DECISION MATHEMATICS (C) UNIT 1 TEST PAPER 6

1. It is required to make 10 litres of paint, by mixing yellow (costing £6 per litre), red (£5 per litre) and blue (£3 per litre). At least a third of the total must be yellow, to keep the mixture light. No more than £45 must be spent altogether, and as little red paint as possible must be used.
   (i) Assuming that x litres of yellow paint, y litres of red and z litres of blue are used, write down two inequalities satisfied by x and y (other than $x \geq 0$ and $y \geq 0$). [4]
   (ii) Write down the objective function which is to be minimised. [1]

2. (i) In the K_4 graph G, with nodes A, B, C and D, list all the paths from A to B. [2]
   (ii) In the K_5 graph H, with nodes A, B, C, D and E, find how many paths there are from A to E. [3]
   (iii) Classify each of G and H as Eulerian, semi-Eulerian or neither. [2]

3. (i) A simple connected graph has 6 vertices, all of degree d. List the possible values of d, and state the total number of edges in each case. [3]
   (ii) Another simple connected graph has 7 vertices, all of degree d. Give all possible values of d and draw an example of such a graph when $d = 4$. [4]

4. A high-speed computer link is being set up to connect six cities. The distances between the cities are given in the following table:

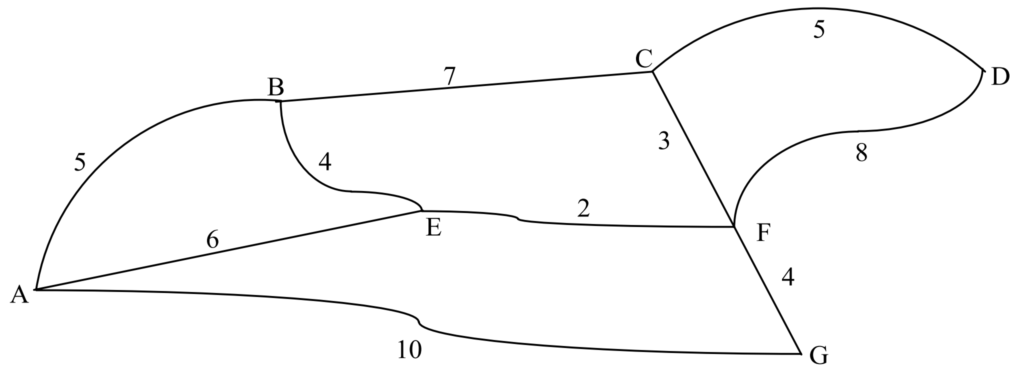
	London	Bath	Bristol	Reading	Oxford	Swindon
London	-	85	102	45	52	77
Bath	85	-	15	36	42	21
Bristol	102	15	-	51	47	35
Reading	45	36	51	-	28	34
Oxford	52	42	47	28	-	15
Swindon	77	21	35	34	15	-

   (i) Use Prim's algorithm, starting from London, to find the minimum spanning tree of these cities. Write down the length of the required computer link. [5]
   (ii) Another city, Exeter, needs to be joined to this system. The city in the existing list which is nearest to Exeter is Bristol, 76 miles away. Sketch the minimum spanning tree for the seven cities. [3]

5. (i) Give one advantage and one disadvantage of using the first-fit decreasing bin-packing algorithm for packing items in containers, in which items are sorted in order of decreasing size and then packed using the first-fit algorithm. [2]
   (ii) Illustrate your answer by packing the following items into bins of size 20, using both the first-fit and the first-fit decreasing algorithms :
   6 3 12 9 8 11 10 [4]
   (iii) If sorting n items requires an $O(n^2)$ algorithm, and packing by the first-fit algorithm is $O(n)$, write down the order of complexity of the first-fit decreasing algorithm.
   Hence estimate the time taken to pack 100 items using the first-fit decreasing algorithm, if it takes 0.03 seconds to sort 20 items. [3]

6. (i) Explain briefly why it is not possible to travel just once along each arc of a network with four odd nodes. [2]

(ii) Use a suitable algorithm to find the minimum distance around the network shown below, travelling at least once along each arc, starting and finishing at A. [5]



(iii) Write down a possible route of minimum length. [1]

(iv) If A and C are now permanently connected by an extra arc of length 15, write down the new length of the shortest closed path that traverses every arc. [2]

7. (i) Give two reasons why the Simplex method might be a better method of solving a linear programming problem than the graphical method. [2]

(ii) It is required to maximise the function $P = 3x + y + 2z$,
subject to the constraints
$x + 2y + 3z \leq 10$, $2x + 3y + z \leq 8$ and $3x + 4y + 2z \leq 15$,
together with non-negativity constraints.
Use the Simplex algorithm to find the maximum value of P, together with the values of x, y and z at which the maximum occurs. [12]

DECISION MATHS 1 (C) PAPER 6 : ANSWERS AND MARK SCHEME

1. (i) Using $x + y + z = 10$, cost is $6x + 5y + 3z = 6x + 5y + 3(10 - x - y)$ M1 A1
$= 3x + 2y + 30 \le 45$ (given), so we get $3x + 2y \le 15$; also $x \ge 3^1/_3$ A1 B1
(ii) $P = y$ must be minimised B1 5

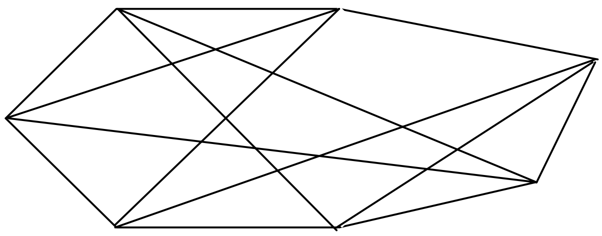
2. (i) AB, ACB, ADB, ACDB, ADCB B2
(ii) 1 direct path AE; 3 with one intermediate node e.g. ABE; M1
6 with two intermediate nodes e.g. ABCE; 6 with three intermediate A1
nodes e.g. ABCDE, total = 16 A1
(iii) G is neither (4 odd nodes); H is Eulerian (5 even nodes) B1 B1 7

3. (i)

d	2	3	4	5
e	6	9	12	15

B1 B1 B1

(ii) $d = 2, 3, 4, 5, 6$ B2
$d = 4$:



B2 7

4. (i)

	1	5	6	2	3	4
	London	Bath	Bristol	Reading	Oxford	Swindon
London	-	~~85~~	~~102~~	~~45~~	~~52~~	~~77~~
Bath	~~85~~	~~-~~	~~15~~	~~36~~	~~42~~	4 21
Bristol	102	5 15	-	51	47	35
Reading	1 45	~~36~~	~~51~~	~~-~~	~~28~~	~~34~~
Oxford	~~52~~	~~42~~	~~47~~	2 28	~~-~~	~~15~~
Swindon	~~77~~	~~21~~	~~35~~	~~34~~	3 15	-

M1 M1 A1 A1
Length = 45 + 28 + 15 + 21 + 15 = 124 A1
(ii)
Ex 76 Br 15 Ba 21 Sw 15 Ox 28 Re 45 Lo M1 A1 A1 8

5. (i) Advantage – prior sorting sometimes enables items to fit into a smaller number of bins B1
Disadvantage – items need to be sorted first B1
(ii) Packing without sorting i.e. first-fit: 4 bins
Bin 1 : 6 + 3 + 9; Bin 2 : 12 + 8; Bin 3 : 11; Bin 4 : 10 M1 A1
Packing with sorting i.e. first-fit decreasing:
Sort: 12 11 10 9 8 6 3
Bin 1 : 12 + 8; Bin 2 : 11 + 9; Bin 3 : 10 + 6 + 3 : Only 3 bins M1 A1
(iii) $O(n^2)$; time = $0.03 \times (100/20)^2 = 0.75$ s B1 M1 A1 9

6. (i) An odd node must be a starting OR finishing point of a tour along every arc. It is not possible to have four starting or finishing points B1 B1

(ii) Odd nodes A B C E; possible pairings: M1

AB + CE = 5 + 3 + 2 =10 AC + BE = 6 + 2 + 3 + 4 = 15 A1

AE + BC = 6 + 7 = 13 so repeat AB and CE (via F) A1

Minimum distance = 5 + 6 + 4 + 7 + 3 + 2 + 5 + 8 + 4 + 10 + 10 = 64 M1 A1

(iii) e.g. A G F D C F E F C B E A B A B1

(iv) Now only two odd nodes, so no choice – must do BE twice M1

Therefore length = 54 + 15 + 4 = 73 A1 10

7. (i) Graphical method can only be used for two variables – Simplex can have any number of variables. Graphical method is imprecise, if vertex co-ordinates are read off graph – Simplex is precise B2

(ii)

P	x	y	z	r	s	t	
1	-3	-1	-2	0	0	0	0
0	1	2	3	1	0	0	10
0	2	3	1	0	1	0	8
0	3	4	2	0	0	1	15

M1 A1 A1

P	x	y	z	r	s	t	
1	0	3.5	-0.5	0	1.5	0	12
0	0	0.5	2.5	1	- 0.5	0	6
0	1	1.5	0.5	0	0.5	0	4
0	0	-0.5	0.5	0	- 1.5	1	3

M1 A1 A1

P	x	y	z	r	s	t	
1	0	3.6	0	0.2	1 .4	0	13.2
0	0	0.2	1	0.4	-0.2	0	2.4
0	1	1.4	0	-0.2	0.6	0	2.8
0	0	-0.6	0	-0.2	-1.4	1	1.8

M1 A1 A1

Therefore the maximum value of P is 13.2, when $x = 2.8$, $y = 0$ and $z = 2.4$ A1 A1 A1

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