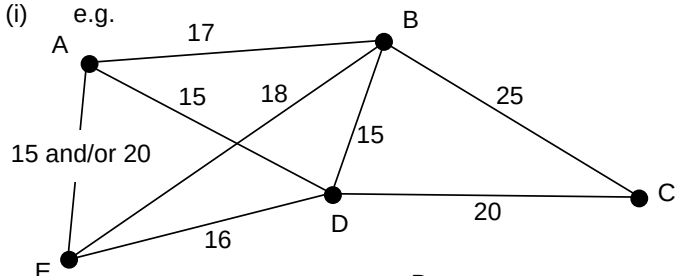
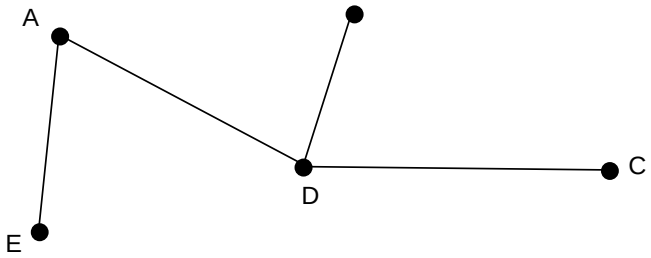
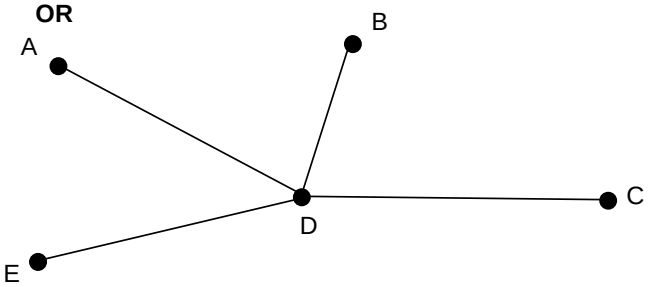
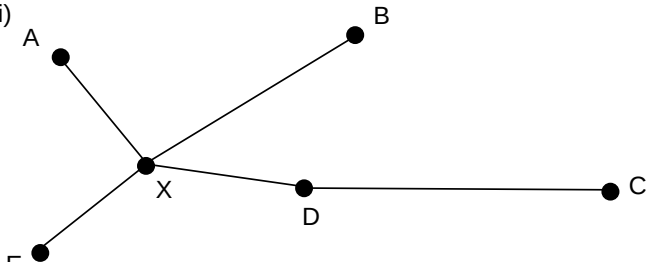
(ii) Intersection of $2x+5y=60$ and $x+y=18$ is at (10,8) M1

$10 + 2\times 8 = 26$ A1

Question 4.

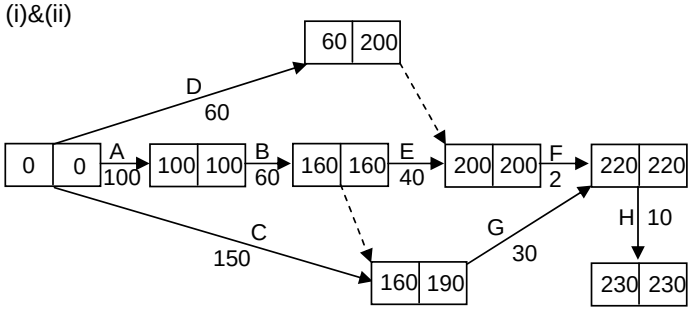
(i)	e.g. 0–4 exit 5–9 other vertex	B1 B1
(ii)	e.g. 1 A ExA 2 A B A B A B ExB 3 A ExA 4 A B A B A ExA 5 A B ExB 6 A B A B A B ExB 7 A B A B ExB 8 A ExA 9 A B ExB 10 A ExA	M1 process with exits A1
	0.5, 0.5, 1.9 (Theoretical answers: 2/3, 1/3, 2) (Gambler's ruin)	B1 probabilities M1 duration A1
(iii)	e.g. 0–2 exit 3–5 next vertex in cycle 6–8 other vertex 9–ignore and re-draw	M1 ignore DM1 conditionality A1 equal prob A1 efficient
(iv)	e.g. 1 A B A B A ExA 2 A C A ExA 3 A ExA 4 A B C B C ExC 5 A ExA 6 A C A B ExB 7 A ExA 8 A B C ExC 9 A ExA 10 A ExA	M1 A2
	0.7, 0.1, 0.2 (Theoretical probs are 0.5, 0.25, 0.25) (Markov chain)	M1 A1

Question 5.

(i) e.g.  	M1 A1 connectivity A1 lengths
(ii)  Order: AE; AD; DB; DC or AD; AE; DB; DC or AD; DB; AE; DC Length: 65 km	M1 connected tree A2 (–1 each error) A1 B1
OR  Order: AD; DB; DE; DC Length: 66 km	
(iii)  Length: 53 km Advice: Close BC, AE and BD	M1 connected tree A2 (–1 each error) B1 B3
(iv) facility (e.g. anglers) distances (e.g. B to C)	B1

Question 6.

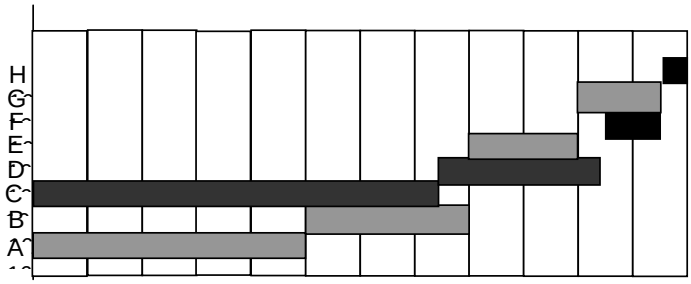
(i)&(ii)



time – 230 minutes
critical – A; B; E; F; H

M1	sca (activity on arc)
A1	single start & end
A1	dummy
A1	rest
M1	forward pass
A1	
M1	backward pass
A1	
B1	
B1	cao

(iii) e.g.



M1	cascade
A2	
B1	Joan/Keith

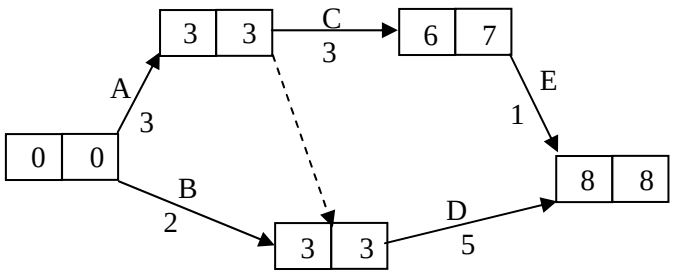
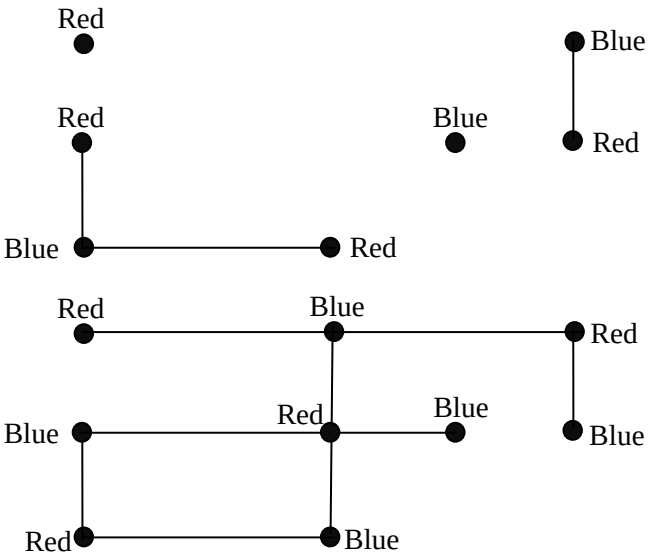
Least time = 240 mins

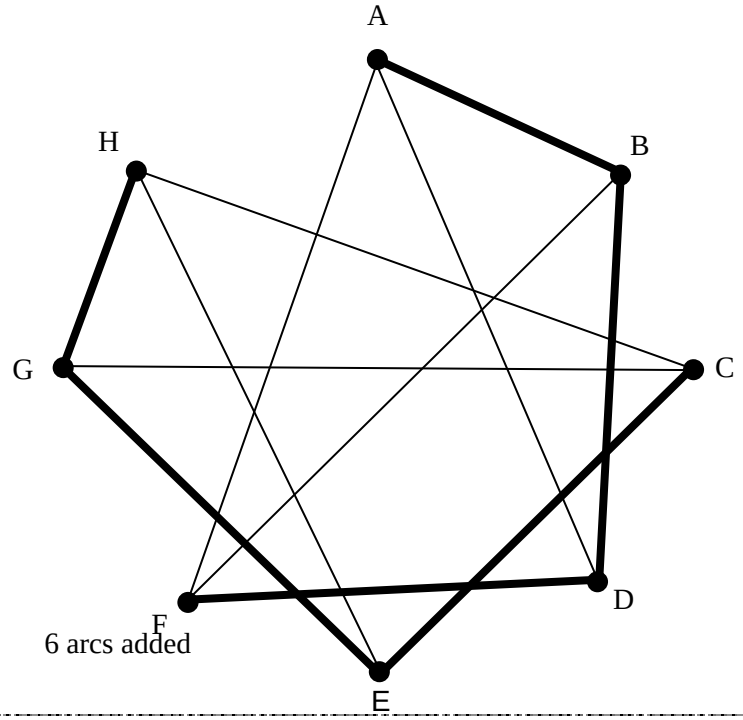
B1

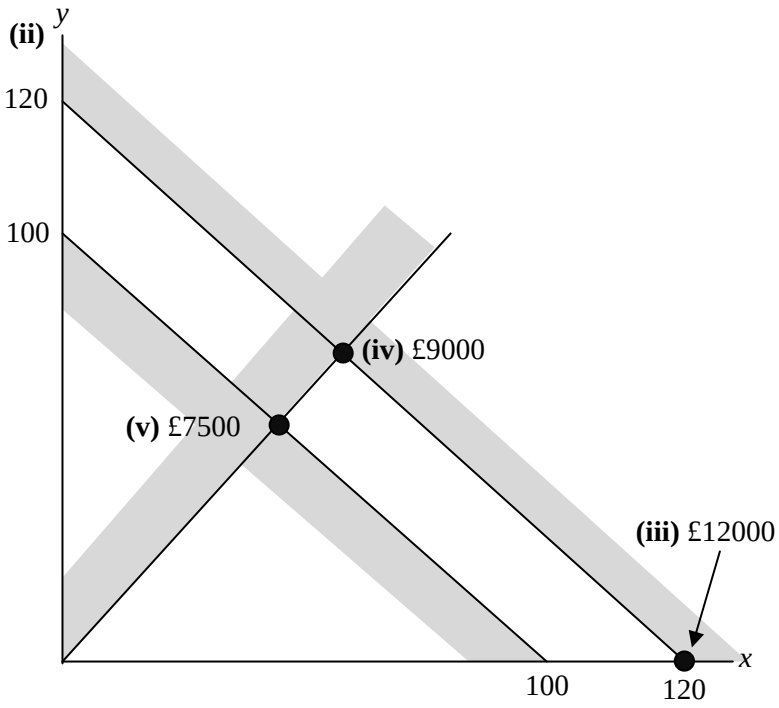
Minimum project completion times assumes no resource constraints.

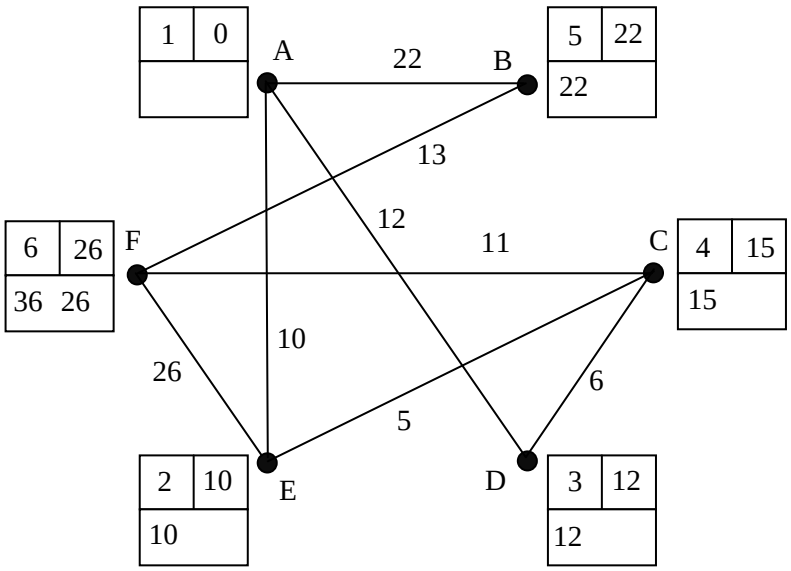
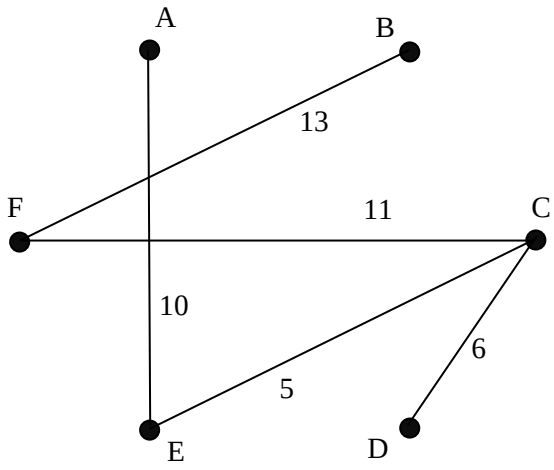
B1

4771 Decision Mathematics 1

1	**(i) & (ii)**	 Critical activities: A and D	M1 activity-on-arc A1 C and E OK A1 D OK M1 forward A1 pass M1 backward A1 pass B1
2	**(i)**		M1 subgraph A1 M1 Changing colours A1 top right A1 bottom left A1 not singletons B1
	(ii)	The rule does not specify a well-defined and terminating set of actions.	B1

3	**(i)**	No repeated arcs. No loops	B1 B1
	(ii)	Two disconnected sets, {A,B,D,F} and {C,E,G,H}	M1 A1
	(iii)	 6 arcs added	M1 A1 B1
	(iv)	$4\times4 = 16$ or $\binom{8}{2} - 12 = 28 - 12 = 16$	B1

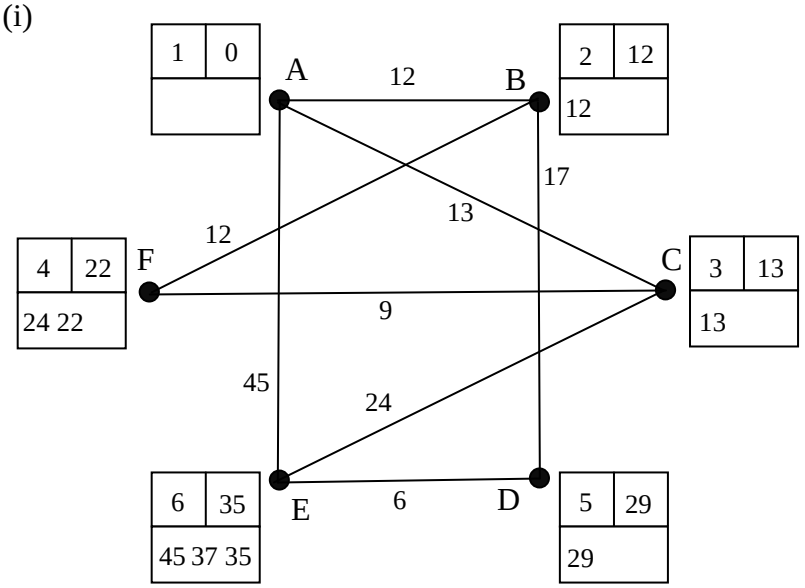
4	**(i)**	e.g. Let x be the number of adult seats sold. Let y be the number of child seats sold. $x + y \leq 120$ $x + y \geq 100$ $x \geq y$	M1 A1 B1 B1 B1
	(ii)		B3 lines (scale must be clear) B1 shading (axes must be clear) B1 point + amount M1 point A1 amount M1 point A1 amount
	(vi)	6000+60c > 10000 => c ≥ 67	M1 A1

5	**(i) & (ii)**	 shortest route: A E C F distance: 26 miles	M1 network A1 arcs A1 lengths M1 Dijkstra A1 working values B1 order of labelling B1 labels B1 B1
	(iii)	CE CD AE CF ~~AD~~ BF ~~AB~~ ~~EF~~  total length of connector = 45	M1 5 arc connector A1 AD not included A1 all OK, inc order B1 B1
	(iv)	A 3 miles (or length = 9) B 2 miles (or length = 10)	B1 B1

6	**(i)**	e.g. 0, 1, 2 → fall 3, 4, 5, 6, 7, 8 → not fall 9 → redraw	M1 ignore at least 1 A1 proportions correct A1 efficient
	(ii)	apple r n fall? 1 1 yes 2 3 no 3 8 no 4 0 yes 5 2 yes 6 7 no Three apples fall in this simulation.	M1 A2 –1 each error B1√
	(iii)	apple r n fall? 2 0 yes 3 1 yes 6 4 no apple r n fall? 6 4 no apple r n fall? 6 8 no apple r n fall? 6 0 yes 5 days before all have fallen	M1 A2 –1 each error A1√
	(iv)	apple r n fall? 1 picked 2 1 yes 3 3 no 4 8 no 5 0 yes 6 2 yes apple r n fall? 3 picked 4 7 no apple r n fall? 4 picked 3 days before none left	M1 A2 –1 each error B1√
	(v)	more simulations	B1

1.

(i)



M1 Dijkstra
A1 working values
B1 order of labelling
B1 labels

AB	12	AB
AC	13	AC
AD	29	ABD
AE	35	ABDE
AF	22	ACF

B1 AB and AC
B1 AD and AF
B1 AE

(ii) 5

B1

2.

(i)

~~3~~	~~8~~
~~6~~	~~4~~
~~12~~	~~2~~
24	1

24

M1 doubling and halving
M1 deleting and summing
A1 cao

(ii)

~~26~~	~~42~~
52	21
~~104~~	~~10~~
208	5
~~416~~	~~2~~
832	1

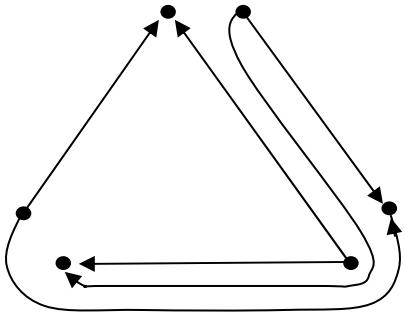
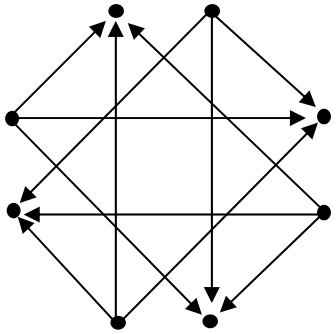
1092

M1 doubling and halving
M1 deleting
DM summing
A1 cao

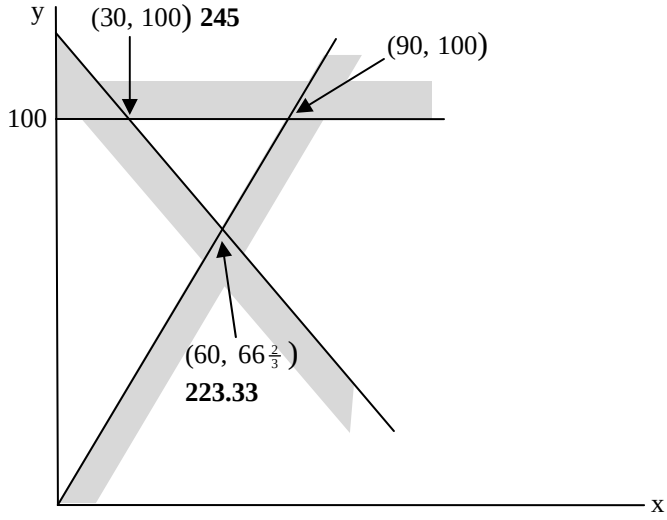
(iii) multiplication

B1

3.

(i) 	B1 B1 B1
(ii) 	B1 12 arcs B1 connectvity B1 3 out of each in vertex B1 3 into each out vertex
(iii) The graphs represent traffic flows within the junctions. They do not take account of flows approaching or leaving the junctions. (Graphs are not planar if these flows are added, so traffic flows have to cross.)	B1

4.

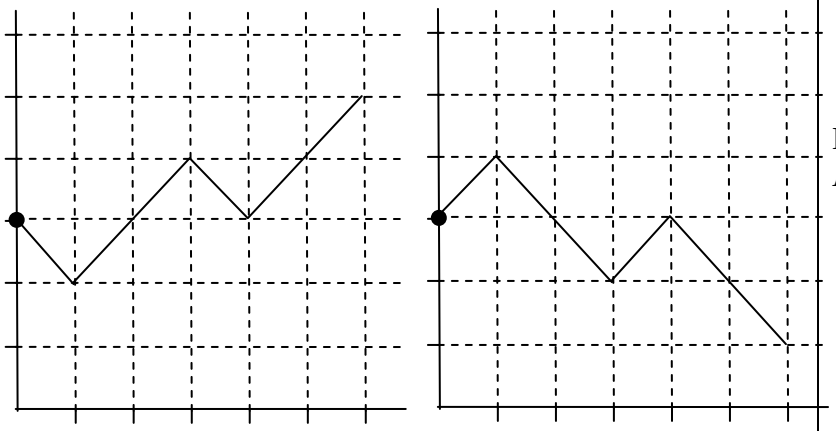
(i) Each small tile has area 100 cm^2 so $1000x$ Similarly $900y$ So $1000x + 900y \geq 400 \times 300 = 120000$	M1 areas A1 tile areas A1
(ii) $y \leq 100$ $10x \leq 9y$	B1 B1 B1
(iii) e.g. minimise $1.5x + 2y$	B1
	B3 lines B1 shading
Integer solution required, so x=60, y=67, cost = 224	M1 solving A1 x = 59-61 y = 66-68 A1 220-228
(iv) wastage or design	B2

5.

(i) e.g. 0 to 4 –> stagger left
5 to 9 –> stagger right
+ accumulation

M1
A1
B1

(ii) probably one of:



M1
A1

(iii) repeat
relative frequency

B1
B1

(iv) e.g. 0 to 2 –> stagger left
3 to 8 –> stagger right
9 reject and redraw

M1 reject some
A1 proportions
A1 efficient

(v) e.g.

run 1	R	L	R	L	L	R		
run 2	R		L		R	R	L	R
run 3	R	R	L	L	L	L		
run 4	L	L	R	L	R	R		
run 5	R	R	R	*				
run 6	L	R	R		R		R	*
run 7	R	R	L	R	R	*		
run 8	R	R	L	R	R	*		
run 9	R		R		R	*		
run 10	L	R	R	L	R	R		

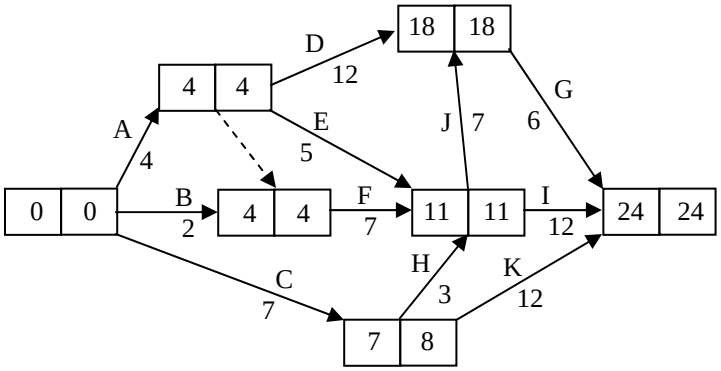
M1
A2 (–1 each wrong row)

Probability estimate = 0.5
(Theoretical = $0.7^3 + 5\times0.7^4\times0.3 = 0.70315$)

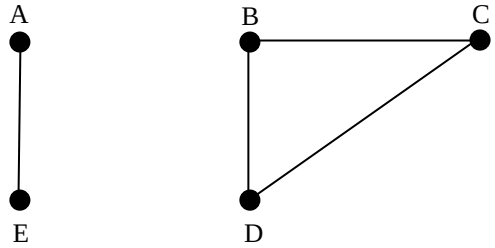
B1 falling in

M1 probability
A1

6.

(i) & (ii)  	M1 activity-on-arc A1 D, E, H and K A1 F A1 I and J A1 G M1 forward pass A1 M1 backward pass A1
Duration = 24 months	B1 cao
Critical : A; F; J; G	B1 cao
(iii) Crash F by 1 month and G by 1 month at a cost of £6m.	B1 F by 1 month B1 G by 1 month B1 £6m
(iv) Crash G by 2 months at a cost of £8m.	M1 G only A1 £8m

1.

(i)  	M1 A1	M1 any and only 3 of the 4 A1 all
(ii) 6	M1 attempt at complete connectivity A1	M1 5, 6 or 7 A1 6
(iii) e.g. 4 arcs and (e.g.) {A}, {B, C, D, E}	B1 B1 B1	4 ... set of 1 ... disjoint set of 4 SC M1 6 arcs A1 appropriate sets ... disjoint of size 2 and 3
(iv) Reference to parts (i) and (ii), in reverse − or similar	B1	

2.

(i)

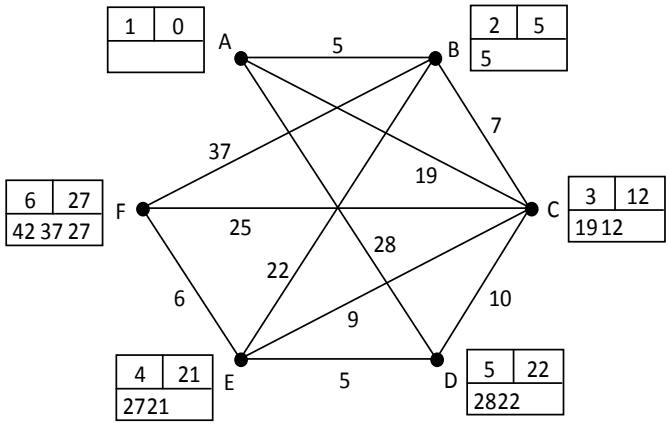
Test number	Sample drawn from flagons numbered	Result (D = dead, A = alive)
1	1, 2, 3, 4	A
2	5, 6	A
3	7	D
4	8	A

(ii)

Test number	Sample drawn from flagons numbered	Result (D = dead, A = alive)
1	1, 2, 3, 4	D
2	5, 6, 7, 8	D
3	1, 2	A
4	3	D
5	4	A
6	5, 6	A
7	7	D
8	8	A

B1		cao
B1		cao ... allow extra second line of 5678 D, but with –1
B1		cao
B1		cao
B1		cao
B1		cao
B1	1,2	award the last two B1s only for contiguous blocks of 3 tests
	3	
	4	
B1	5,6	from line 3 allow extraneous lines but –1 once only, and only from the last two B1s
	7	
	8	

3.

(i)		M1 Dijkstra A1 working values B1 order of labelling B1 labels	
	Shortest distance = 27	B1	
	Shortest route … ABCEF	B1	
(ii)	Because F was the final vertex labelled.	B1	cao
(iii)	Because if there were to be a shorter route than BCEF from B to F, then A to B followed by it would give a shorter route from A to F. or "B is en route"	B1	

4.

(i)

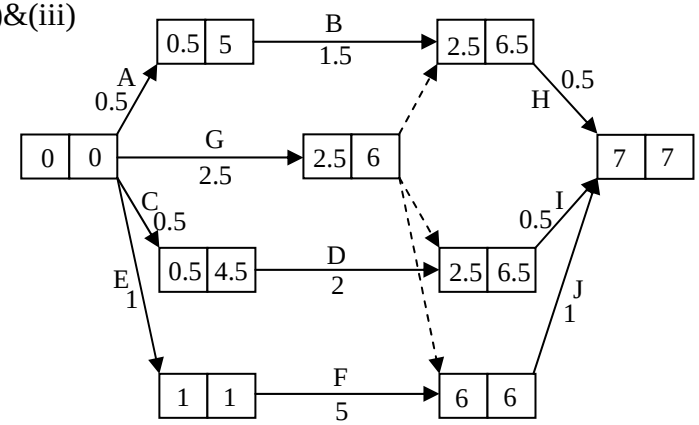
Task	Description	Duration (mins)	Immediate predecessor(s)
A	Fill kettle and switch on	0.5	–
B	Boil kettle	1.5	A
C	Cut bread and put in toaster	0.5	–
D	Toast bread	2	C
E	Put eggs in pan of water and light gas	1	–
F	Boil eggs	5	E
G	Put tablecloth, cutlery and crockery on table	2.5	–
H	Make tea and put on table	0.5	B; G
I	Collect toast and put on table	0.5	D; G
J	Put eggs in cups and put on table	1	F; G

B1 A, C, E and G

B1 B, D and F

B1 H, I and J

(ii)&(iii)



M1 activity-on-arc
A1 A, G, C ,E, B, D, F
A1 H, I, J

no follow through
no multiple starts

no multiple ends

M1 A1 forward pass
M1 A1 backward pass

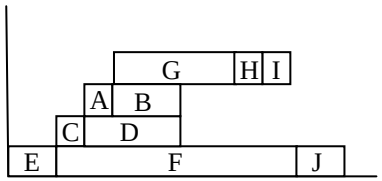
√ but no follow of activity-on-node
√ ditto

(iv) critical activities: E; F; J
duration: 7 minutes

B1 cao
B1 cao

task: A B C D E F G H I J
float: 4.5 4.5 4 4 0 0 3.5 4 4 0

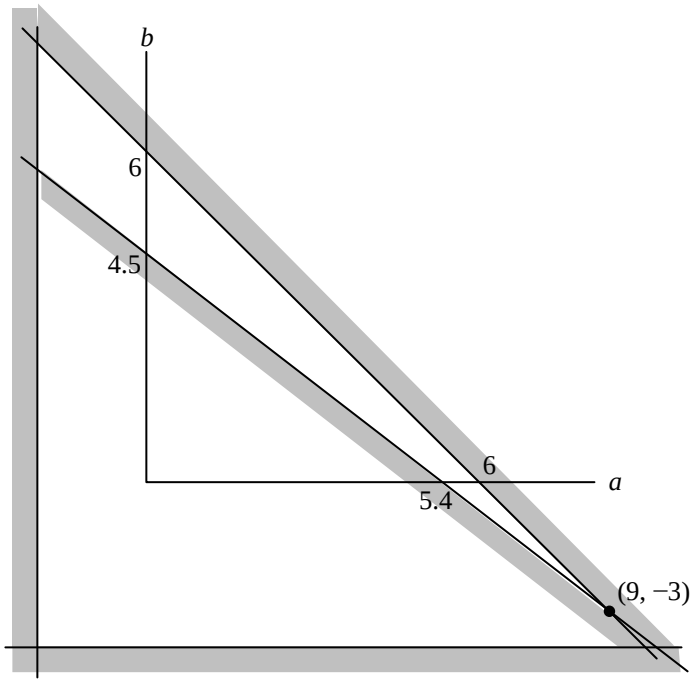
B1 cao blank=0

(v) e.g. 	M1 A1 A1	cascade or condensed cascade activities other than B, D and F non-overlapping correctly finish in 7	need to have 9 or 10 activities E C A G H I J cao

5.

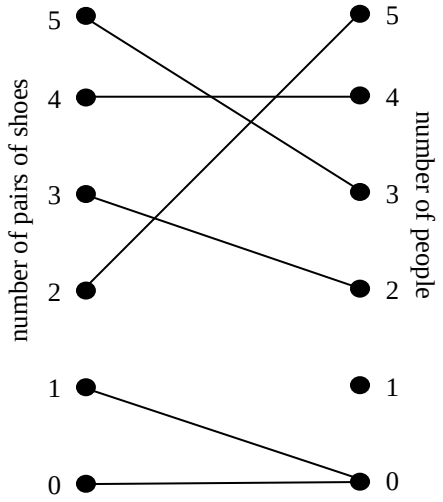
(i)	e.g. 00–04 6 05–29 7 30–79 8 80–99 9	M1 rule using 2-digit nos A1 correct proportions A1 efficient	
(ii)	e.g. 00–09 goal 10–99 no goal	B1	complete rule required
(iii)	e.g. 8 0 1 0 0 0 0 0 0 so 1 goal	B1 B1 B1	√ rule (i) √ need to see which are converted ... their 8 and rule (ii) √ their 8 and rule (ii) ... ignore previous line
(iv)	e.g. 00–31 5 32–63 6 64–79 7 80–95 8 96–99 reject and redraw	M1 2 or more rejected A1 correct proportions A1 efficient	allow part (iv) if seen elsewhere 3 or 4 rejected in part (v) below expect either 00–11 or 88–99 for goal any other rule must be declared to score marks
(v)	e.g. 6 0 0 1 0 0 0 so 1 goal	B1 M1 A1	√ rule (iv) √ their 6 ... need to see which are converted √
(vi)	Each scored 10 goals. Nothing to choose between them.	M1 A1	goals scored one, the other or indifferent, depending on goals scored
(vii)	More repetitions	B1	"greater number of random numbers" → 0 "more accurate data" → 0 Also no "or"s! 3-digit RNs → 0

6.

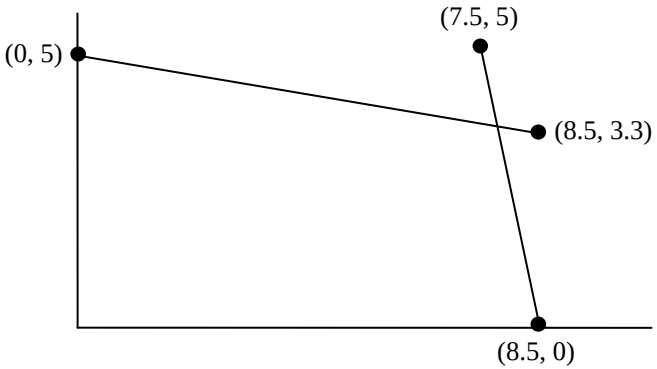
(i) Thousands of litres of A in stock = 2 $b \geq -4$	B1 B1	 cao
(ii) $5(a+2) + 6(b+4) \geq 61$ $(a+2) + (b+4) \leq 12$ giving $a + b \leq 6$	M1 A1 M1 A1	 watch for fluke
(iii) 	B4 lines B1 shading	√ their negative gradient stock line √ shape = or
(iv) Increase stock levels of A by 9000 litres. Reduce stock levels of B by 3000.	B1 B1	Give the marks for 9000, –3000, or equivalent ±200 litres on both
(v) New stock levels are 11000 of A and 1000 of B. $5\times11000 + 6\times1000 = 61000$ $11000 + 1000 = 12000$	B1 B1 B1	√ (iv) SC correct answer from nowhere OK Allow comment only for the "fully stocked" B1.

4771, June 2011, Markscheme

1.

(i)		B1 3 to 4 deleted B1 1 to 4 deleted B1 4 to 4 added	-1 for each arc in error
(ii)	14	B1	
(iii)	47	M1 A1 cao	Award method mark if answer correct, or if wrong but with a sum of products shown.
(iv)	(0, 0) and (1, 0)	B1	Award only if correct **points** are specified in some way.
(v)	Explanation should recognise that a line is a set of points − not appropriate in this context.	B1	e.g. “Intermediate points have no meaning.” e.g. “Can’t have one and a half pairs of shoes.” (sic)

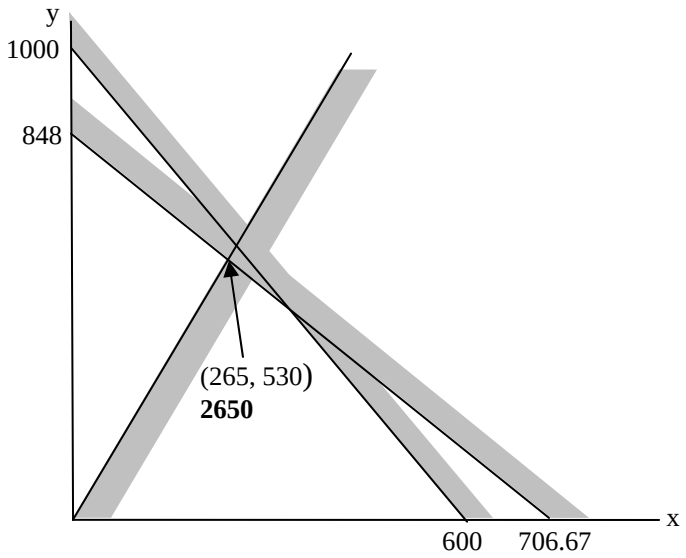
2.

(i) X = min(25, 8.5) = 8.5 or equivalent	B1	cao	OK if only seen once or more on graph
Y = min(5, 42.5) = 5 oe	B1	cao	OK if only seen once or more on graph
X^* = (85–10)/10 = 7.5 oe	B1	cao	OK if only seen on graph
Y^* = (25–8.5)/5 = 3.3 oe	B1	cao	OK if only seen on graph
	B1	allow ft	sensibly scaled for their X and Y e.g. disallow if either of the lines in the question could intersect both axes.
	B1	cao	lines - can extend to beyond segment
	B1	cao	condone minor errors in plotting (e.g. 8.5 plotted at 9)
(ii) Avoids tiny feasible regions.	B1		need comment on size of region

3.

(i) e.g. 1, 2, 3 → 1 4 → 2 5, 6 → 3	M1 A1 A1	function with domain {1,2,3,4,5,6} and range {1,2,3} (special cases are possible – if correct!) proportions 3:2:1 all OK
(ii) e.g. 1, 2 → 1 3 → 2 4 → 3 (5, 6 → reject and throw again)	M1 reject some A1 reject two A1 rest	(Special cases are possible – if correct! e.g. allow throwing die twice and allocating correct proportions of 36.)
(iii) non uniform allows 100	B1 B1	"101 values" OK no credit for, e.g. "3 is not a two-digit number"

4.

(i)	e.g. x = number of large houses y = number of standard houses	M1 A1	M1 for variables for large and for standard A1 for "number"
	land: 200x + 120y <= 120000 oe cash: 60x + 50y <= 42400 oe market: x <= 0.5y oe	B1 B1 B1	use "isw" for incorrect simplifications -1 once only for any " < "
(ii)		B1 line 1, allow ft B1 line 2, allow ft B1 line 3, allow ft B1 feasible region	for instance, if x <= 2y in part (i), then allow correct graph of x <= 0.5y or ft graph of x <= 2y plotting tolerance on axis intersection points – within correct small square must consider 3 lines ft if region includes y-axis interval from origin upwards allow any clear indication of feasible region ignore any indication(s) of boundary lines included or excluded
(iii)	intersection of y=2x and 6x+5y=4240, (265, 530) 2650	M1 correct point, cao A1	identification only - coordinates not required here their 4x+3y from (260-280, 520-540)
(iv)	their 60x + 50y <= 45000 or line from their (0, 900) to (750, 0)	B1 ft	can be implied from final M1 working
	Best point is at the intersection of the land constraint and the new cash constraint, and not on y=2x	M1 comparison of two (or more) points A1	not just ringing points their identified best point is not on y=2x or an axis
	(214, 643) 2785	M1 correct point, cao A1	identification, coordinates not required here bedrooms - their 4x+3y from (200-220, 620-660)

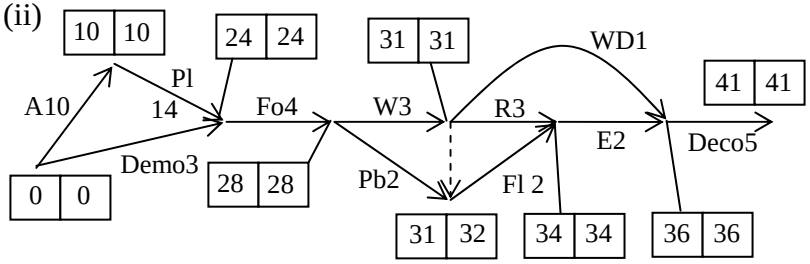
5.

(i)

Activity	Immediate predecessors
A	–
Pl	A
Demo	–
Fo	Pl; Demo
W	Fo
Pb	Fo
R	W
Fl	Pb; W
E	R; Fl
WD	W
Deco	WD; E

M1 Fl correct
A1 rest

(ii)



M1 at least one correct nontrivial join
A1 forward pass

M1 at least one correct nontrivial burst
A1 backward pass

excluding start node

(iii) critical activities: A; Pl; Fo; W; R; E; Deco
project duration = 41 days

B1 cao
B1 cao

act	A	Pl	Dm	Fo	W	Pb	R	Fl	E	WD	Dc
float	0	0	21	0	0	2	0	1	0	4	0

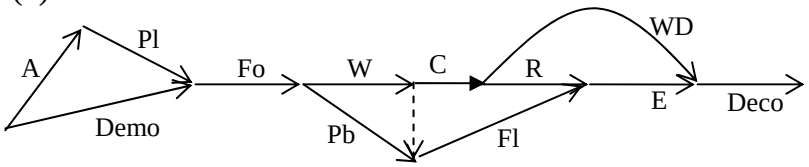
B1 A, Pl, Dm, Fo, W
B1 rest

cao
cao – most see zeros, dashes or empty spaces won't do

(iv) Fl has both W and Pb as immediate predecessors.
R and WD have only W as immediate predecessor.

B1
B1 one of R/WD

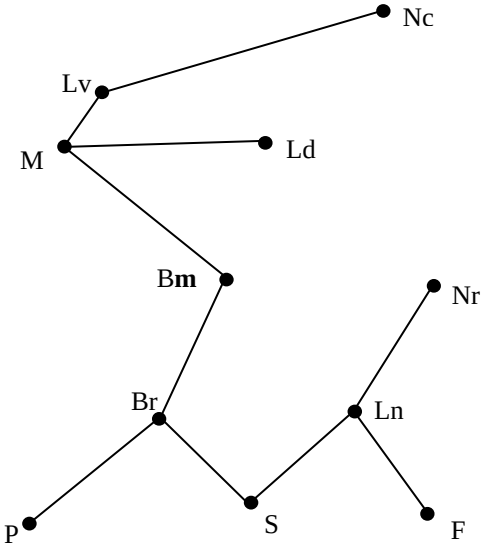
SC1 for a convincing but not specific answer, e.g. "A dummy is needed to cater for both joint and separate precedences".

(v)  	M1 C between W and R A1 Fl + dummy OK A1 WD OK	
(vi) new duration = 42 days critical activities: A; Pl; Fo; W; C; R; E; Deco	B1 cao	both needed

6.

(i)

	1	7	9	8	2	10	3	6	11	5	4
	P	S	F	Ln	Br	Nr	Bm	Ld	Nc	Lv	M
P	–	150	–	240	125	–	–	–	–	–	–
S	150	–	150	80	105	–	135	–	–	–	–
F	–	150	–	80	–	–	–	–	–	–	–
Ln	240	80	80	–	120	115	120	–	–	–	–
Br	125	105	–	120	–	230	90	–	–	–	–
Nr	–	–	–	115	230	–	160	175	255	–	–
Bm	–	135	–	120	90	160	–	120	–	–	90
Ld	–	–	–	–	–	175	120	–	210	100	90
Nc	–	–	–	–	–	255	–	210	–	175	–
Lv	–	–	–	–	–	–	–	100	175	–	35
M	–	–	–	–	–	–	90	90	–	35	–



Length = 985 miles

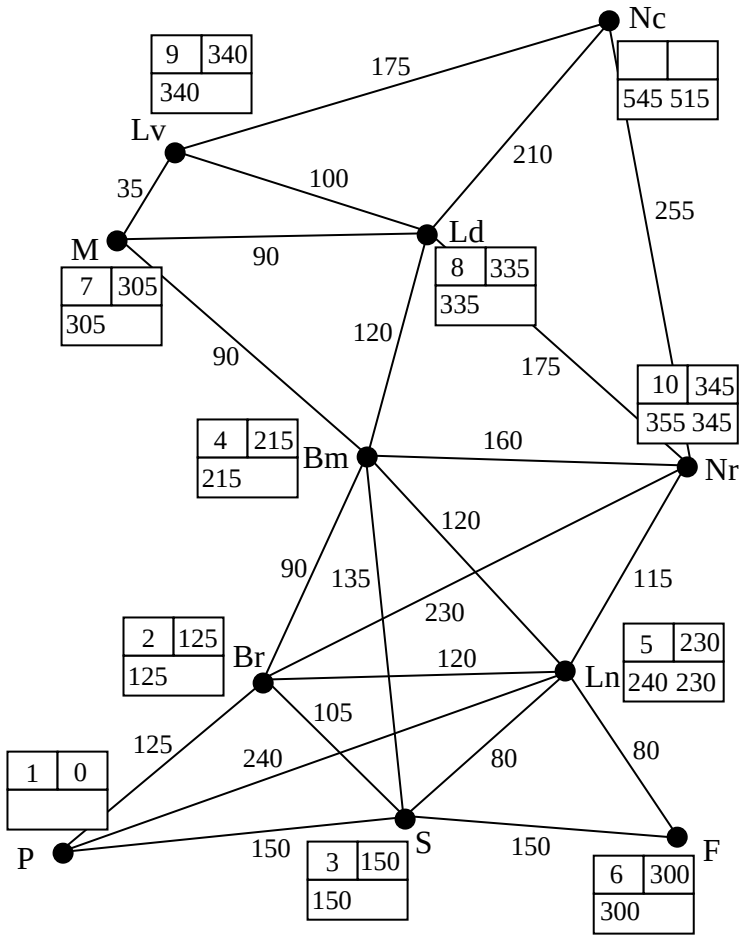
M1 tabular Prim
A2 choosings
A1 crossings

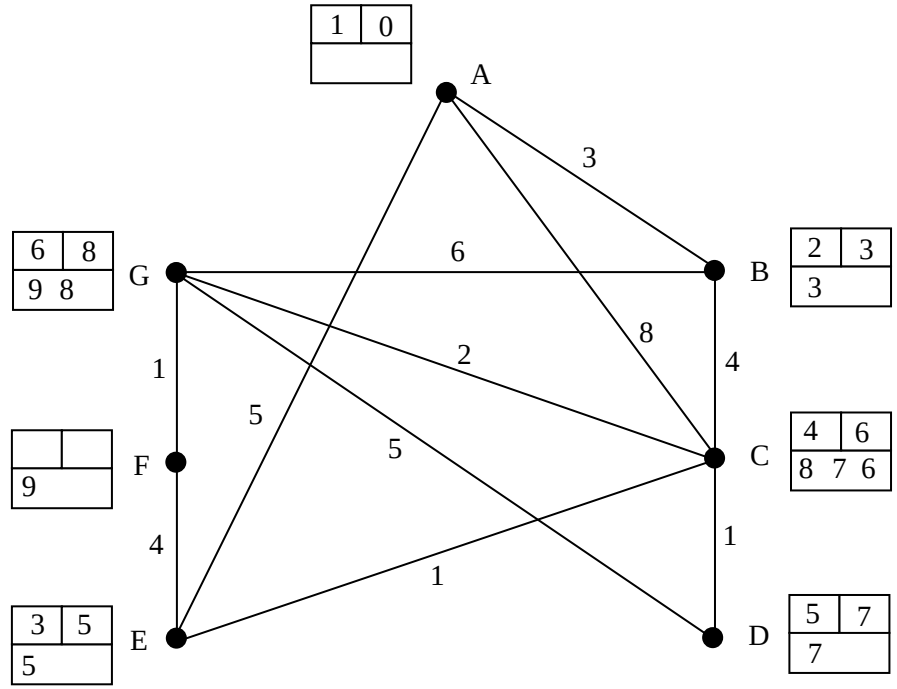
125 in P column and 90 in Br column ringed, with both rows crossed
all circles in correct place; -1 each error (watch for one error making two changes to a row)
all rows crossed out except, possibly, Nc row.

accept convincing transpose

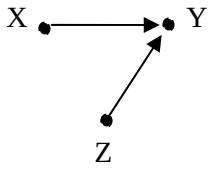
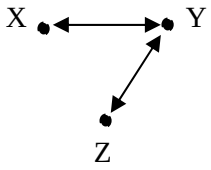
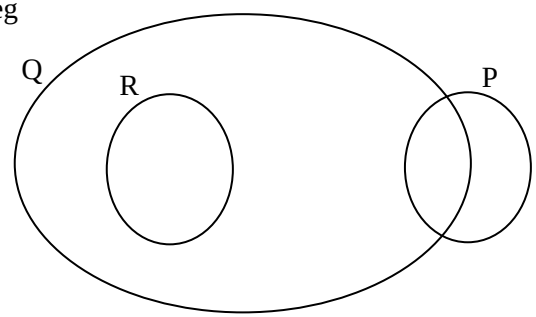
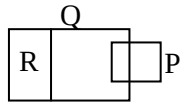
B1 cao

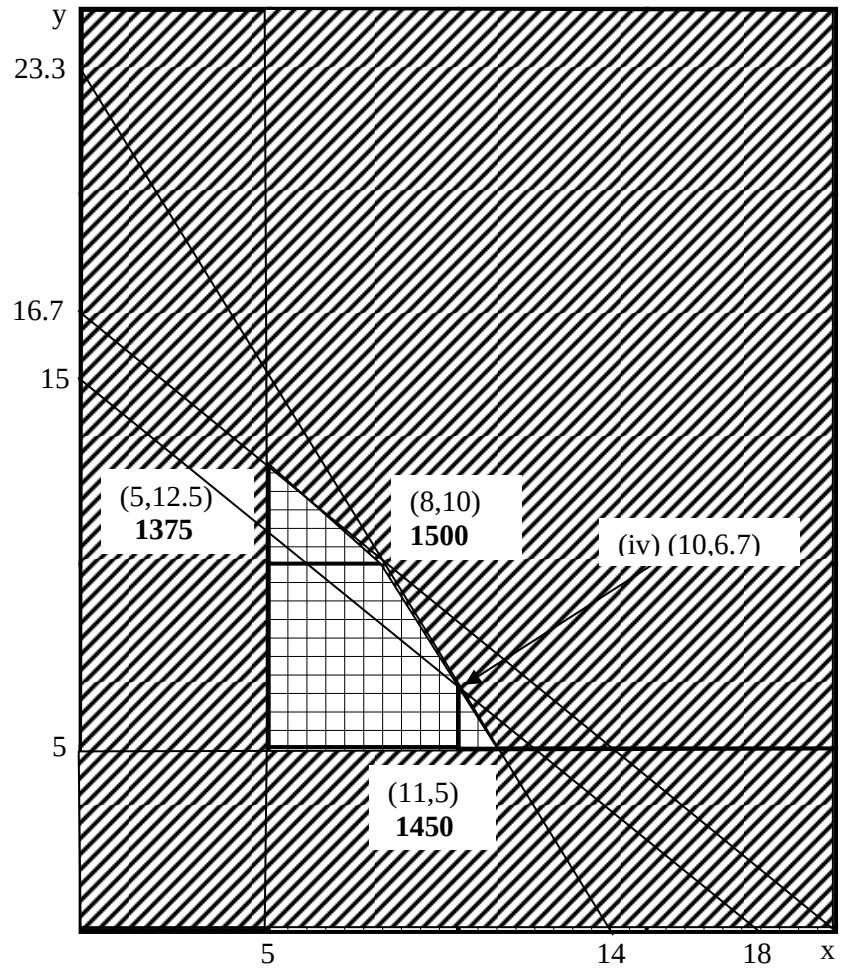
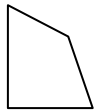
B1 cao

(ii) Advantage: shortest length of track Disadvantage: tree, no redundancy ≡ fragility (breakdown et al) Disadvantage: some journeys are not shortest paths	B1 cao B1 B1	allow cost minimisation could say "no cycles" disallow comments relating to direct connectivity, or relating to more stops "longer journeys" or "takes longer" allowed allow "min connector arcs may be more expensive" oe don't allow two marks for the same point described differently. e.g. longer journeys/more time/more upkeep
(iii) 	M1 Dijkstra A1 working values B1 labels B1 order of labelling	correct working values (no extras) at Ln and Nr, and working values only superseded at Ln and Nr (ignore Nc for this M) (need to check Nc here)
Route: P S Ln Nr Distance: 345 miles	B1 cao B1 cao	
(iv) Distance by min connector = 425 miles	B1 ft their mc	

Question	Answer	Marks	Guidance
1 (i) & (ii)	 Route: AECG Distance: 8	B1 B1 B1 B1 B1 B1 B1 B1 **[8]**	connectivity lengths Dijkstra working values other than at C order of labelling labels	 Award if wv's OK at C. allow legitimate later and larger wv's which are listed, but not used. Disregard F. SC ... If possible follow for these two marks. following errors in network

Question			Answer	Marks	Guidance	
2	(i)		A L R B f(L) f(R) 3 3.382 3.618 4 2.146 1.910 3.382 3.618 3.764 4 1.910 1.875 3.618	B1 B1 B1 B1 B1 B1 **[6]**	R and L f(R) and f(L) A L and R f(L) and F(R) A	-1 once only for incorrect accuracy, but condone 1.91. Surds OK, but lose the accuracy mark. (Q says 3dp.)
2	(ii)		Saves a function evaluation	B1 **[1]**		Has to be a comment about function values.
2	(iii)		eg Setting the control on a gas fire to achieve a room temperature of 20C. Function could be $(temp–20)^2$. (This example shows that optimising can be used to "achieve".) Note that the domain cannot be time based ... i.e finding when something occurred. One cannot go back in time to take a reading!	B1 **[1]**	Optimisation with need to sample at discrete intervals.	"Deepest point in seabed" example seen. This is acceptable, assuming that depth soundings are taken at points, and ignoring the fact that the domain is two dimensional rather than one dimensional.

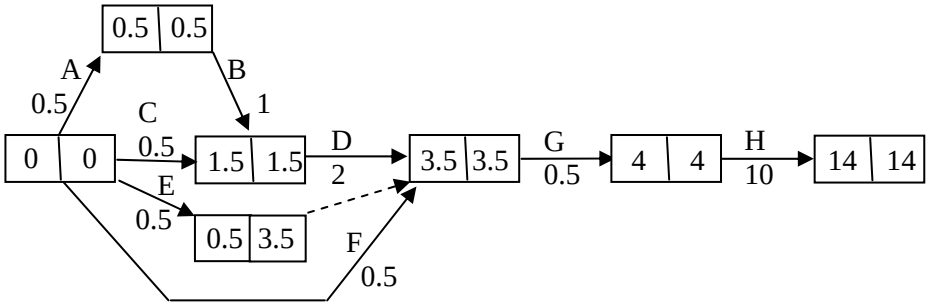
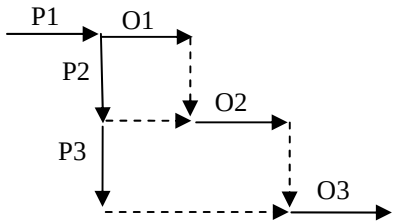
3	**(i)**		"is a subset of"  "shares at least one element with" 	M1 A1 M1 A1 **[4]**	directed graph on 3 vertices all correct undirected on 3 vertices all correct	Arcs must either have an arrow at each end. or no arrows.
3	**(ii)**		eg 	M1 A1 B1 B1 **[4]**	R subset of Q no other subsets $P \cap Q$ $P \cap Q'$ If P and R shown intersecting then can score M1 A1 B0 B0.	Allow area split in two, with third area. eg 

Question			Answer	Marks	Guidance	
4	(i)		Let x be the number of type X motors produced. Let y be the number of type Y motors produced. $10x + 12y \leq 200$ $x \geq 5$ and $y \geq 5$ $0.5x + 0.3y \leq 7$	M1 A1 B1 B1 B1 **[5]**	adequate definition "number of"	Strict inequalities are equally OK
4	(ii)			B1 B1 B1 B1 **[4]**	inclined line inclined line x=5 and y=5 shading follow line errors if shape is 	The guidance level of accuracy throughout this question is ±0.25 on the x coordinate and ±0.25 on the y coordinate. (Look at (8,10) first.) Inaccurate sketch with axis intercepts given is OK.

Question			Answer	Marks	Guidance	
4	(iii)		Profit = 100X + 70Y	B1		
			(5,12.5) or (5,12) 1375 or 1340 (8,10) 1500 (11,5) 1450	M1	optimisation	either profit line or evaluating and comparing at their 3 appropriate points (OK if on graph)
			£1500 profit.	A1	1500 seen cao SC B1 for 1500 without the preceding M mark	
				[3]		
4	(iv)		Solution in range $\left(10 \pm \frac{1}{4},\ 6\frac{2}{3} \pm \frac{1}{4}\right) = \left(9.75 - 10{,}25,\ 6.41\dot{6} - 6.91\dot{6}\right)$	B1	cao	looking for $\left(10,\ 6\frac{2}{3}\right)$
			Identification of one of (9,7), (10,6) and (11,5).	B1	cao	
			Evaluation at all three of (9,7) (10,6) (11,5) **1390** **1420** **1450**	M1		
			So 11 of X and 5 of Y	A1	cao	
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		eg 0–7 → double 8 → single 9 reject and re-draw	M1 A1 **[2]**	reject correct proportions	Rejection can be implied.
5	(ii)		eg 0–5 → double 6,7 → single 8,9 reject and re-draw	M1 A1 **[2]**	reject correct proportions	Rejection can be implied. Ignore rule for (4,0).
5	(iii)		e.g. day selection / doubles / singles / random number 1 5 0 2 4 1 5 double 3 3 2 9, 4 double 4 2 3 0 double 5 1 4 Probability of drawing a single bag on day 5 is now 4/6.	 M1 A1 M1 A1 M1 A1 M1 A1 **[8]**	 allow 5 shown as used on RN list. selection must show RN(s) explicitly new scenario seen explicitly, not implied by day 4 rule a correct day 4 rule selection and new scenario denominator = 6 numerator	For the simulation M1's you need to see a random number being used with their rules Follow a candidate who manages correctly to go from (4,1) to (4,0). It will then gain M1 if it correctly goes to (3,1) on day 4, with A1 if shows no simulation needed. rule must be seen needs RN explicit. Allow new scenario if seen in subsequent probability calculation. Can be implied by 2/3 or 1/3 if correct for their simulation.

Question			Answer	Marks	Guidance
5	(iv)		4 simulations, each ending with 6 bags all scenarios correct	M1 A1 **[2]**	Condone one slip. Condone simulating at (4,0) if correctly done. 6 bags can be implied by probs of thirds or sixths.
5	(v)		Either averaging correct probabilities or sum of singles/30	M1 A1 **[2]**	Correct computation, but allow 1 slip or omission. Correct answer for their simulations.

Question			Answer	Marks	Guidance
6	(i) & (ii)		 Minimum completion time = 14 mins Critical activities ... A, B, D, G, H	M1 A1 A1 A1 A1 M1 A1 M1 A1 B1 B1 **[11]**	activity on arc at least 1 dummy for E and F precedences for D precedences for G rest eg. penalise multiple starts forward pass backward pass If OK at start of dummy. If there is no dummy then these two marks are not available.
6	(iii)		2 people	B1 **[1]**	
6	(iv)		1 person ... 15.5 mins	B1 **[1]**	
6	(v)		 time = 35.5 minutes	B1 B1 **[2]**	network time with small oven
6	(vi)		revised time = 26.5 minutes	B1 **[1]**	time with large oven

RECOGNISING ACHIEVEMENT

GCE

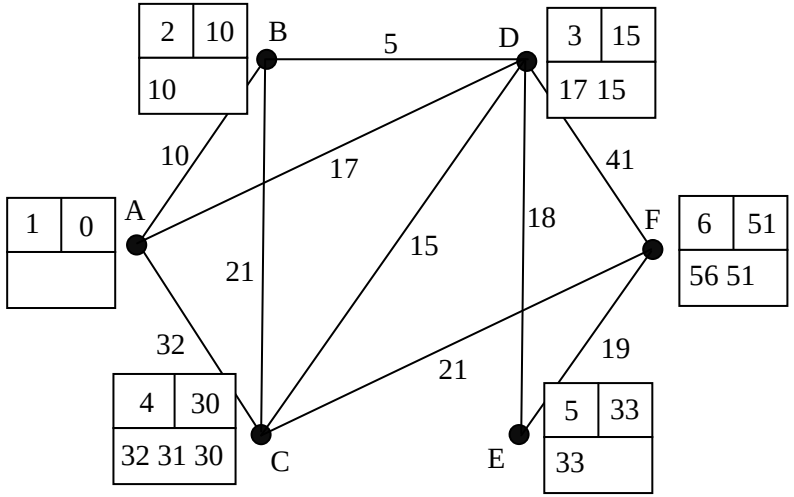
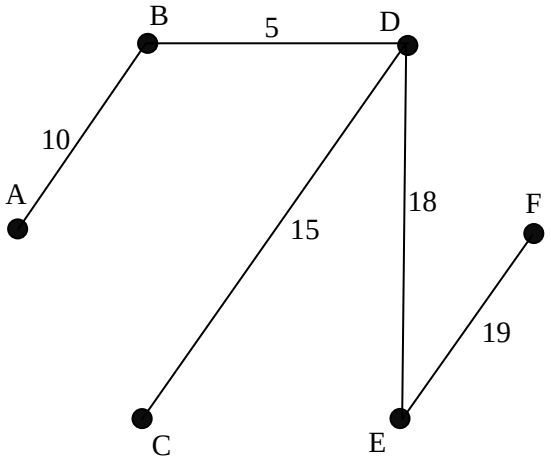
Mathematics (MEI)

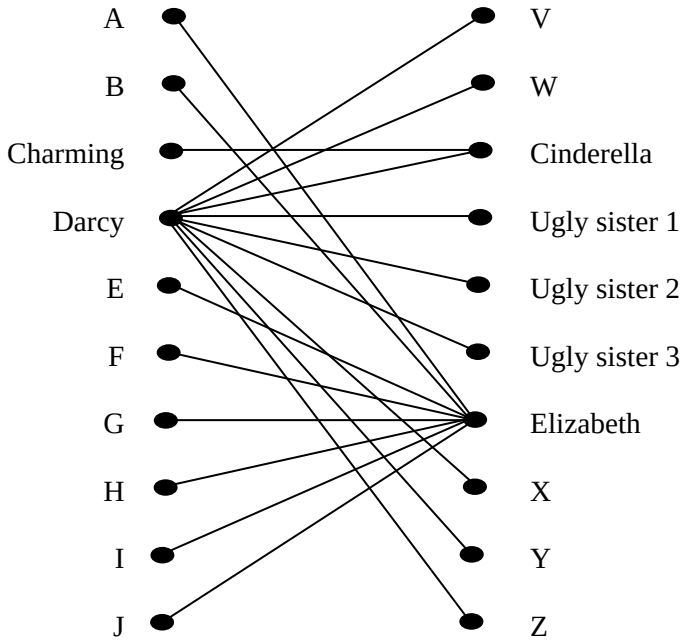
Advanced Subsidiary GCE

Unit **4771:** Decision Mathematics 1

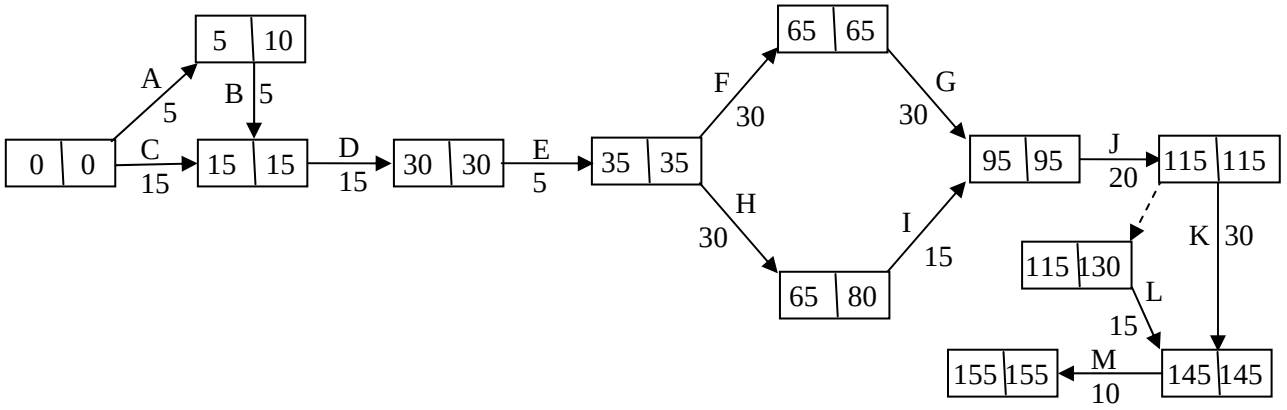
Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

Question			Answer	Marks	Guidance
1	(i)		 Route ... ABDCF Time ... 51 minutes	M1 A1 B1 B1 B1 [5]	Dijkstra (if working values correct at D) working values order of labelling labels route and time
	(ii)		 Time ... 52 minutes	B1 B1 B1 [3]	methodology indicated correct min connector cao

Question			Answer	Marks	Guidance
2	(i)		bipartite	B1 **[1]**	cao
	(ii)		100	M1 A1 **[2]**	allow for 200 cao
	(iii)			B1 B1 B1 **[3]**	Darcy correct Elizabeth correct Panto characters correct
	(iv)		58	M1 A1 **[2]**	$18 + (8 \times 5)$ allow for 98 cao

Question			Answer	Marks	Guidance
3	(i)		Step 1 $x = 0.44$ Step 2 $oldr = 1$ Step 345 $i = 1, j = 0.5, k = 0.5$ Step 6 $change = 0.22$ Step 7 $newr = 1.22$ Step 9 $oldr = 1.22$ Steps 10 11 12 $i = 2, j = –0.5, k = –0.125$ Step 13 $change = –0.0242$ Step 14 $newr = 1.1958$ Step 15 $\|change\| = 0.0242$ Step 9 $oldr = 1.1958$ Steps 10 11 12 $i = 3, j = –1.5, k = 0.0625$ Step 13 $change = 0.005324$ Step 14 $newr = 1.201124$ Step 15 $\|change\| = 0.005324$ Step 9 $oldr = 1.201124$ Steps 10 11 12 $i = 4, j = –2.5, k = –0.03906$ Step 13 $change = –0.0014641$ Step 14 $newr = 1.1996599$ Step 15 $\|change\| = 0.0014641$ Step 17 1.1996599	B1 B1 B1 B1 B1 **[6]**	cao set-up (i.e. as far as 1.22) 3 steps correct new estimate (1.1958) iteration (to 1.201124) iteration and end
	(ii)		$1 – 0.22 – 0.0242 – 0.005324 – 0.0014641 = 0.7490119$	M1 A1 **[2]**	use of –0.44 as shown SC1 (cao) for algorithm repeated or answer only

Question			Answer	Marks	Guidance
4	(i) & (ii)		 Minimum completion time = 155 minutes Critical activities are C, D, E, F, G, J, K and M	M1 A1 A1 A1 A1 **[5]** M1 A1 M1 A1 B1 B1 **[6]**	activity on arc single start and end A, B, C OK J, K, L OK rest OK forward pass (must have at least one join correct backward pass (must have at least one burst correct) cao cao
4	(iii)		eg Kate: C C C D D D E1 F1 F1 F1 F1 F1 F1 H1 H1 H1 H1 H1 H1 Pete: A B E2 F2 F2 F2 F2 F2 F2 H2 H2 H2 H2 H2 H2 cont.: G1 G1 G1 G1 G1 G1 I1 I1 I1 J1 J1 J1 J1 K1 K1 K1 K1 K1 K1 cont: G2 G2 G2 G2 G2 G2 I2 I2 I2 J2 J2 J2 J2 K2 K2 K2 K2 K2 K2 cont: L1 L1 L1 M1 M1 cont: L2 L2 L2 M2 M2 215 minutes (3 hours and 35 minutes)	 B1 B1 B1 **[3]**	 ABCD rest ... watch for M's after K's and L's cao
4	(iv)		Two more people would be needed, so that the H's and I's could be done at the same time as the F's and G's, and so that the two L's could be done at the same time as the two K's	B1 B1 **[2]**	cao reasoning

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5</td><td>(i)</td><td></td><td>e.g. 0 → 0

1, 2 → 1
3, 4, 5 → 2
6, 7 → 3
8, 9 → 4</td><td>M1

A1

[2]</td><td>either 0.2 for 1 or 0.3 for 2

all proportions correct</td></tr>
<tr><td>5</td><td>(ii)</td><td></td><td>random number 5 3 0 2 4 7 9 1 1 8
number of occupants 2 2 0 1 2 3 4 1 1 4</td><td>M1
A1
[2]</td><td>8 outcomes correct
all correct</td></tr>
<tr><td>5</td><td>(iii)</td><td></td><td>e.g. 0, 1 → child
2 – 9 → adult</td><td>B1

[1]</td><td>must use all 10 digits
cao</td></tr>
<tr><td>5</td><td>(iv)</td><td></td><td>random number
child (C) or adult (A)
<table><tr><td>chair</td><td colspan="2">1</td><td colspan="2">2</td><td colspan="2">3</td><td colspan="2">4</td><td colspan="2">5</td><td colspan="2">6</td><td colspan="2">7</td><td colspan="2">8</td><td colspan="2">9</td><td colspan="2">10</td></tr><tr><td>occ1</td><td>6</td><td>A</td><td>0</td><td>C</td><td>9</td><td></td><td>6</td><td>A</td><td>2</td><td>A</td><td>9</td><td>A</td><td>1</td><td>C</td><td>5</td><td>A</td><td>6</td><td>A</td><td>2</td><td>A</td></tr><tr><td>occ2</td><td>2</td><td>A</td><td>6</td><td>A</td><td>5</td><td></td><td>2</td><td></td><td>1</td><td>C</td><td>1</td><td>C</td><td>4</td><td>A</td><td>8</td><td></td><td>1</td><td></td><td>9</td><td>A</td></tr><tr><td>occ3</td><td>3</td><td></td><td>7</td><td></td><td>2</td><td></td><td>1</td><td></td><td>3</td><td></td><td>6</td><td>A</td><td>6</td><td>A</td><td>5</td><td></td><td>3</td><td></td><td>5</td><td>A</td></tr><tr><td>occ4</td><td>3</td><td></td><td>1</td><td></td><td>1</td><td></td><td>2</td><td></td><td>8</td><td></td><td>0</td><td></td><td>6</td><td>A</td><td>0</td><td></td><td>5</td><td></td><td>1</td><td>C</td></tr></table>
number of children = 5
number of adults = 15</td><td>M1

A1

[2]</td><td>8 chairs OK

all OK</td></tr>
<tr><td>5</td><td>(v)</td><td></td><td>40 children and 120 adults</td><td>B1
[1]</td><td>FT... × by 8</td></tr>
<tr><td>5</td><td>(vi)</td><td></td><td>e.g. 00 – 06 → 0
07 – 13 → 1
14 – 34 → 2
35 – 55 → 3
56 – 90 → 4
91 – 99 ignore and “redraw”</td><td>M1
A1
A1

[3]</td><td>ignore some
proportions correct
efficient</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5</td><td>(vii)</td><td></td><td>random number 23 65 07 99 37 45
number of occupants 2 4 1 – 3 3</td><td>M1
A1
[2]</td><td>3 OK
all correct FT</td></tr>
<tr><td>5</td><td>(viii)</td><td></td><td>
<table>
<tr><td>chair</td><td colspan="2">1</td><td colspan="2">2</td><td colspan="2">3</td><td colspan="2">4</td><td colspan="2">5</td></tr>
<tr><td>occ1</td><td>1</td><td>C</td><td>9</td><td>A</td><td>6</td><td>A</td><td>8</td><td>A</td><td>1</td><td>C</td></tr>
<tr><td>occ2</td><td>2</td><td>A</td><td>2</td><td>A</td><td>8</td><td></td><td>0</td><td>C</td><td>8</td><td>A</td></tr>
<tr><td>occ3</td><td>6</td><td></td><td>3</td><td>A</td><td>2</td><td></td><td>2</td><td>A</td><td>1</td><td>C</td></tr>
<tr><td>occ4</td><td>4</td><td></td><td>6</td><td>A</td><td>1</td><td></td><td>9</td><td></td><td>4</td><td></td></tr>
</table>
number of children = 4
number of adults = 9

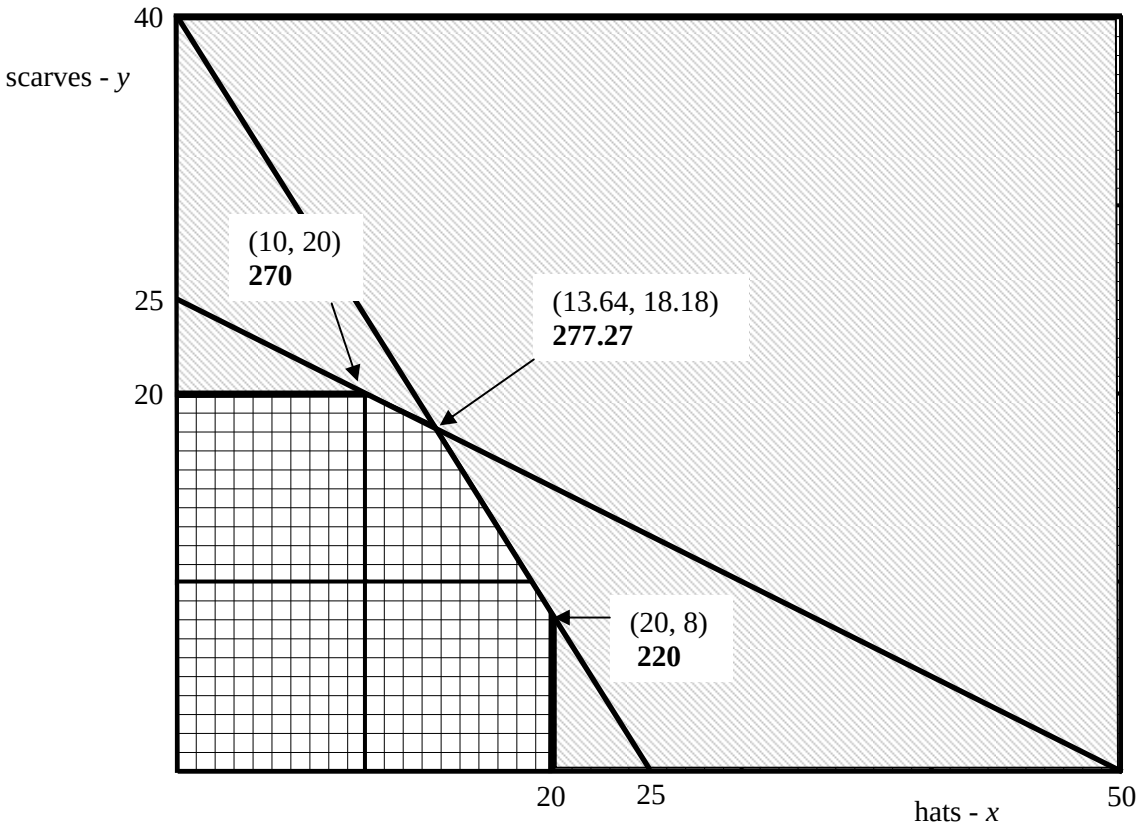
64 children and 144 adults</td><td>

B1

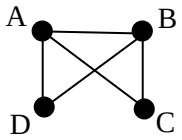
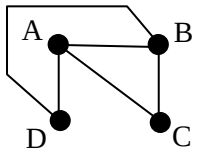
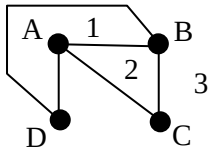
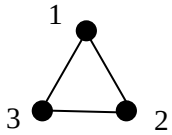
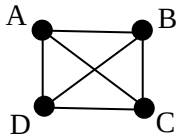
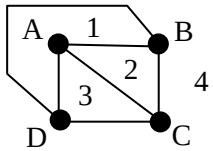
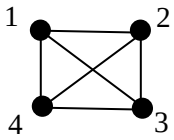
B1
[2]</td><td>

FT ... all correct

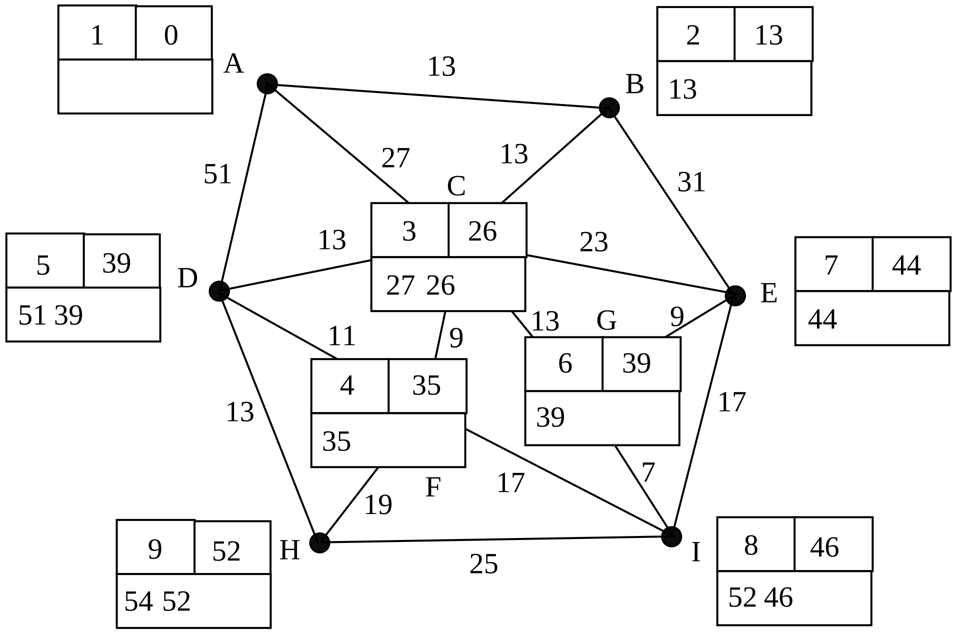
FT ... × by 16</td></tr>
<tr><td>5</td><td>(ix)</td><td></td><td>greater reliability or more representative</td><td>B1
[1]</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance
6	(i)		e.g. Let x be the number of hats which Jean knits Let y be the number of scarves which Jean knits $1.5x + 3y \leq 75$, i.e. $x + 2y \leq 50$ $4x + 2.5y \leq 100$, i.e. $8x + 5y \leq 200$ $x \leq 20$ and $y \leq 20$	B1 B1 B1 B1 B1	must say "number of" or vice-versa of course simplification not required both
			 	B1 B1 B1 B1 B1	lines (cao) shading ... follow any set of two horizontal, two vertical and two negatively inclined lines which give a hexagon in the bottom left corner.
				[10]	

Question			Answer	Marks	Guidance
6	(ii)		Objective = $7x + 10y$ Best non-integer point Solution ... (12, 19) **274**, (13, 18) **271** or (14, 17) **268** So 12 hats and 19 scarves	B1 M1 A1 B1 **[4]**	objective considering profits at their three points as indicated cao cao
6	(iii)		10 hats and 20 scarves £34	B1 B1 **[2]**	cao FT ... *their answer* – 240

Question			Answer	Marks	Guidance
1	(i)		 e.g. 	M1 A1 B1 **[3]**	simple and connected but not complete. (Ignore directions) cao planar - cao
1	(ii)		e.g.  	M1 A1 **[2]**	exactly 3 vertices cao
1	(iii)		  	B1 M1 A1 **[3]**	complete graph on 4 letters 4 regions cao (planar OK)

Question			Answer	Marks	Guidance
2	(i)		<table><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>comps</td><td>swaps</td></tr><tr><td>i=1</td><td>9</td><td>7</td><td>3</td><td>11</td><td>5</td><td>13</td><td></td><td>5</td><td>3</td></tr><tr><td>i=2</td><td>7</td><td>3</td><td>9</td><td>5</td><td>11</td><td>13</td><td></td><td>4</td><td>3</td></tr><tr><td>i=3</td><td>3</td><td>7</td><td>5</td><td>9</td><td>11</td><td>13</td><td></td><td>3</td><td>2</td></tr><tr><td>i=4</td><td>3</td><td>5</td><td>7</td><td>9</td><td>11</td><td>13</td><td></td><td>2</td><td>1</td></tr><tr><td>i=5</td><td>3</td><td>5</td><td>7</td><td>9</td><td>11</td><td>13</td><td></td><td>1</td><td>0</td></tr></table>	B1 B1 B1 B1 B1 **[5]**	i=2 row OK i=3 row OK FT i=4 and 5 rows OK cao comparisons swaps
2	(ii)		comparisons 6 swaps 3	B1 B1 **[2]**	cao (OK if in 2 parts) cao (OK if in 2 parts)
2	(iii)		further swaps 6	B1 **[1]**	cao

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>3</td><td>(i)</td><td></td><td> 

AB 13
ABC 26
ABCD 39
ABE 44
ABCF 35
ABCG 39
ABCDH 52
ABCGI 46</td><td>B1
B1
B1
B1

B1
B1

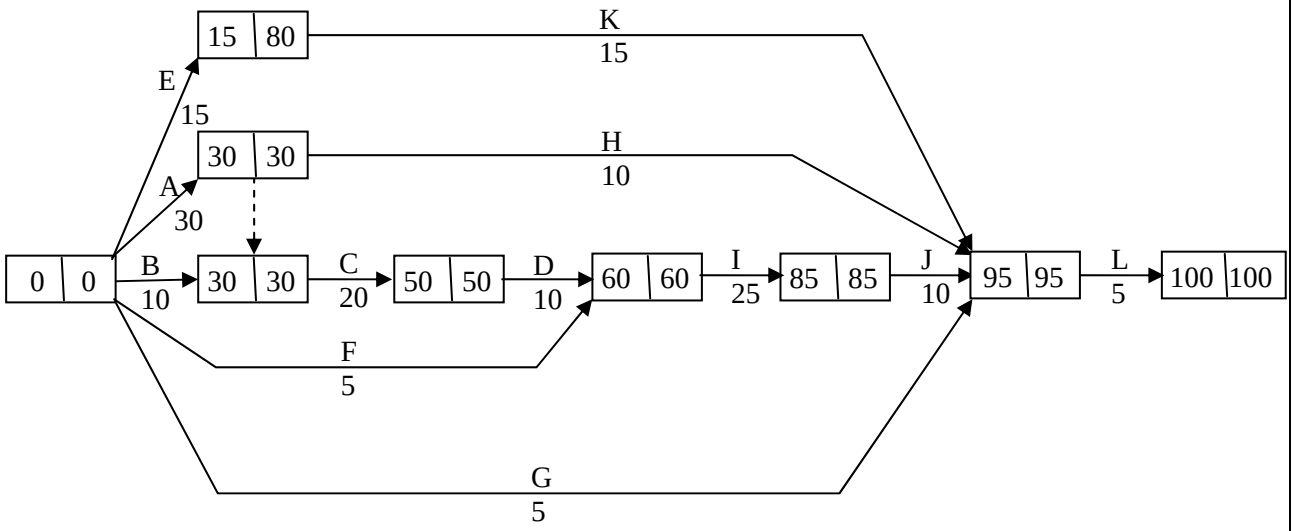
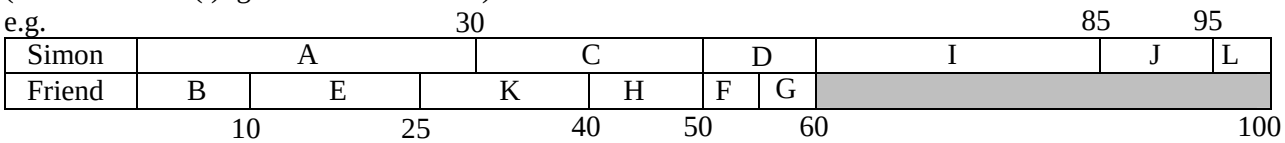
[6]</td><td>Dijkstra – C correct
other working values
order of labelling
labels

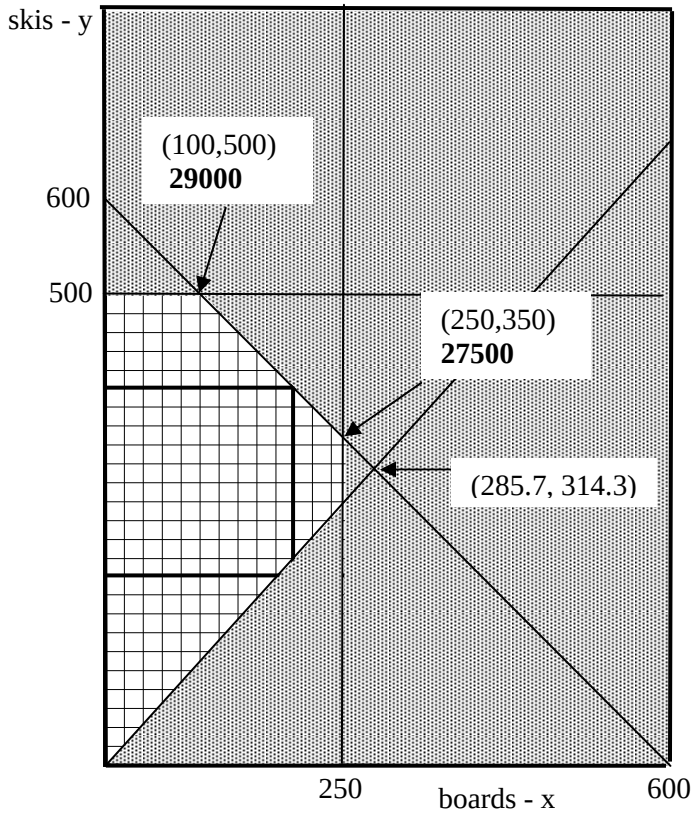
Note that D and G could be labelled in the reverse order.

first 4 pairs
second 4 pairs</td></tr>
<tr><td>3</td><td>(ii)</td><td></td><td>Turn distances to times throughout the network.
Add 10 mins to every arc incident upon C.
(or do Dijkstra twice, once with C deleted, and compare with the adjusted time through C)</td><td>E1

E1

[2]</td><td>Explanations needed, not answers
any correct logic</td></tr>
</table>

Question			Answer	Marks	Guidance
4	(i) & (ii)		 Minimum completion time = 100 minutes Critical activities are A, C, D, I, J and L	M1 A1 A1 A1 A1 **[5]** M1 A1 M1 A1 B1 B1 **[6]**	activity on arc single start and end A, B, C OK D, F, I OK rest OK forward pass (must have at least one join correct FT backward pass (must have at least one burst correct) FT cao cao
4	(iii)		e.g. Critical activities (100 mins) + others. e.g. B has to be done whilst A is underway.	B1 **[1]**	Needs a comparison of times, possibly implied.
4	(iv)		(If L omitted in (i) ignore omission here.) e.g. 	M1 A1 A1 A1 **[4]**	diagram like this or attempted cascade ... no more than 1 omitted activity nowhere needing more than 2 people precedences correct fully correct, inc who does what

Question			Answer	Marks	Guidance
5	(i)		e.g. Let x be the number of snowboards Let y be the number of (pairs of) skis $x + y \leq 600$ $x \leq 250$ and $y \leq 500$ $1.1x \leq y$	B1 B1 B1 B1 B1	 or vice-versa of course both
			 	B1 B1 B1 B1 B1	FT horizontal line FT vertical line FT positive slope line x+y = 600 Note ... error tolerance of +/- half a small square within feasible region. shading ... follow any pentagon bounded by the y-axis, a horizontal line, a vertical line, a negatively inclined line and a positively inclined line
				[10]	

Question			Answer	Marks	Guidance
5	(ii)		Objective = 40x + 50y 29000 at (100,500) 27500 at (250,350) Solution ... 100 snowboards and 500 pairs of skis	B1 M1 A1 **[3]**	objective considering profits at the two indicated points of their pentagon (or using a profit line) cao www
5	(iii)		€10 or more	B1 **[1]**	cao (allow €51 etc)
5	(iv)		35 snowboards	M1 A1 **[2]**	moving to appropriate new feasible point on their negatively inclined line cao... integer! (allowing 30 to 40 for graphical inaccuracy)

Question			Answer	Marks	Guidance
6	(i)		e.g. 0, 1, 2 → 1 3, 4, 5, 6, 7 → 2 8 → 3 9 → 4	M1 A1 **[2]**	either 3 numbers for 1 or 5 numbers for 2 all proportions correct
6	(ii)		random number 5 3 2 4 7 9 1 1 8 time interval (mins) 2 2 1 2 2 4 1 1 3 arrival times 0 2 4 5 7 9 13 14 15 18	M1 A1 B1 **[3]**	all outcomes achieved with first 2 correct for their rule all correct FT accumulation
6	(iii)		e.g. 00 – 13 → 0.1 14 – 41 → 0.25 42 – 83 → 1 84 – 97 → 2 98, 99 ignore and "redraw"	M1 A1 A1 **[3]**	ignore some proportions correct efficient (fewer than 7 rejected)
6	(iv)		random number 23 15 01 32 45 47 86 71 17 83 processing time 0.25 0.25 0.1 0.25 1 1 2 1 0.25 1	M1 A1 **[2]**	first 4 customers correct for their rule all correct FT
6	(v)		e.g. 0 – 5 → 1 6 – 9 → 0.25	B1 **[1]**	
6	(vi)		random number 8 3 0 1 4 0 2 5 7 6 payment time 0.25 1 1 1 1 1 1 1 0.25 0.25	B1 **[1]**	FT
6	(vii)		arrival 0 2 4 5 7 9 13 14 15 18 departure 0.5 3.25 5.1 6.35 9 11 16 18 18.5 19.75	M1 A1 **[2]**	deals with a wait correctly all correct FT
6	(viii)		arrival 0 2 4 5 7 9 13 14 15 18 departure 0.5 3.25 5.1 6.35 9 11 16 18 15.5 19.25	M1 A1 **[2]**	deals with last 3 correctly all correct FT