

D1 May 2012

Write your answers in the D1 answer book for this paper.

1. A carpet fitter needs the following lengths, in metres, of carpet.

20 33 19 24 31 22 27 18 25

He cuts them from rolls of length 50 m.

- (a) Calculate a lower bound for the number of rolls he needs.
You must make your method clear. (2)
- (b) Use the first-fit bin packing algorithm to determine how these lengths can be cut from rolls of length 50 m. (3)
- (c) Carry out a bubble sort to produce a list of the lengths needed in **descending** order.
You need only give the state of the list after each pass. (4)
- (d) Apply the first-fit decreasing bin packing algorithm to show how these lengths may be cut from the rolls. (3)

(Total 12 marks)

$$1. \text{ Lower Bound} = \frac{219}{50} = 4.38$$

\Rightarrow 5 rolls needed

b) Bin 1: 20, 19 — (11) 6 rolls
 Bin 2: 33, — (17)
 Bin 3: 24, 22 — (4) (81m of waste)
 Bin 4: 31, 18 — (1)
 Bin 5: 27 — (23)
 Bin 6: 25 — (25)

c)

	20	33	19	24	31	22	27	18	25
1 st pass	33	20	24	31	22	27	19	25	18
2 nd pass	33	24	31	22	27	20	25	19	18
3 rd pass	33	31	24	27	22	25	20	19	18
4 th pass	33	31	27	24	25	22	20	19	18
5 th pass	33	31	27	25	24	22	20	19	18
final pass	33	31	27	25	24	22	20	19	18

d) Bin 1: 33 — (17) 5 rolls
 Bin 2: 31, 19 — (0)
 Bin 3: 27, 22 — (1) (31m of waste)
 Bin 4: 25, 24 — (1)
 Bin 5: 20, 18 — (12)

2.

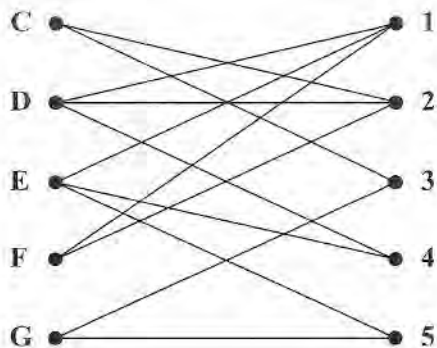


Figure 1

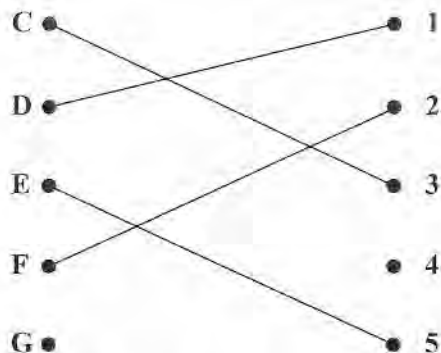


Figure 2

Figure 1 shows the possible allocations of five workers, Charles (C), David (D), Ellie (E), Freya (F) and Georgi (G), to five tasks, 1, 2, 3, 4 and 5.

Figure 2 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. State clearly the alternating path that you use and list your final matching. (4)

(b) Find another solution starting from the given initial matching. You should state the alternating path and list the complete matching it gives. (3)

(Total 7 marks)

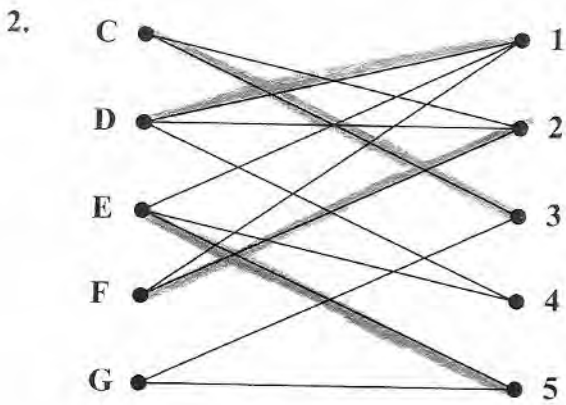


Figure 1

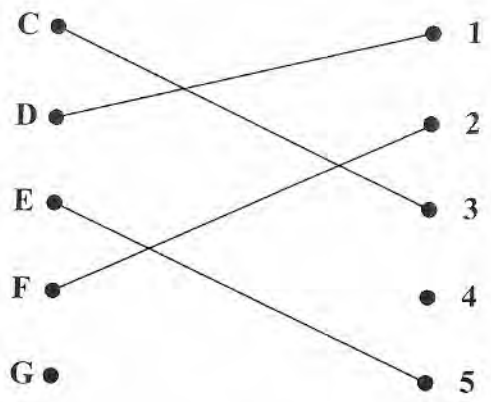


Figure 2

a) $G - 5 = E - 4$

b) $G - 5 = E - 1 = D - 4$

change status

change status

$G = 5 - E = 4$

$G = 5 - E = 1 - D = 4$

Complete Matching

Complete Matching

$C = 3$

$C = 3$

$D = 1$

$D = 4$

$E = 4$

$E = 1$

$F = 2$

$F = 2$

$G = 5$

$G = 5$

3.

	A	B	C	D	E	F	G
A	-	15	19	-	22	24	-
B	15	-	-	8	13	-	-
C	19	-	-	12	-	16	-
D	-	8	12	-	10	-	18
E	22	13	-	10	-	15	16
F	24	-	16	-	15	-	17
G	-	-	-	18	16	17	-

The table shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

- (a) Complete the drawing of the network in Diagram 1 of the answer book by adding the necessary arcs from vertex D together with their weights. (2)
- (b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree. (3)
- (c) Draw the minimum spanning tree using the vertices provided in Diagram 2 in the answer book. (1)
- (d) State the weight of the minimum spanning tree. (1)

(Total 7 marks)

3.

	A	B	C	D	E	F	G
A	-	15	19	-	22	24	-
B	15	-	-	8	13	-	-
C	19	-	-	12	-	16	-
D	-	8	12	-	10	-	18
E	22	13	-	10	-	15	16
F	24	-	16	-	15	-	17
G	-	-	-	18	16	17	-

(a)

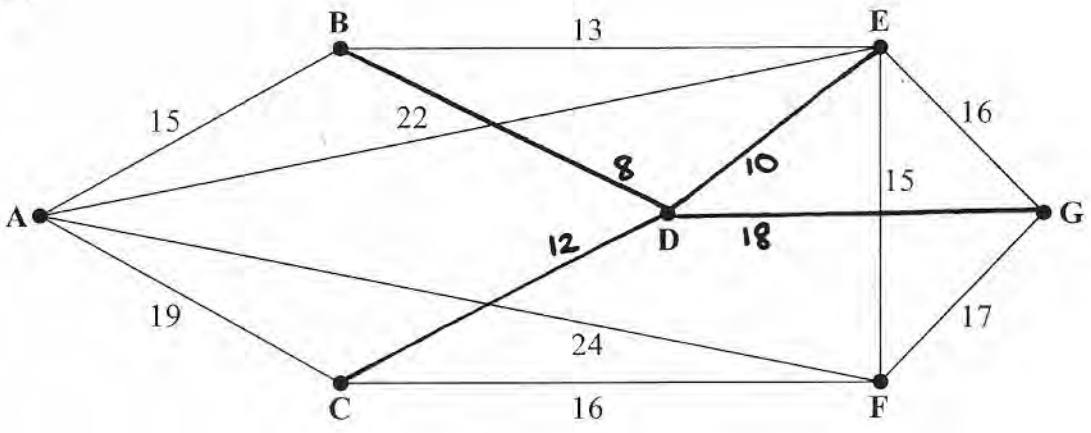


Diagram 1

(b)

DB, DE, DC, EB reject, $[AB]$, $[EG]$, $[EF]$, $[CF \text{ reject}]$
 reject all other.

(c)

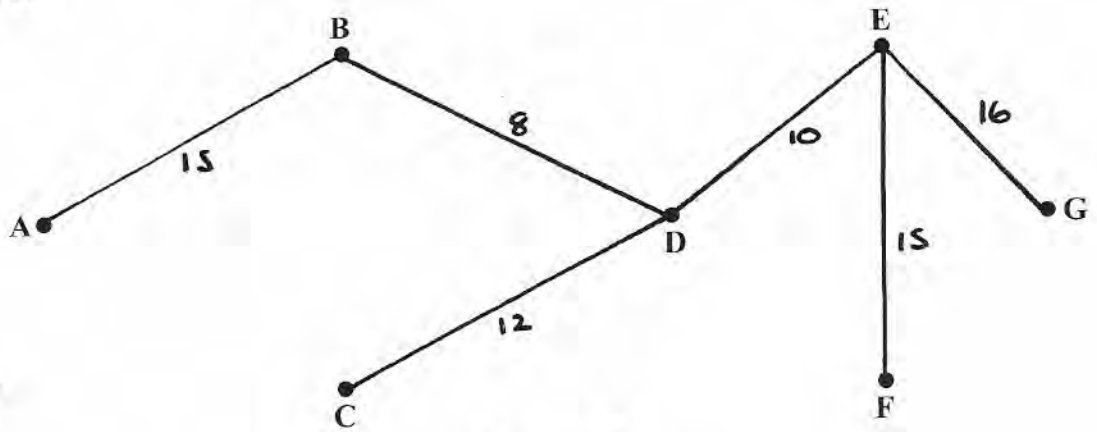


Diagram 2

(d) Weight of minimum spanning tree 76

4.

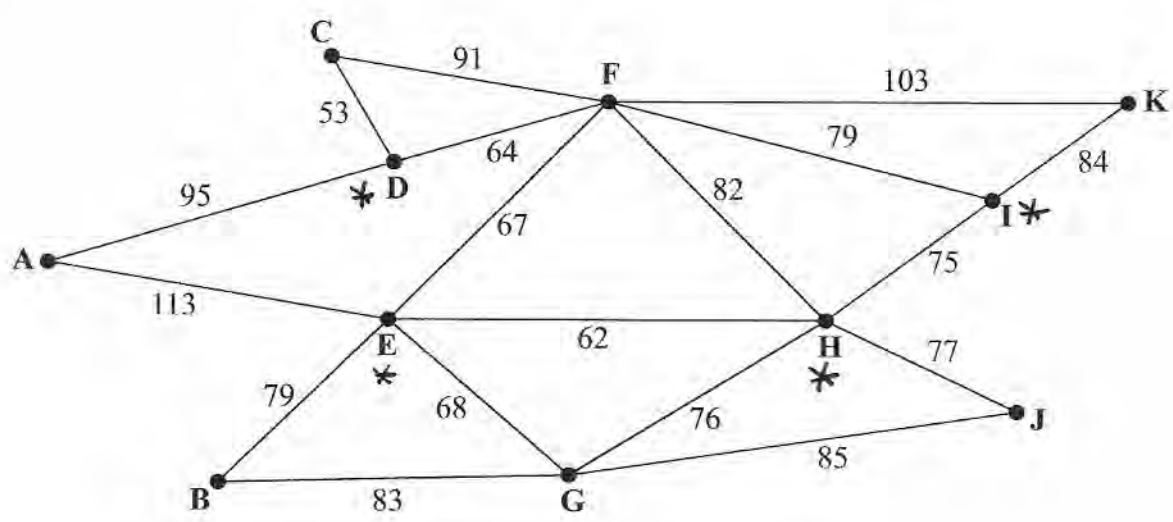


Figure 3

[The total weight of the network is 1436 m]

(a) Explain the term **valency**. (2)

Figure 3 models a system of underground pipes. The number on each arc represents the length, in metres, of that pipe.

Pressure readings indicate that there is a leak in the system and an electronic device is to be used to inspect the system to locate the leak. The device will start and finish at A and travel along each pipe at least once. The length of this inspection route needs to be minimised.

(b) Use the route inspection algorithm to find the pipes that will need to be traversed twice. You should make your method and working clear. (5)

(c) Find the length of the inspection route. (1)

Pipe HI is now found to be blocked; it is sealed and will not be replaced. An inspection route is now required that excludes pipe HI. The length of the inspection route must be minimised.

(d) Find the length of the minimum inspection route excluding HI. Justify your answer. (2)

(e) Given that the device may now start at any vertex and finish at any vertex, find a minimum inspection route, excluding HI. (2)

(Total 12 marks)

a) The valency of a node is the number of arcs incident to it.

b)

DE F	131	HI	75	(206)	Traverse DF, FI, EH twice
DI F	143	EH	62	(205) *	
DH F	146	EI	137	(283)	

c) $1436 + 205 = \underline{1641m}$

d) traverse DF, FE twice + 131 to route 75 removed.

$1436 + 131 - 75 = \underline{1492m}$

e) [C start/end D/E, length = 1361]

DCFKIFHJGBEADFEHGE

5.

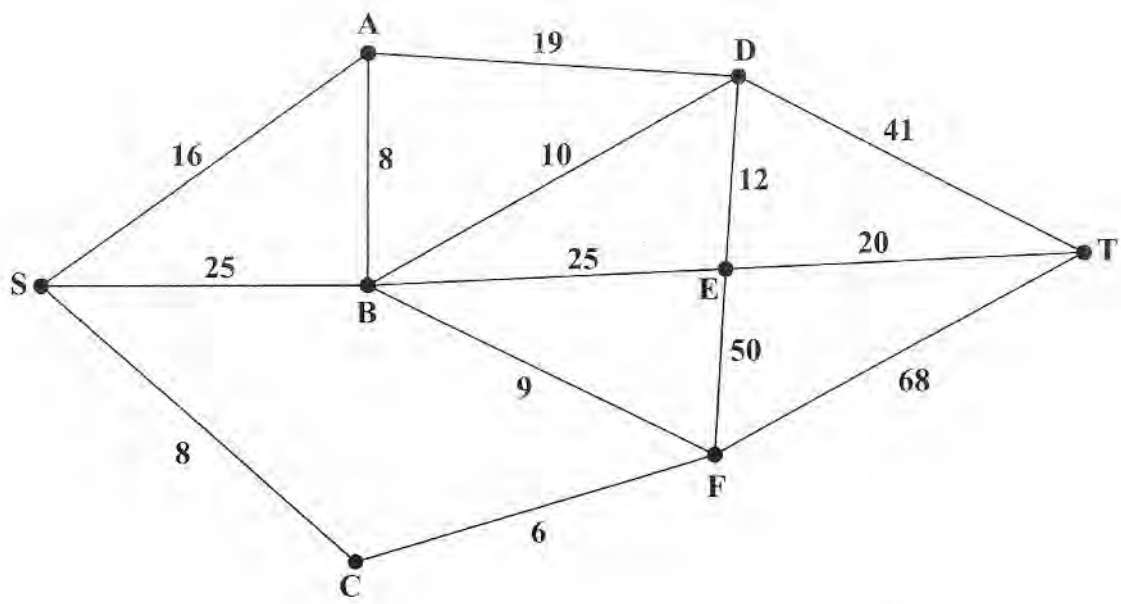


Figure 4

Figure 4 shows a network of roads. The number on each arc represents the length, in miles, of the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest route from S to T. State your shortest route and its length. (6)

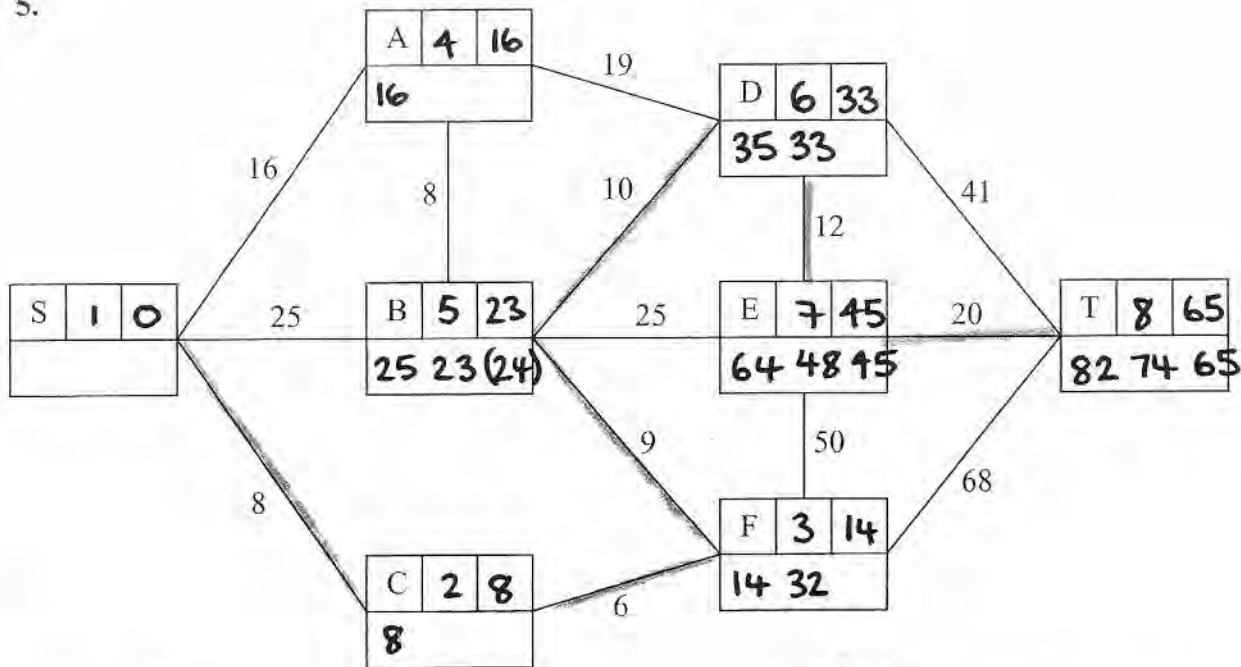
(b) Explain how you determined your shortest route from your labelled diagram. (2)

Due to flooding, the roads in and out of D are closed.

(c) Find the shortest route from S to T avoiding D. State your shortest route and its length. (2)

(Total 10 marks)

5.



$65 - 20 = 45$ ET
 $45 - 12 = 33$ DE
 $33 - 10 = 23$ BD
 $23 - 9 = 14$ FB
 $14 - 8 = 8$ CF
 $8 - 8 = 0$ SC

Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route: SCFBDET

Length of shortest route: 65

c) SCFBET (+3) 68

6.

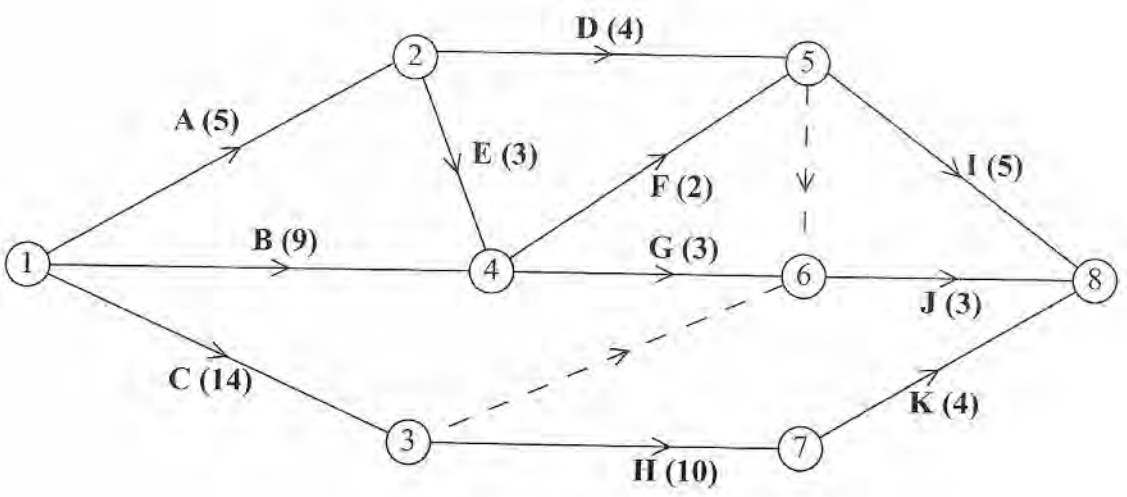


Figure 5

Figure 5 is the activity network relating to a development project. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Complete the precedence table in the answer book. (2)
- (b) Complete Diagram 1 in the answer book to show the early event times and late event times. (4)
- (c) Calculate the total float for activity E. You **must** make the numbers you use in your calculation clear. (2)
- (d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working. (2)
- (e) Schedule the activities using the minimum number of workers so that the project is completed in the minimum time. (4)

(Total 14 marks)

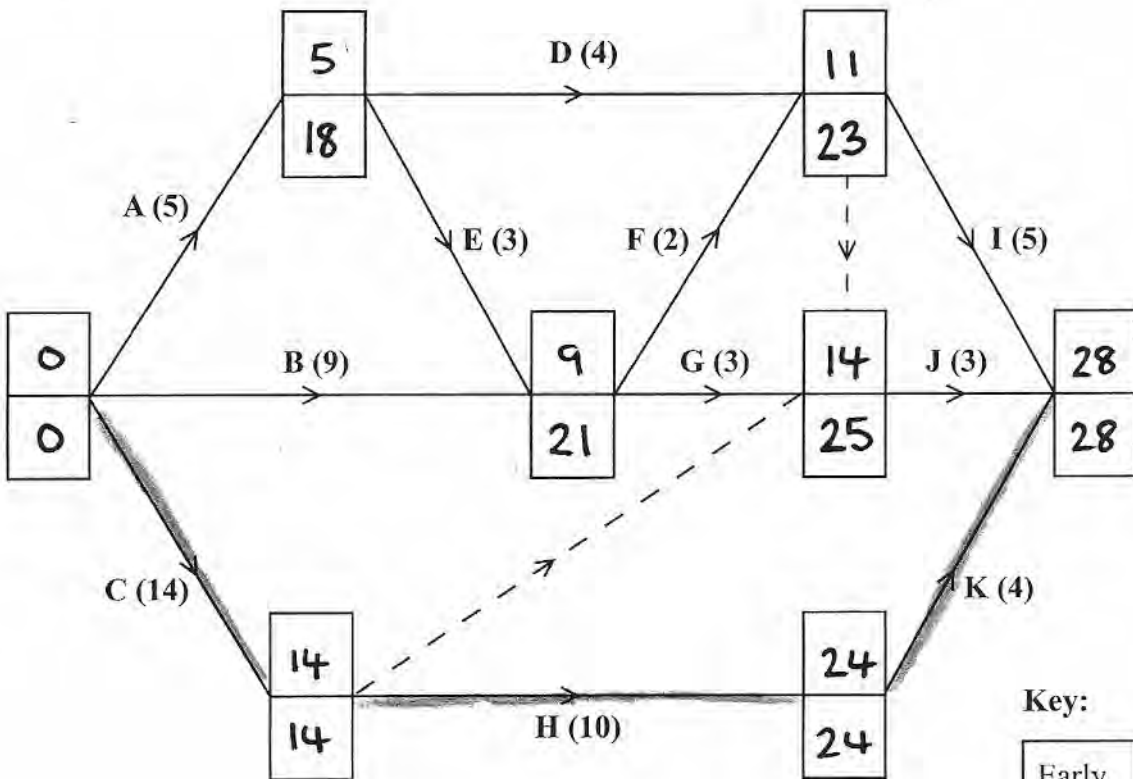
6. (a)

Activity	Immediately preceding activities
A	-
B	-
C	-
D	A

Activity	Immediately preceding activities
E	A
F	B, E
G	B, E
H	C

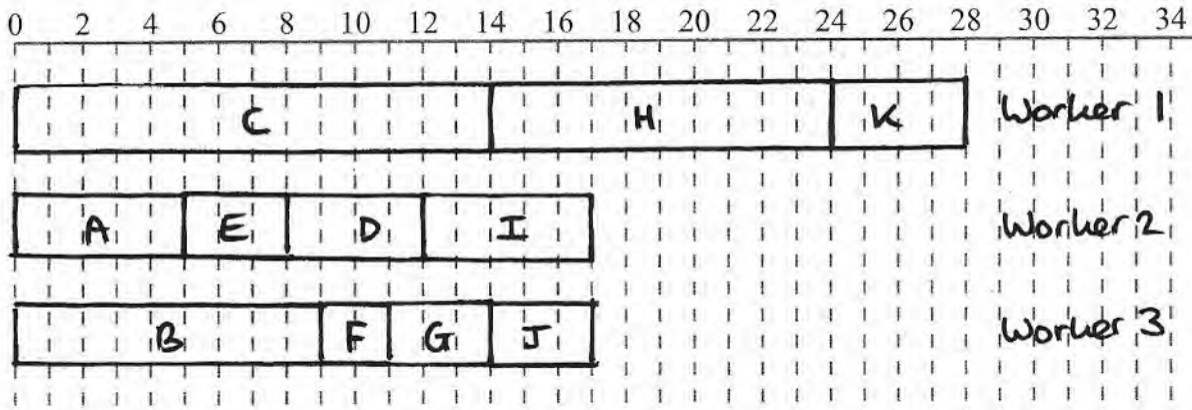
Activity	Immediately preceding activities
I	D, F
J	D, F, G, C
K	H

(b)



c) float on E = $21 - 3 - 5 = 13$

d) lower bound = $\frac{62}{28} = 2.2 \rightarrow 3$ Workers.



A company is going to hire out two types of car, standard and luxury.

Let x be the number of standard cars it should buy.

Let y be the number of luxury cars it should buy.

Figure 6 shows three constraints, other than $x, y \geq 0$

Two of these are $x \geq 20$ and $y \geq 8$

(a) Write, as an inequality, the third constraint shown in Figure 6.

(1)

The company decides that at least $\frac{1}{6}$ of the cars must be luxury cars.

(b) Express this information as an inequality and show that it simplifies to

$$5y \geq x$$

You must make the steps in your working clear.

(2)

Each time the cars are hired they need to be prepared. It takes 5 hours to prepare a standard car and it takes 6 hours to prepare a luxury car. There are 300 hours available each week to prepare the cars.

(c) Express this information as an inequality.

(1)

(d) Add two lines and shading to Diagram 1 in the answer book to illustrate the constraints found in parts (b) and (c).

(2)

(e) Hence determine the feasible region and label it R.

(1)

The company expects to make £80 profit per week on each car.

It therefore wishes to maximise $P = 80x + 80y$, where P is the profit per week.

(f) Use the objective line (ruler) method to find the optimal vertex, V, of the feasible region. You must clearly draw and label your objective line and the vertex V.

(3)

(g) Given that P is the expected profit, in pounds, per week, find the number of each type of car that the company should buy and the maximum expected profit.

(3)

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

7.

$x = 20$

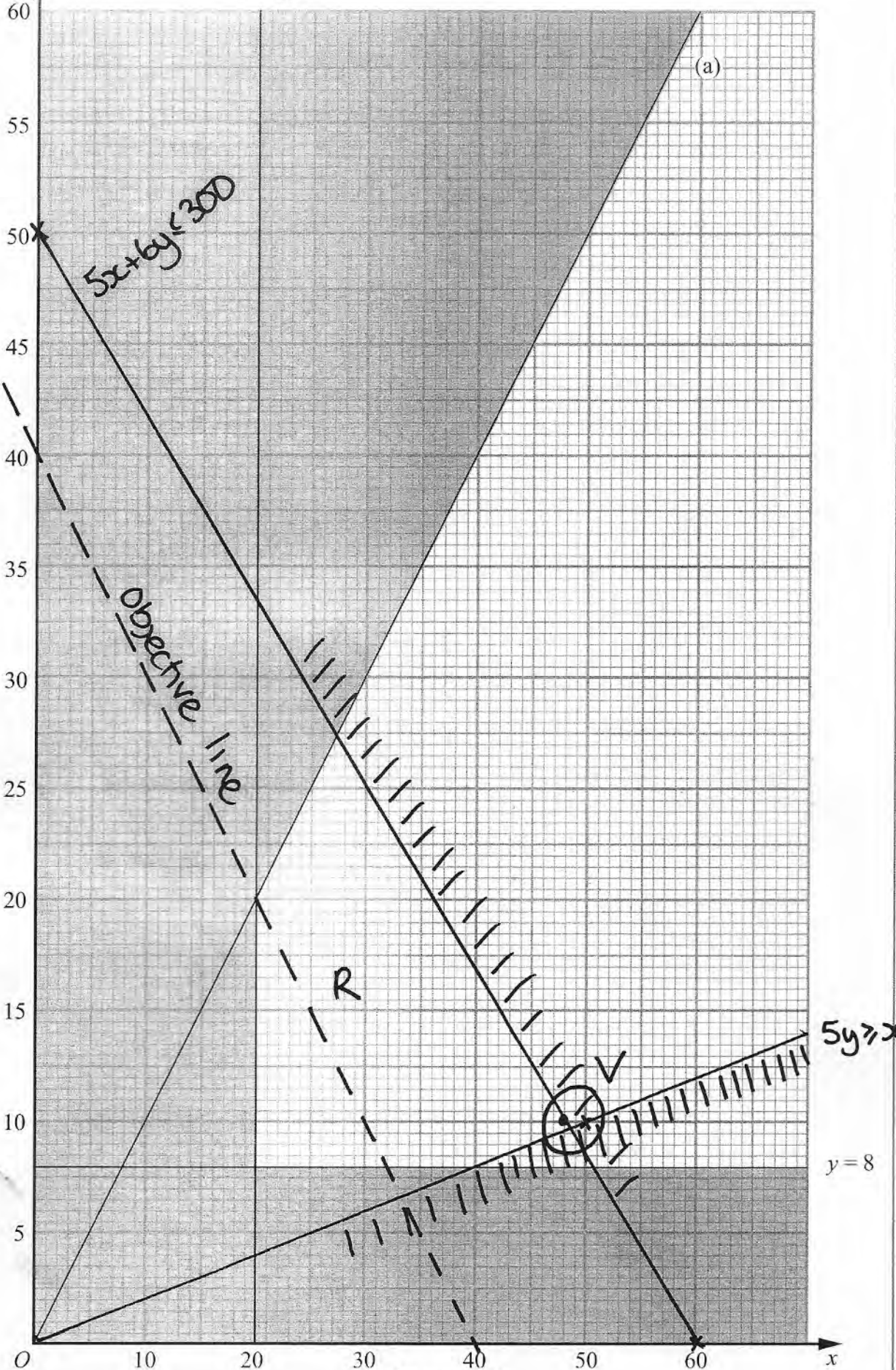


Diagram 1

a) $y \leq x$

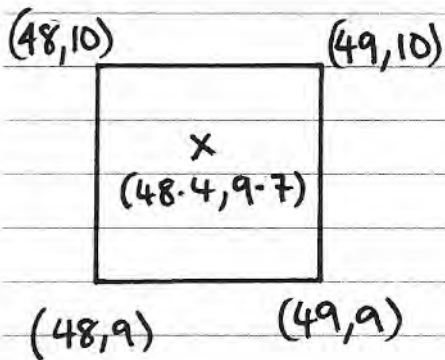
b) $y \geq \frac{1}{6}(x+y) \Rightarrow 6y \geq x+y \Rightarrow 5y \geq x$
 (x6) (y)

c) $5x + 6y \leq 300$

g) Point testing (probably unnecessary!)

$x = 5y \Rightarrow 25y + 6y = 300 \Rightarrow y = 9.7$

$x = 48.4$



Point	$5y \geq x$	$5x + 6y \leq 300$	Profit
(48, 9)	$45 < 48$ X		
(48, 10)	$50 \geq 48$ ✓	$240 + 60 \leq 300$ ✓	* £4640
(49, 9)	$45 < 49$ X		
(49, 10)	$50 \geq 49$ ✓	$245 + 60 \leq 300$ X	

$x = 48, y = 10$ Profit = £4640