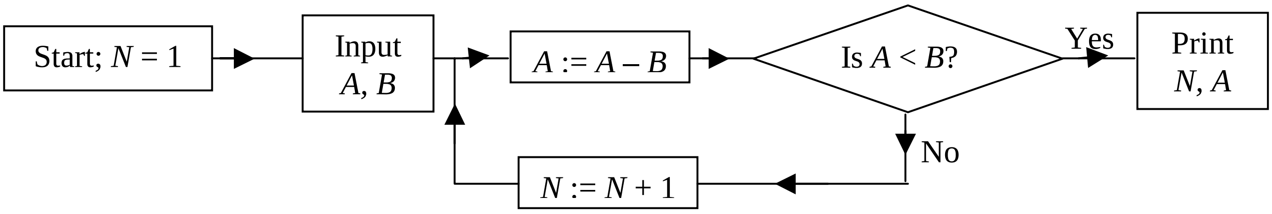
1. An algorithm is described by the following flow chart :



(i) Carry out the algorithm when the input values are A = 87 and B = 13, listing the values of A and N at each stage. [4]

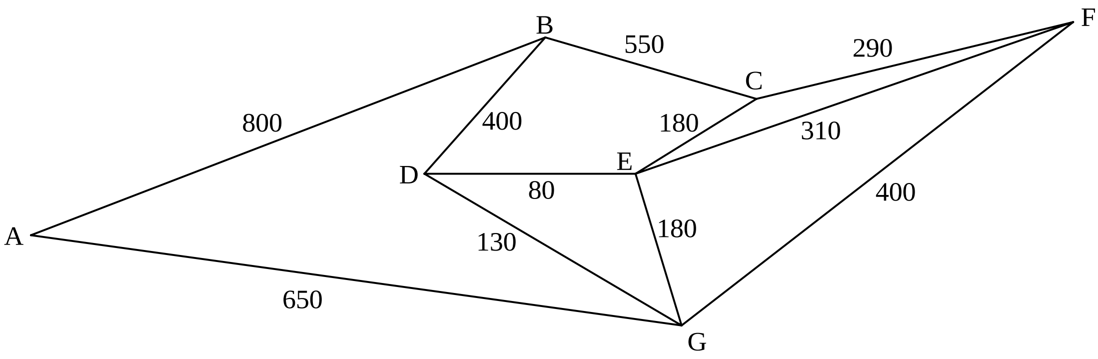
(ii) State the purpose of this algorithm. [2]

2. Six computer terminals need to be connected on a network. The cost of wiring between each pair is shown on the table

	A	B	C	D	E	F
A	-	56	48	32	89	65
B	56	-	37	54	38	49
C	48	37	-	29	46	45
D	32	54	29	-	39	51
E	89	38	46	39	-	54
F	65	49	45	51	54	-

Use Prim's algorithm, starting at A, to find the cheapest way of connecting them all together. Indicate the order in which links are chosen, and sketch the final layout. [7]

3. The diagram shows a set of roads; distances are given in metres. After any fall of snow, the council sends out a gritting lorry, which has to go at least once down the middle of each road.



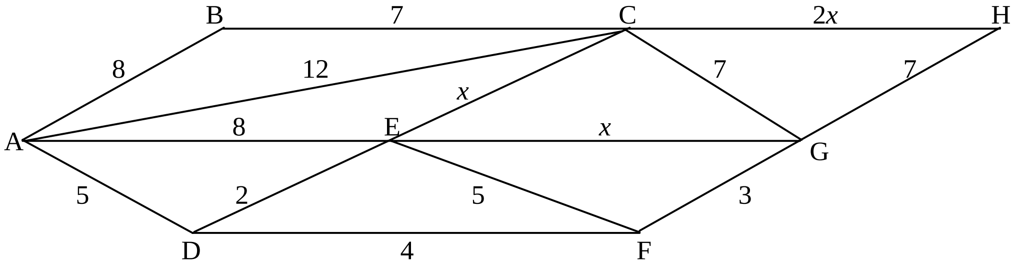
(i) Write down the valency of each node in the network. [2]

(ii) Find the minimum distance the lorry must travel, starting and finishing at G. [5]

(iii) The lorry could travel directly from B to F, along a disused alley of length 410 m. Explain why this would shorten the total distance that it would travel. [2]

4. In the diagram for Question 3, the council surveyor, who is based at G, has to visit all six sites A, B, C, D, E and F and then return to G.
   (i) Find an upper bound for the total distance that he must travel. Briefly explain why your method gives an upper bound. [5]
   (ii) By deleting E, find a lower bound for the distance that he must travel. [3]
   (iii) Show that it is possible to find a lower bound that is greater than 2.5 km. [2]

5. The diagram shows a network of towns, with the travel time between each pair (in hours) shown on each connecting arc, where x is an integer.



   (i) Use Dijkstra's algorithm to show that the quickest route from A to C takes either 12 hours or $(7 + x)$ hours, depending on the value of x. [4]
   (ii) Find the four possible expressions for the shortest time from A to H. [6]
   (iii) Given that ADEGH is the quickest route, find the value of x. [3]

6. (i) Describe the purpose of slack variables in the Simplex Algorithm. [2]

Three types of tree are to be used in a garden. They each take up different amounts of ground and provide different amounts of shade, as shown in the table together with their costs :

	Ash	Beech	Cedar
Ground (m^2)	2	3	4
Shade (m^2)	3	2	4
Cost (£)	15	12	18

Letting x be the number of Ash trees, y the number of Beech and z the number of Cedars,
   (ii) write down two inequalities for x, y, and z, given that the garden is 60 m^2 in area, and that there is a budget of £300. [2]

It is required to maximise the total amount of shade.
   (iii) Write down a Simplex tableau to model the situation as a linear programming problem. [3]
   (iv) Find the maximum amount of shade available under these constraints. [8]

DECISION MATHS 1 (C) PAPER 10 : ANSWERS AND MARK SCHEME

1. (i)

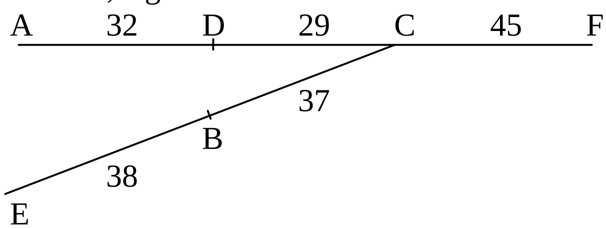
A	87	74	61	48	35	22	9
N	1	2	3	4	5	6	

M1 M1

Print 6, 9 — A1 A1

(ii) The algorithm divides A by B (by successive subtraction), giving the integer answer, together with the remainder — B2 — 6

2. Order of adding arcs is A, D, C, B, E and F, — M1 M1
with arcs AD, DC, CB, BE, CF — A1 A1
Total length = 181 — A1
Network sketched, e.g. :



M1 A1 — 7

3. (i) A 2, B 3, C 3, D 3, E 4, F 3, G 4 — M1 A1
(ii) Pairing of odd nodes: BC + DF = 550 + 390 = 940, — B1
BD + CF = 400 + 290 = 690, BF + CD = 840 + 260 = 1100 — B1 B1
Shortest to repeat BD and CF, giving total distance 3970 + 690 = 4660 — M1 A1
(iii) The arc BF would make B and F into even nodes, so now only CD needs to be repeated. The new total is 3970 + 260 + 410 = 4640 — M1 A1 — 9

4. (i) By any suitable method, M.S.T. consists of arcs DE, DG, EC, CF, DB, AG; length = 1730 m — M1 A1
U. bound = 2 x 1730 = 3460 m — M1 A1
This is an upper bound because traversing the M.S.T. twice from G certainly visits every town and returns to G, so it is a solution — B1
(ii) M.S.T. without E has length 1870 — B1
Add shortest arcs to rejoin E, giving 1870 + 80 + 180 = 2130 m — M1 A1
(iii) Deleting and rejoining A, get 1730 + 800 = 2530 m — M1 A1 — 10

5. (i) Shortest routes to C are AC, 12, or ADEC, $7 + x$ — M1 M1 A1 A1
(ii) Shortest routes to H are: via C : $12 + 2x$, or 26, or $7 + 3x$ or $21 + x$; — M1 A1
via G : $14 + x$ or 19, so 4 possible shortest times are 19, $14 + x$, $12 + 2x$ and $7 + 3x$ — M1 A1 M1 A1
(iii) ADEGH takes $14 + x$; if this is less than 19, then $x < 5$;
if $14 + x$ is less than $7 + 3x$, then $7 < 2x$ i.e. $3.5 < x$ — M1
Therefore $3.5 < x < 5$, i.e. $x = 4$ (this also gives $14 + x < 12 + 2x$) — M1 A1 — 13

6. (i) Slack variables enable inequalities to be written as equations B2

(ii) $2x + 3y + 4z \le 60$, $15x + 12y + 18z \le 300$ i.e. $5x + 4y + 6z \le 100$ B1 B1

(iii) To maximise $P = 3x + 2y + 4z$:

P	*x*	*y*	*z*	*r*	*s*	
1	-3	-2	-4	0	0	0
0	2	3	4	1	0	60
0	5	4	6	0	1	100

M1 A1 A1

(iv)

P	*x*	*y*	*z*	*r*	*s*	
1	-1	1	0	1	0	60
0	0.5	0.75	1	0.25	0	15
0	2	-0.5	0	-1.5	1	10

M1 A1

P	*x*	*y*	*z*	*r*	*s*	
1	0	0.75	0	0.25	0.5	65
0	0	0.875	1	0.625	-0.25	12.5
0	1	-0.25	0	-0.75	0.5	5

M1 A1

The objective row is all positive, so there is a maximum amount of shade of 65 m^2, when $x = 5$, $y = 0$ and $z = 12\frac{1}{2}$. M1 A1

Clearly, there cannot be a half tree, so the practical answer is 5 Ash trees and 12 Cedar trees, giving 63 m^2 of shade. (The saving of ½ of a cedar tree does not allow the purchase of a whole ash or beech, to increase the amount of shade.) M1 A1

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