D1 JAN13 **Write your answers in the D1 answer book for this paper.**

1.

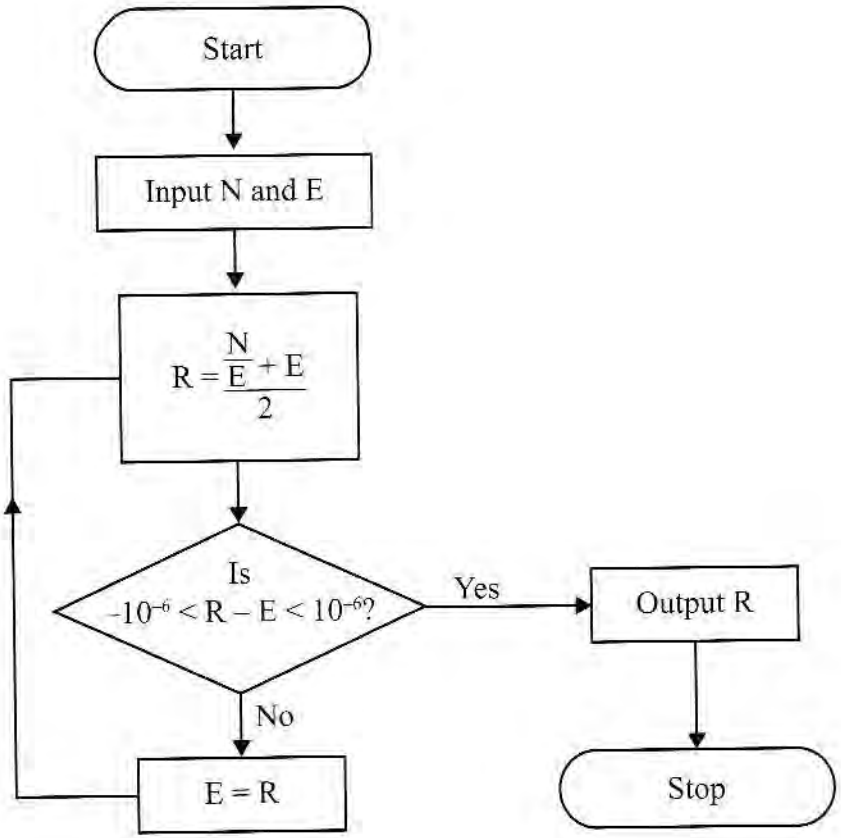


Figure 1

Hero's algorithm for finding a square root is described by the flow chart shown in Figure 1.

Given that N = 72 and E = 8,

(a) use the flow chart to complete the table in the answer book, working to at least seven decimal places when necessary. Give the final output correct to seven decimal places.

(4)

The flow chart is used with N = 72 and E = – 8,

(b) describe how this would affect the output.

(1)

(c) State the value of E which cannot be used when using this flow chart.

(1)

(Total 6 marks)

1. *You may not need to use all the rows in this table*

N	E	R	Is $-10^{-6} < R - E < 10^{-6}$?
72	8	8·5	0·5; NO
	8·5	8·4852941	0·015 ; NO
	8·4852941	8·4852814	0·0000013; NO
	8·4852814	8·4852814	YES
		output = 8·4852814	
	b)	output would be negative	
	c)	zero.	

2. (a) Starting with a list of all the letters of the alphabet in alphabetical order, demonstrate how a binary search is used to locate the letter P. In each iteration, you must make clear your pivot and the part of the list you are retaining.

(4)

(b) Find the maximum number of iterations needed to locate any particular letter of the alphabet. Justify your answer.

(2)

(Total 6 marks)

2.

1 A
2 B
3 C
4 D
5 E
6 F
7 G
8 H
9 I
10 J
11 K
12 L
13 M
14 N
15 O
16 P
17 Q
18 R
19 S
20 T
21 U
22 V
23 W
24 X
25 Y
26 Z

(i) $\frac{1+26}{2} = 13.5 \rightarrow 14^{th} = N$

P after N $\Rightarrow$ delete 1-14

(ii) $\frac{15+26}{2} = 20.5 \rightarrow 21^{st} = U$

P before U $\Rightarrow$ delete $21^{st} - 26^{th}$

(iii) $\frac{15+20}{2} = 17.5 \rightarrow 18^{th} = R$

P before R $\Rightarrow$ delete $18^{th} - 20^{th}$

(iv) $\frac{15+17}{2} = 16^{th} = P$

$\therefore$ P is 16^{th} in the list

b) 4 iterations.

after 1 iteration; delete 13 $\Rightarrow$ 13 remain
after 2 iterations; delete 7 $\Rightarrow$ 6 remain
after 3 iterations; delete 3 $\Rightarrow$ 3 remain
after 4 iterations; delete 2 $\Rightarrow$ 1 remains.

3.

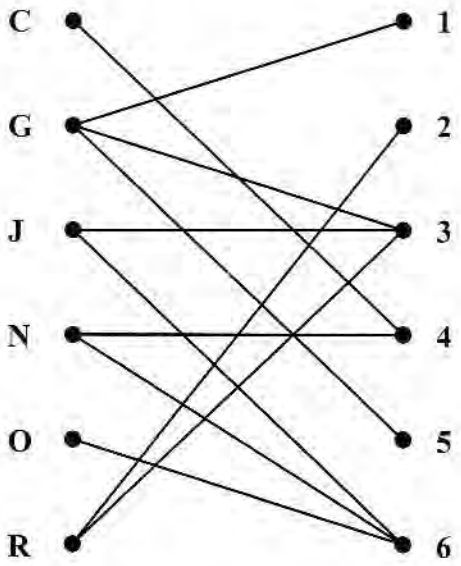


Figure 2

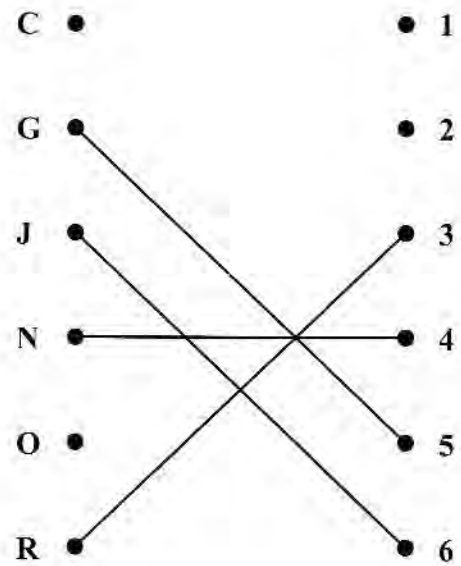


Figure 3

Figure 2 shows the possible allocations of six workers, Charlie (C), George (G), Jack (J), Nurry (N), Olivia (O) and Rachel (R), to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 3 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You should give the alternating path you use and list your improved matching.

(3)

(b) Explain why it is not possible to find a complete matching.

(2)

After training, Charlie adds task 5 to his possible allocations.

(c) Taking the improved matching found in (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. Give the alternating path you use and list your complete matching.

(3)

(Total 8 marks)

3.

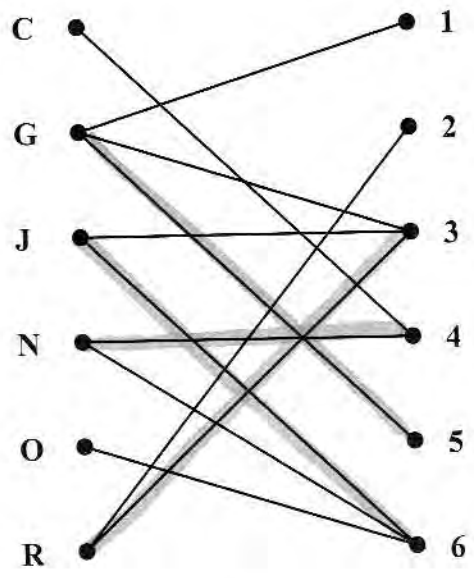


Figure 2

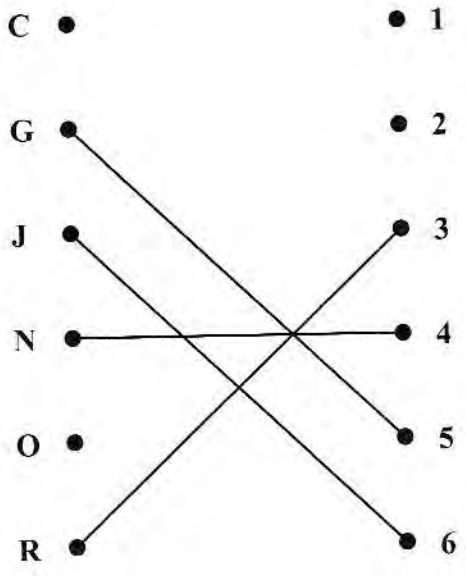


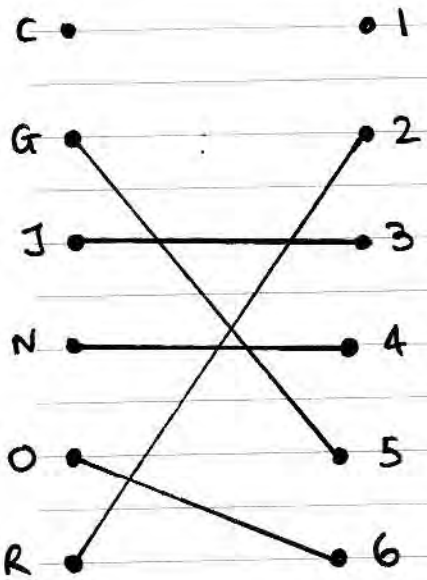
Figure 3

O-6=J-3=R-2

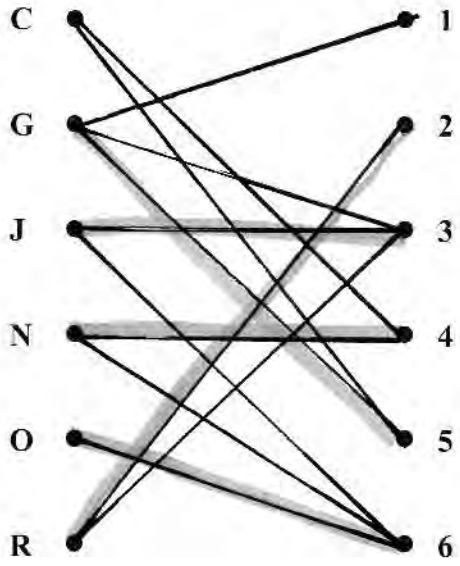
change status

O=6-J=3-R=2

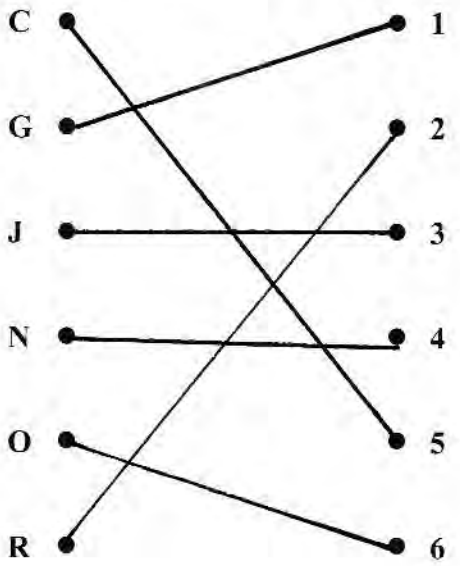
Improved Matching



Question 3 continued



Complete Matching



b) George is the only worker that could be allocated to tasks 1 and 5. He cannot do both, so a ~~maximum~~ complete matching is not posible from the original bipartite graph.

c) $C-5 = G-1$

change status

$C = 5 - G = 1$

4.

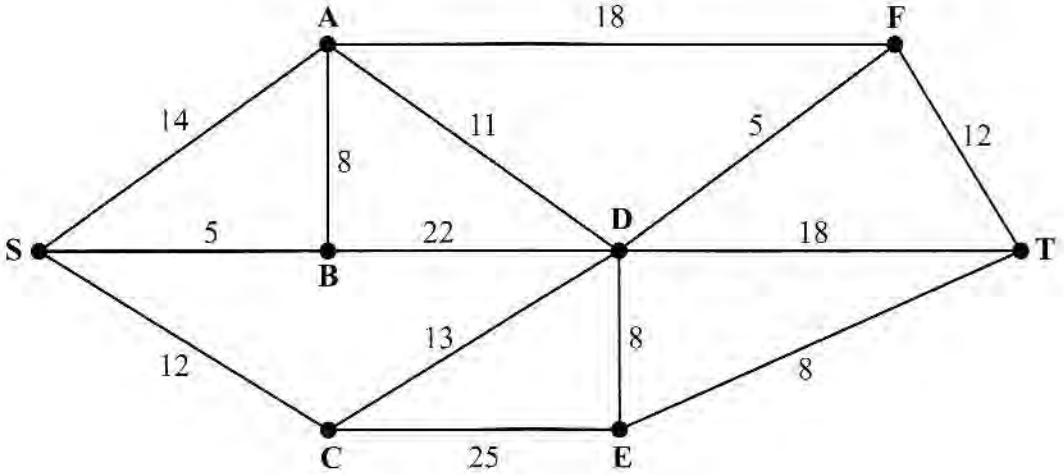


Figure 4

(a) Explain what is meant, in a network, by the term **path**.

(2)

Figure 4 represents a network of canals. The number on each arc represents the length, in miles, of the corresponding canal.

(b) Use Dijkstra's algorithm to find the shortest path from S to T. State your path and its length.

(6)

(c) Write down the length of the shortest path from S to F.

(1)

Next week the canal represented by arc AB will be closed for dredging.

(d) Find a shortest path from S to T avoiding AB and state its length.

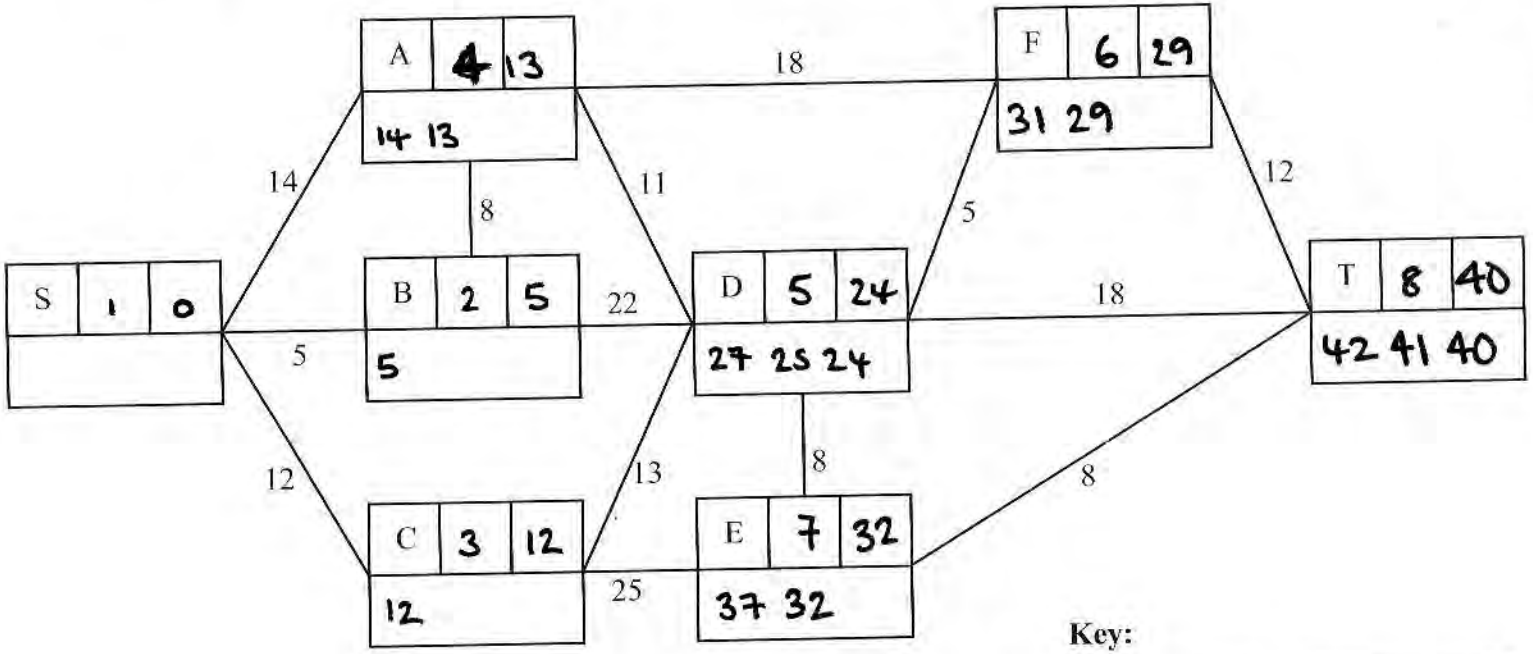
(2)

(Total 11 marks)

4. (a) A path is a finite sequence of edges, such that the end vertex of one edge is the start vertex of the next. No vertex appears more than once.

(b)

40-8=32 ET
32-8=24 DE
24-11=13 AD
13-8=5 BA
5-5=0 SB



Key:

Vertex	Order of labelling	Final value
Working value		

Shortest path SBADET

Length of shortest path 40

c) SF → SBADF (29)

d) SADET (+1) 41 = length.

5.

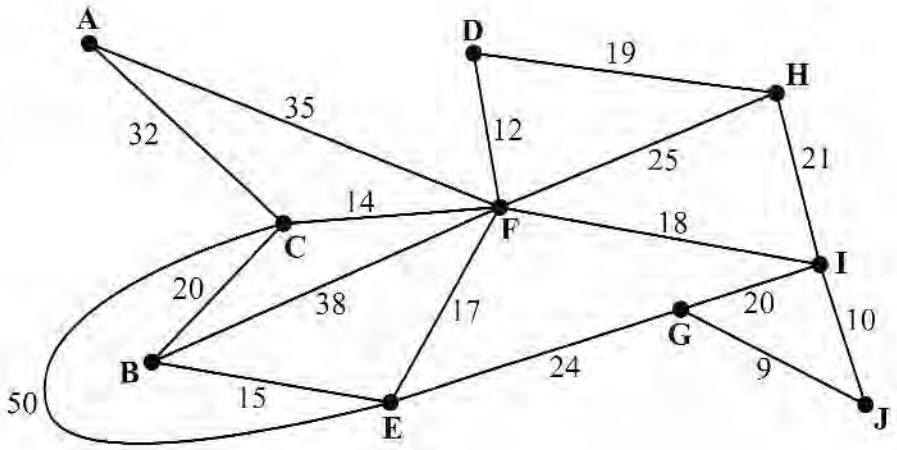


Figure 5

[The weight of the network is 379]

Figure 5 represents the roads in a highland wildlife conservation park. The vertices represent warden stations. The number on each arc gives the length, in km, of the corresponding road.

During the winter months the park is closed. It is only necessary to ensure road access to the warden stations.

(a) Use Prim's algorithm, starting at A, to find a minimum connector for the network in Figure 5. You must state the **order** in which you include the **arcs**.

(3)

(b) Given that it costs £80 per km to keep the selected roads open in winter, calculate the minimum cost of ensuring road access to all the warden stations.

(2)

At the end of winter, Ben inspects all the roads before the park re-opens. He needs to travel along each road at least once. He will start and finish at A, and wishes to minimise the length of his route.

(c) Use the route inspection algorithm to find the roads that will be traversed twice. You must make your method and working clear.

(6)

(d) Find the length of the shortest inspection route.

(1)

If Ben starts and finishes his inspection route at different warden stations, a shorter inspection route is possible.

(e) Determine the two warden stations Ben should choose as his starting and finishing points in order that his route has minimum length. Give a reason for your answer and state the length of the route.

(3)

(Total 15 marks)

5.

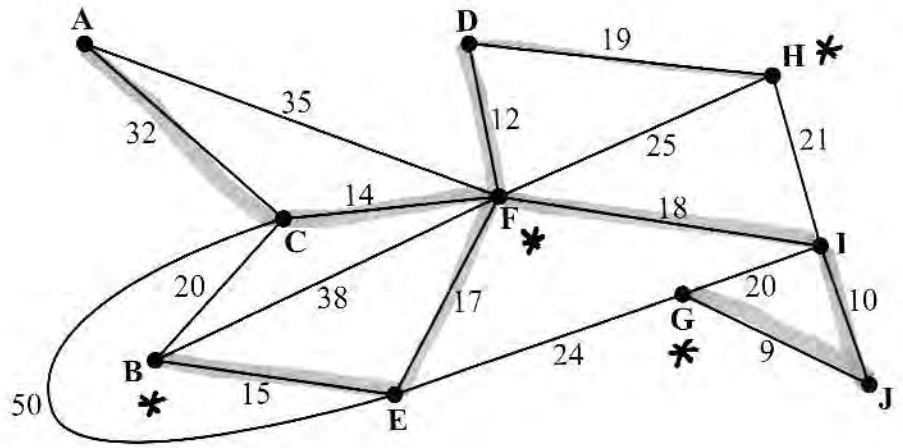


Figure 5

[The weight of the network is 379]

a) AC ; CF ; FD ; FE ; EB ; FI ; IJ ; GJ ; DH

b) weight = 146 km (× £80) £11680

c) BF 32 (E) GH 40 (JI) → 72

BG 39 (E) FH 25 → 64 *

BH 57 (EF) FG 37 (IJ) → 94

traverse BE, EG, FH twice + 64 to route

d) ∴ min route = 443 km.

e) traverse FH twice, + 25 to route

∴ min route = 404 km

start B end at G or start G, end B.

starting and ending at different points means that graph only needs to be semi-eulerian.

6.

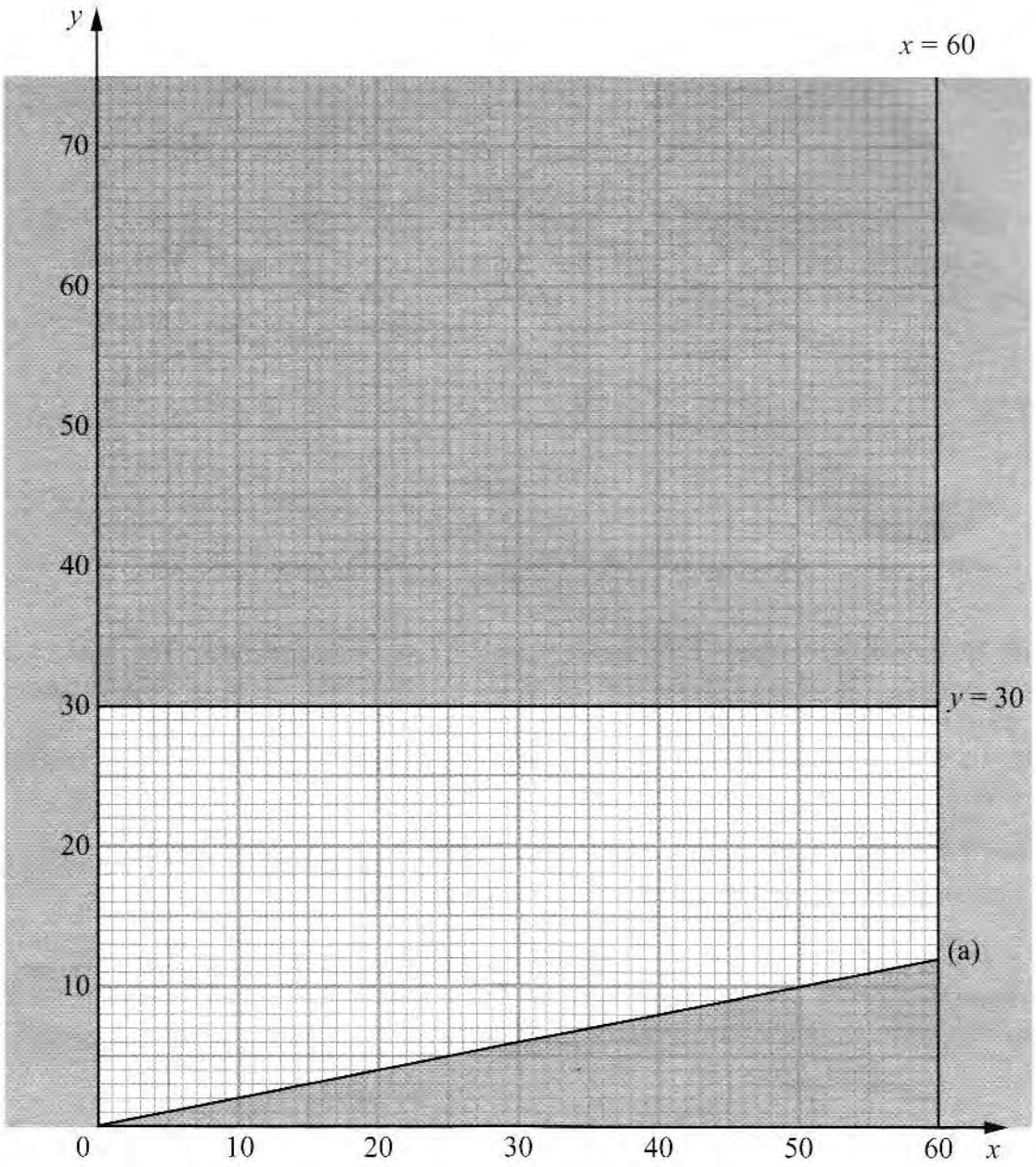


Figure 6

Lethna is producing floral arrangements for an awards ceremony.
She will produce two types of arrangement, Celebration and Party.
Let x be the number of Celebration arrangements made.
Let y be the number of Party arrangements made.

Figure 6 shows three constraints, other than $x, y \geqslant 0$

The rejected region has been shaded.

Given that two of the three constraints are $y \leqslant 30$ and $x \leqslant 60$,

(a) write down, as an inequality, the third constraint shown in Figure 6.

(2)

Each Celebration arrangement includes 2 white roses and 4 red roses.
Each Party arrangement includes 1 white rose and 5 red roses.

Lethna wishes to use at least 70 white roses and at least 200 red roses.

(b) Write down two further inequalities to represent this information.

(3)

(c) Add two lines and shading to Diagram 1 in the answer book to represent these two inequalities.

(2)

(d) Hence determine the feasible region and label it R.

(1)

The times taken to produce each Celebration arrangement and each Party arrangement are 10 minutes and 4 minutes respectively. Lethna wishes to minimise the total time taken to produce the arrangements.

(e) Write down the objective function, T, in terms of x and y.

(1)

(f) Use point testing to find the optimal number of each type of arrangement Lethna should produce, and find the total time she will take.

(4)

(Total 13 marks)

6. a) line $y = \frac{10}{50}x + 0 \Rightarrow y = \frac{1}{5}x$

$\therefore y \geq \frac{1}{5}x$

b) $2x + y \geq 70$ $\quad 4x + 5y \geq 200$

c) minimise T (min) $\quad T = 10x + 4y$

4 10

20 50

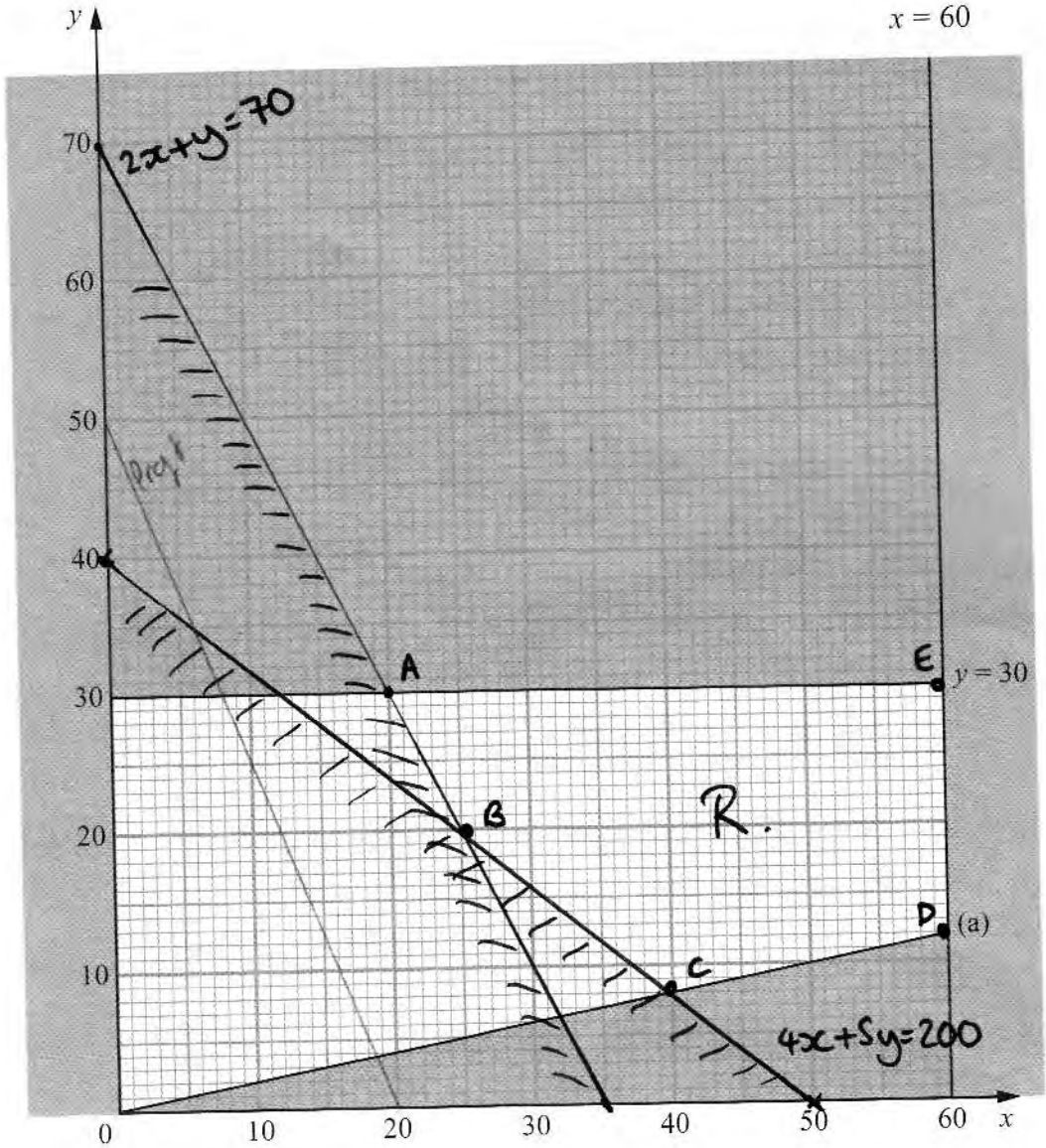


Diagram 1

Vertex	Coordinates	time (= 10x+4y)
A	(20,30)	320 *
B	(25,20)	330
C	(40,8)	432
D	(60,12)	648
E	(60,30)	720

$x = 20$ 'celebration'

$y = 30$ 'party'

time = 320 min

7.

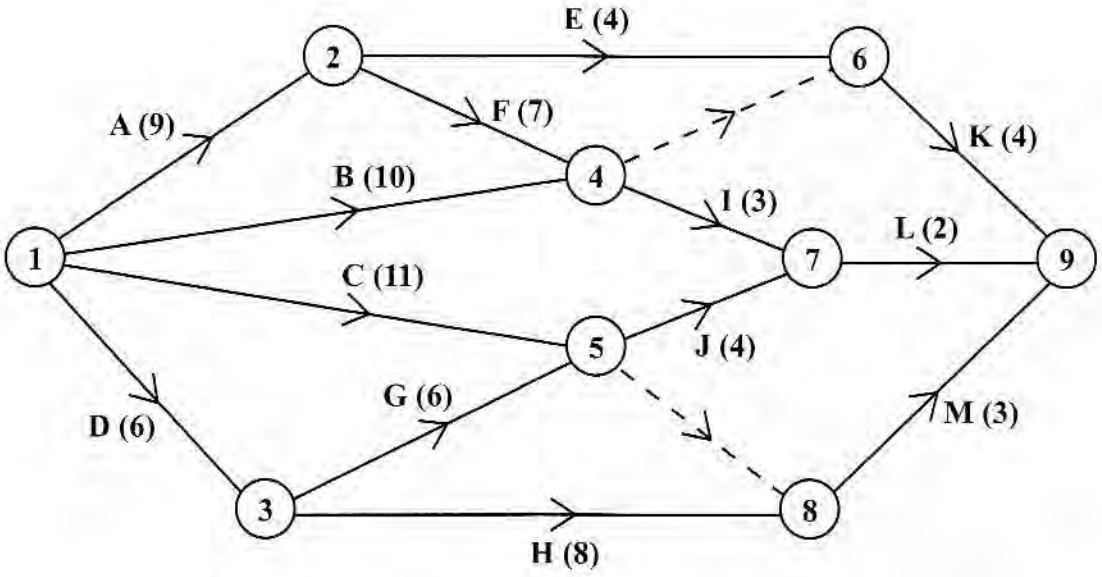


Figure 7

Figure 7 is the activity network relating to a building project. The activities are represented by the arcs. The number in brackets on each arc gives the time to complete the activity. Each activity requires one worker.

The project **must** be completed in the shortest possible time.

(a) Explain the reason for the dotted line from event 4 to event 6 as shown in Figure 7.

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(c) State the critical activities.

(1)

(d) Calculate the total float for activity G. You must make the numbers you use in your calculation clear.

(2)

(e) Draw a Gantt chart for this project on the grid provided in the answer book.

(4)

(f) State the activities that **must** be happening at time 5.5

(1)

(g) Use your Gantt chart to determine the minimum number of workers needed to complete the project in the minimum time. You must justify your answer.

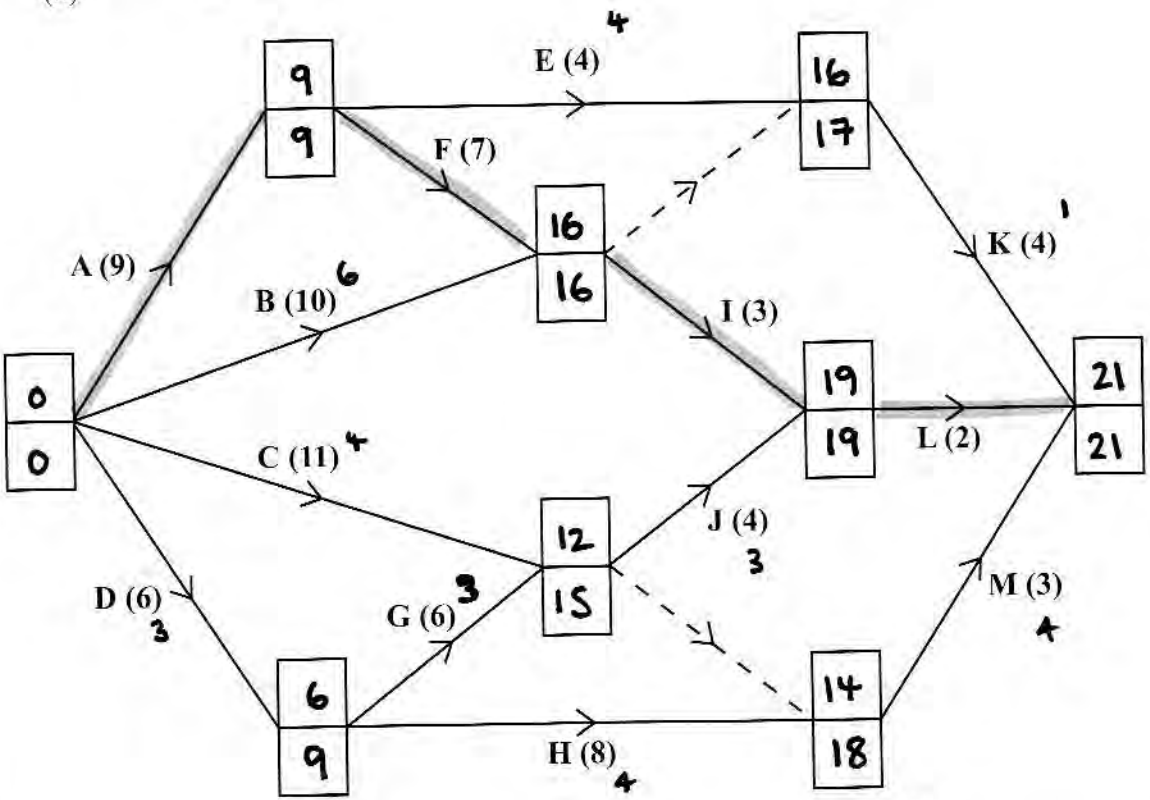
(2)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

7. (a) activity K depends on activities B, E and F. But activity I depends on activities B and F only.

(b)



Key:

Early event time
Late event time