MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

DECISION MATHEMATICS 1, D1

Practice Paper D1-B

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- There is an insert for this paper for questions 3, 4 and 5.
- The total number of marks for this paper is **72**.

Section A (24 marks)

1 **(a)** Fig.1.1 shows part of a family tree. The vertices represent individuals, with males denoted by capital letters and females by lower case letters. An individual at a lower level of the graph is connected to one male at a higher level and to one female at a higher level – the individual's parents. Thus, for instance, j's father's parents (j's paternal grandparents) are A and b. j's mother is c, and j's mother's parents (j's maternal grandparents) are not shown.

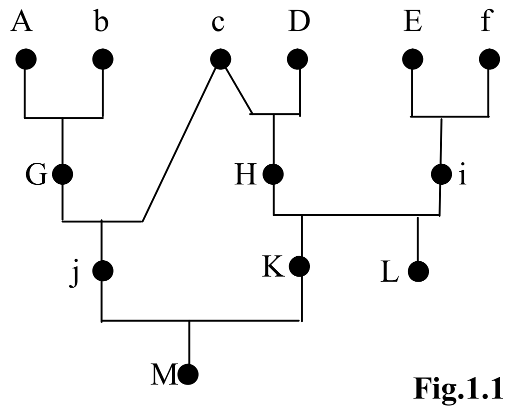


Fig.1.1

(i) List the four grandparents of M. [2]

The male X is not shown on the graph. X's father is M's paternal grandfather. X's mother is M's paternal grandmother.

(ii) Who are X's parents? [1]

(iii) What is the relationship between X and K? [1]

(iv) What grandparent is shared by X and M? [1]

(b) The octahedron shown in Fig.1.2 is represented by a graph whose nodes correspond to the eight faces of the octahedron. Nodes are joined by an arc in the graph if the corresponding faces share an edge in the octahedron.

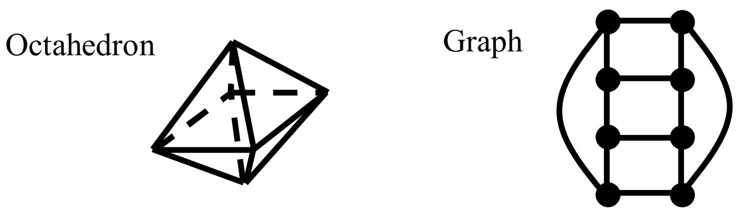
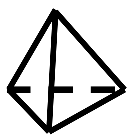
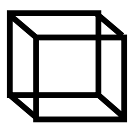


Fig.1.2

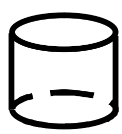
(i) Draw the graph representing a tetrahedron (i.e. a triangular based pyramid). [1]



(ii) Draw the graph representing a cube. [1]



(iii) Draw the graph representing a cylinder. (This has three faces.) [1]



2 The following algorithm counts the number of complete days from the beginning of the year 2000 to a date given in the form $d/m/y$, where d is the day number, m is the month number and y is the year number, $y \geq 2000$.

- start with $d - 1$
- add on $365 \times (y - 2000)$
- if $m > 2$ add on the integer part of $\frac{1}{4}(y - 1996)$,
  otherwise add on the integer part of $\frac{1}{4}(y - 1997)$
- if $m = 2$ then add on 31
- if $m = 3$ then add on 59
- if $m = 4$ then add on 90
- if $m = 5$ then add on 120
- if $m = 6$ then add on 151
- if $m = 7$ then add on 181
- if $m = 8$ then add on 212
- if $m = 9$ then add on 243
- if $m = 10$ then add on 273
- if $m = 11$ then add on 304
- if $m = 12$ then add on 334

(i) Apply the algorithm to 14/12/2008. [4]

(ii) Apply the algorithm to 29/09/2012. [2]

(iii) How many complete days are there between 14/12/2008 and 29/09/2012? [2]

3 **There is an insert for this question. Answer all of the question on the insert.**

The network in Fig.3 shows direct distances between nodes.

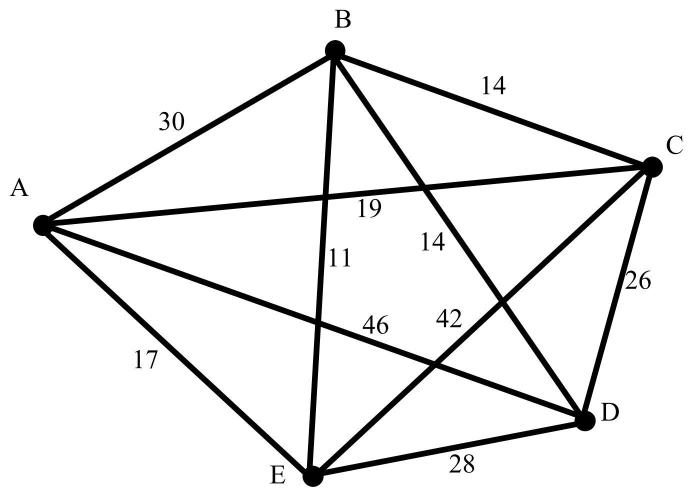


Fig.3

(a) Use Dijkstra's algorithm to find the shortest routes from A to B and from A to D. Give the routes and their lengths. [6]

(b) Dijkstra's algorithm has quadratic complexity. A computer implementation of Dijkstra takes 5 seconds to solve a shortest distance problem for a complete network on 20 vertices. Approximately how long will it take for a complete network on 50 vertices? [2]

Section B (48 marks)

4 **There is an insert for part (a) of this question.**

(a) Activity X is part of a large project. Fig.4.1 shows that part of the project activity network relating to X. Each activity relating to X is shown, together with its relevant early event time or late event time. Activity durations are shown on the arcs.

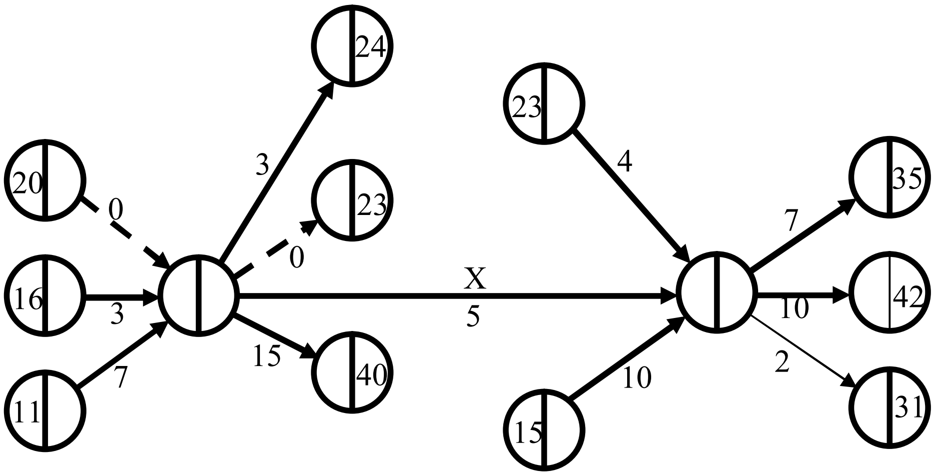


Fig.4.1

(i) Compute the early time and the late time for the "i" event for X, and the early time and the late time for the "j" event for X. [4]

(ii) Calculate the total float for X and the independent float for X. [3]

(b) The precedences and durations for the activities of a project are shown in table 4.2.

Activity	Immediate predecessors	Duration
A	–	2
B	–	3
C	A, B	4
D	B	2
E	C	7
F	C, D	3
G	C, D	5
H	F, G	2

Table 4.2

(i) Draw an activity-on-arc network for the project. [6]

(ii) Find the critical activities and the minimum duration. [3]

5 **There is an insert for parts (v) and (vi) of this question.**

The time that Alf the greengrocer takes to serve a customer is 3.5 minutes with probability 0.25, 3 minutes with probability 0.5, or 2.5 minutes with probability 0.25.

(i) Give an efficient rule for using two-digit random numbers to simulate Alf's service times. [2]

(ii) Use the following random numbers to simulate the service times for 10 customers.

Random numbers: 23; 34; 04; 89; 61; 76; 57; 72; 34; 42 [2]

The intervals between customers arriving at Alf's shop follow the distribution given in table 5.

Arrival Intervals (minutes)	1	2	3	4
Probability	$\frac{1}{7}$	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

Table 5

(iii) Give an efficient rule for using two-digit random numbers to simulate customer arrival intervals. [2]

(iv) Use the following random numbers to simulate the arrival **times** for 10 customers.

Random numbers: 62: 01; 99; 36; 54; 82; 89; 07; 09; 11; 50 [2]

(v) Use the table given in the insert to simulate the arrival, queuing and service of ten customers at Alf's shop. Use your simulated service times from part (ii) and your simulated arrival times from part (iv).

What do you think will happen to the queue in the long run? [5]

Suppose now that potential customers "balk" (i.e. do not enter the shop, but turn and leave) if there are already three customers in the shop.

(vi) In the simulation, which customers will balk? [2]

(vii) What effect does this have on the simulated queuing time for the 10^{th} customer? [1]

6 John wants to spend £5 of his Christmas money on plain and milk chocolates.

He can buy loose plain chocolates at 6p each and loose milk chocolates at 7p each.

He can also buy surplus Christmas presentation boxes at £2 each. These each contain 25 plain chocolates and 25 milk chocolates.

John wants to have at least twice as many milk chocolates as plain chocolates.

Let x be the number of loose plain chocolates that John buys, y be the number of loose milk chocolates, and z the number of presentation boxes.

(i) Give an expression in terms of x, y and z for the total number of chocolates that John buys. [2]

(ii) Give an expression in terms of x, y and z for the cost of John's purchases. [2]

(iii) Explain why the constraint that there should be at least twice as many milk chocolates as plain chocolates can be written as $y \geq 25z + 2x$. [1]

John wishes to maximise the number of chocolates he can buy, subject to his £5 limit and subject to buying at least twice as many milk chocolates as plain chocolates.

(iv) Draw a graph to show that, if $z = 2$, then there is no feasible solution to the problem. [3]

(v) Use a graphical method to solve the problem when $z = 1$. Show that John can purchase 93 chocolates in total, and give all of the available solutions. [8]