DECISION MATHEMATICS (C) UNIT 1 TEST PAPER 8

1. (i) G_1 is a simple connected graph, with 4 vertices. Write down the minimum and maximum number of arcs that G_1 can have. [2]
   (ii) G_n is a simple connected graph, with n vertices. Write down the minimum and maximum number of arcs that G_n can have. [3]

2. (i) Give two reasons, illustrated with examples, to show how algorithms sometimes fail to give appropriate solutions. [4]
   (ii) A graph-theorist misreads Kruskal's algorithm as "choose the shortest arc, then the next shortest, and so on until $n - 1$ arcs have been selected" in order to find a minimum spanning tree. Explain why this may not work. [2]

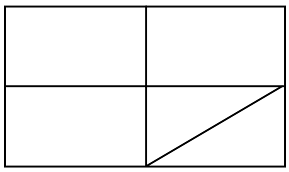
3. A car-hire firm has £300,000 to spend on its fleet of vehicles. It has a 150 m length of parking space available, and it can choose three types of vehicle :

	Rental (£)	Length (m)	Cost (£)
Small hatchback	30	3.6	7000
Family saloon	40	4.2	12000
People carrier	50	6.1	15000

If the firm buys x small hatchbacks, y family saloons and z people carriers,
   (i) write down two constraints satisfied by x, y and z. [2]
   (ii) Write down the objective function for maximising the firm's rental income. [1]
   (iii) State (without solving) the linear programming version of this problem. [3]
   (iv) State which aspect of the problem has not been considered so far in this formulation. [1]

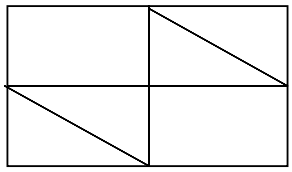
4. (i) Classify each of the following graphs as Eulerian, semi-Eulerian or neither.

(a)



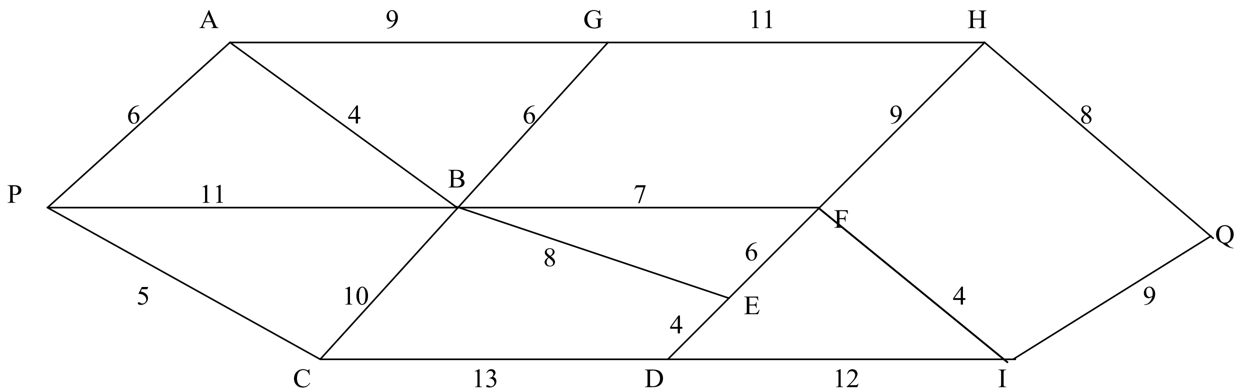
(b)

(c)



[4]
   (ii) Explain why a graph must be Eulerian in order for it to be possible to travel along every arc and return to the starting point. [3]

5. The diagram shows a weighted network.



Turn over …

5. continued …

(i) Use Dijkstra's algorithm to find the path of minimum weight from P to Q. Clearly show the order in which the nodes are labelled, and how you use the labelling to find the path of minimum weight. [10]

(ii) Give an example of a practical situation that could be modelled by this network and the solution to part (i). [1]

6. **Fig. 1**

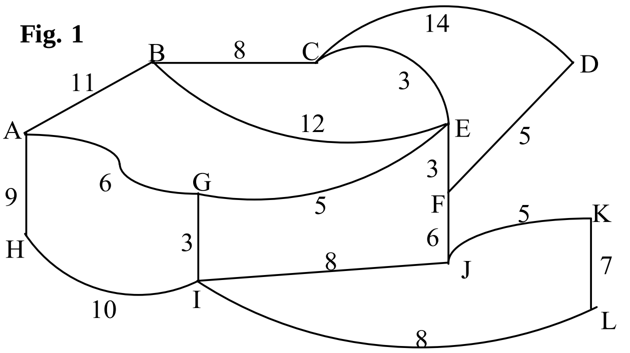
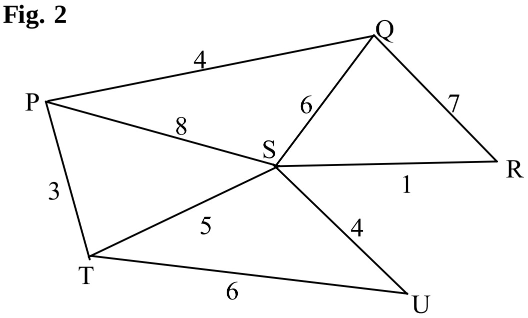


Fig. 2



(i) Use Prim's algorithm to find the minimum spanning tree for the network in Fig. 1, starting from A. [4]

(ii) Use Kruskal's algorithm to find the minimum spanning tree for the network in Fig. 2. [4]

(iii) The two networks above are joined by arcs KT, of length 4, and DP, of length 4. Show that the minimum spanning tree for the complete set is not formed by connecting the two original minimum spanning trees. [3]

7. A furniture manufacturer makes chairs and settees, producing up to 80 chairs and 48 settees per week. The items are sold in suites: Mini – two chairs,
Family – three chairs and one settee,
Grand – three chairs and two settees.

The profits are £20, £30 and £70 per suite, respectively. The total profit is to be maximised.

(i) Writing x, y and z for the number of Mini, Family and Grand suites respectively produced each week, write down equations involving slack variables to represent two inequalities. [2]

(ii) Write down the Simplex tableau for this linear programming problem. [2]

(iii) By first increasing z, use the Simplex algorithm to find the maximum profit that can be made in one week. State the profit and the number of each type of suite that should be made. [9]

DECISION MATHS 1 (C) PAPER 8 : ANSWERS AND MARK SCHEME

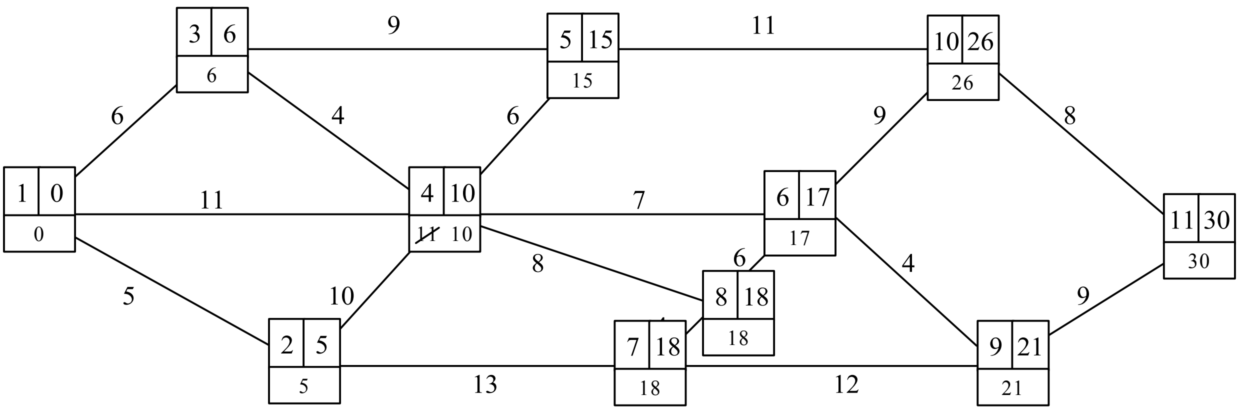
1. (i) Min = 3 max = 6 — B1 B1
   (ii) Min = $n - 1$ max = $n(n - 1)/2$ — B1 M1 A1 — 5

2. (i) They do not allow for unforeseen situations (e.g. negative discriminant in algorithm for quadratic equation); they may not finish in a reasonable time e.g. complete enumeration for TSP — B2 B2
   (ii) It may give a cycle, which cannot be a minimum connector — B2 — 6

3. (i) Cost : $7x + 12y + 15z \leq 300$; space : $3.6x + 4.2y + 6.1z \leq 150$ — B1 B1
   (ii) $P = 30x + 40y + 50z$ — B1
   (iii) Maximise $P = 30x + 40y + 50z$, subject to — B1
   $7x + 12y + 15z \leq 300$, $3.6x + 4.2y + 6.1z \leq 150$, and $x, y, z \geq 0$ — B1 B1
   (iv) x, y, z must be integers — B1 — 7

4. (i) (a) is semi-Eulerian, (b) is neither, (c) is Eulerian — M1 A1 A1 A1
   (ii) In order to travel along each arc exactly once and return to the starting point, every node must be even, so the graph must be Eulerian — M1 M1 A1 — 7

5. (i) Labelling :



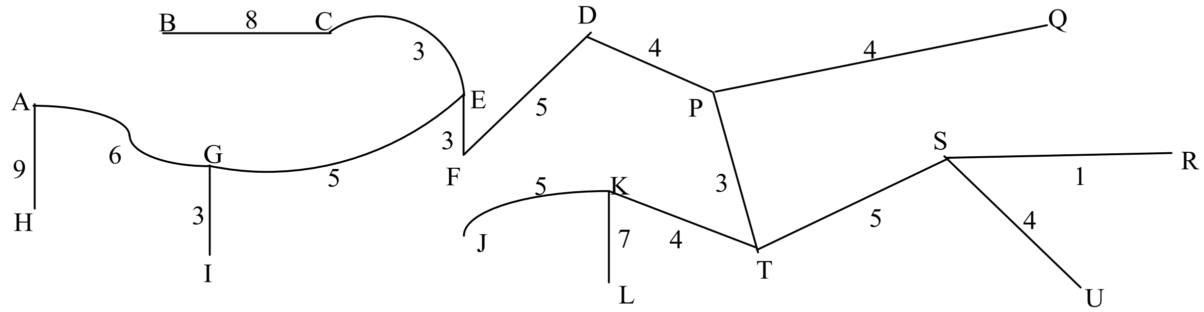
M1 A1 M1 A1 M1 A1

Minimum weight path consists of those arcs for which the start and finish labels differ by an amount equal to their weight, — M1
i.e. P A B F I Q, of weight 30 — M1 A1 A1
   (ii) Finding the quickest route from P to Q, where the weights are the times to travel between each two nodes — B1 — 11

6. (i) Starting with A, select G, I, E, {C, F}, D, J, K, L, B, H — M1 M1 A1 A1

(ii) Select arcs in order SR, PT, {PQ, SU}, ST — M1 M1 A1 A1

(iii) K is nearer T than J, so it links that way, followed by J. Get : — M1 A1



A1 11

7. (i) $2x + 3y + 3z + r = 80$, $y + 2z + s = 48$ — B1 B1

(ii) To maximise $P = 20x + 30y + 70z$:

P	x	y	z	r	s	
1	-20	-30	-70	0	0	0
0	2	3	3	1	0	80
0	0	1	2	0	1	48

(iii) M1 A1

P	x	y	z	r	s	
1	-20	5	0	0	35	1680
0	2	1.5	0	1	-1.5	8
0	0	0.5	1	0	0.5	24

M1 A1 A1

P	x	y	z	r	s	
1	0	20	0	10	20	1760
0	1	0.75	0	0.5	-0.75	4
0	0	0.5	1	0	0.5	24

M1 A1 A1

Max. profit is £1760, making 4 Mini Suites and 24 Grand suites each week — M1 A1 A1 13